Version 1.0: 0608

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2008 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1 **(a)**	$r = \dfrac{1.3781}{\sqrt{7.0036 \times 0.8464}} =$	M1		Used
	0.56 to 0.57	A1	2	AWFW (0.56602)
(b)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1		Both
	SL α = 0.01 (1%) CV r = **0.515 to 0.516**	B1		AWFW (0.5155)
	Calculated r > Tabulated r	M1		Comparison
	Evidence, at 1% level, of a positive correlation between x and y	A1√	4	ft on r and CV
	Special Case for part (b)			
	CV $t_{n-2}(0.99)$ 2.552	(B1)		
	$r\sqrt{\dfrac{n-2}{1-r^2}} = 2.913$	(B1)		
(c)	(Strong) evidence of a positive correlation between best performances of junior athletes in the long jump and in the high jump	B1√	1	ft on **(b)**; or equivalent
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2 **(a)**	$\hat{p} = \frac{132}{200} = \mathbf{0.66}$	B1		CAO; or equivalent
	$98\% \Rightarrow z = \mathbf{2.32 \text{ to } 2.33}$	B1		AWFW (2.3263)
	CI for p: $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		Variance term
		M1		CI expression used
	ie $0.66 \pm 2.3263 \times \sqrt{\frac{0.66 \times 0.34}{200}}$	A1√		ft on $\hat{p}$ and z
	ie **0.66 ± 0.08** or **(0.58, 0.74)**	A1	6	AWRT; or equivalent
(b)	Value of 0.6 (60%) is **within** CI	B1√		ft on **(a)**
	Reason to **doubt** claim of more than 60%	B1√	2	dependent on previous B1 ft on **(a)**; or equivalent
	Total		**8**	
3	$H_0 : \mu_X = \mu_Y$ $H_1 : \mu_X \neq \mu_Y$	B1		Both
	SL α = 0.01(1%) CV $z = (\pm)$ **2.57 to 2.58**	B1		AWFW (2.5758)
	$z = \frac{\lvert 157 - 162 \rvert}{\sqrt{\frac{4.5^2}{10} + \frac{5.7^2}{15}}} =$	M1		Numerator
		M1		Denominator
	$(\pm)$ **2.44 to 2.445**	A1		AWFW (2.4424)
	No evidence, at 1% level, to reject hypothesis that $\mu_X = \mu_Y$	A1√	6	ft on z, CV and signs; or equivalent
	Total		**6**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4 **(a)**	B(0.25) — E(0.30) 0.0750; M(0.50) 0.1250; W(0.20) 0.0500	B1		B, S & D with 3 probabilities
	S(0.60) — E(0.40) 0.0240; M(0.45) 0.2700; W(0.15) 0.0900	B2	3	$3 \times$ (E, M & W) each with 3 probabilities
	D(0.15) — E(0.55) 0.0825; M(0.35) 0.0525; W(0.10) 0.0150	(B1)		$\geq 1 \times$ (E, M & W) (each) with 3 probabilities
(b)(i)	P(E) = $(0.25 \times 0.3) + (0.6 \times 0.4) + (0.15 \times 0.55)$ $= 0.075 + 0.24 + 0.0825 =$	M1		≥ 1 term correct
	0.397 to 0.398 or 159/400	A1	2	AWFW/CAO (0.3975)
(ii)	$\text{P(D} \mid \text{E)} = \dfrac{0.0825}{\text{(b)(i)}} =$	M1		Or equivalent
	0.207 to 0.208 or 11/53	A1	2	AWFW/CAO (0.2075)
(c)	$X \sim \text{B}(10,\ \text{(b)(ii)})$	M1		Used
	$\text{P}(X = 4) = \binom{10}{4}(0.2075)^4(0.7925)^6 =$	A1√		ft on **(b)(ii)**
	0.0955 to 0.0975	A1	3	AWFW (0.09645)
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5 **(a)**	$\hat{\lambda}_A = \frac{3312}{184} = \mathbf{18}$ $\hat{\lambda}_B = \frac{2760}{184} = \mathbf{15}$	B1		CAO both
	95% $\Rightarrow$ z = **1.96**	B1		CAO
	CI for $(\lambda_A - \lambda_B)$:	M1		Variance term
	$(\hat{\lambda}_A - \hat{\lambda}_B) \pm z\sqrt{\frac{\hat{\lambda}_A}{n_A} + \frac{\hat{\lambda}_B}{n_B}}$	M1		CI expression used
	ie $(18-15) \pm 1.96 \times \sqrt{\frac{18}{184} + \frac{15}{184}}$	A1√		ft on $\hat{\lambda}_A$, $\hat{\lambda}_B$ and z
	ie **3 ± 0.83** or **(2.17, 3.83)**	A1	6	AWRT
(b)	Calls from A and B are independent	B1	1	Or equivalent
(a)	*Alternative Solution*			
	$(3312 - 2760) \pm 1.96 \times \sqrt{3312 + 2760}$ =	(M2) (B1)		1.96
	ie 552 ± 152.73	(A1)		
	Dividing by 184	(M1)		
	ie **3 ± 0.83** or **(2.17, 3.83)**	(A1)		AWRT
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6 **(a)(i)**	$E(F) = 128 + 112 = \mathbf{240}$	B1		CAO
(ii)	$Cov(X, Y) = -0.4 \times \sqrt{50 \times 50} = \mathbf{-20}$	M1		Used; or equivalent
	$Var(F) = 50 + 50 + (2 \times -20) = \mathbf{60}$	M1 A1	4	$V(X) + V(Y) + 2Cov(X,Y)$ used CAO; **AG**
(b)(i)	$E(T) = 240 + 75 = \mathbf{315}$	B1√		ft on **(a)(i)**
	$Var(T) = 60 + 36 = \mathbf{96}$	B1	2	CAO
(ii)	$E(M) = 240 - (3 \times 75) = \mathbf{15}$	B1√		ft on **(a)(i)**
	$Var(M) = 60 + \{(-3^2) \times 36\}$ $= 60 + 324 = \mathbf{384}$	M1 A1	3	$V(F) + 3^2V(S)$ used CAO
(c)(i)	$P(T > 300) = P\left(Z > \frac{300 - 315}{\sqrt{96}}\right)$	M1		Standardising 300 or 300.5 using **(b)(i)**
	$= P(Z > -1.53) = P(Z < 1.53)$	m1		Area change
	$=$ **0.936 to 0.938**	A1	3	AWFW
(ii)	$P\left(S > \frac{X + Y}{3}\right) =$	M1		Used; or equivalent
	$P(3S > X + Y) = P(3S > F) =$	M1		Attempt to change to M
	$P(F - 3S < 0) = P(M < 0)$	A1		Or equivalent
	$= P\left(Z < \frac{0 - 15}{\sqrt{384}}\right)$	M1		Standardising 0 using **(b)(ii)**
	$= P(Z < -0.765) = 1 - P(Z < 0.765)$	m1		Area change
	$=$ **0.22(0) to 0.225**	A1	6	
	Total		**18**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\mathrm{E}\big(X(X-1)\big)=\sum_{x=0}^{\infty}x(x-1)\times\frac{\mathrm{e}^{-\lambda}\lambda^x}{x!}$ =	M1		$\sum x(x-1)\times\mathrm{P}(X=x)$ used Ignore limits until A1
	$\sum_{x=2}^{\infty}\frac{\mathrm{e}^{-\lambda}\lambda^x}{(x-2)!}$ =	M1		$\frac{x(x-1)}{x!}=\frac{1}{(x-2)!}$ used
	$\lambda^2\mathrm{e}^{-\lambda}\sum_{x=2}^{\infty}\frac{\lambda^{x-2}}{(x-2)!}$ =	M1		Factor of $\lambda^2\mathrm{e}^{-\lambda}$ used
	$\left(\lambda^2\mathrm{e}^{-\lambda}\right)\times\left(\mathrm{e}^{\lambda}\right)$ = $\boldsymbol{\lambda^2}$	A1	4	Fully correct derivation; **AG**
(ii)	Var(X) = E(X(X – 1)) + E(X) – $(\mathrm{E}(X))^2$	M1		Used
	= $\lambda^2+\lambda-\lambda^2$ = $\boldsymbol{\lambda}$	A1	2	Fully correct derivation; **AG**
(b)(i)	E(D) = 5 – 2 = **3** Var(D) = 5 + 2 = **7**	B1 B1	2	CAO CAO
(ii)	E(F) = (2 × 5) + 10 = **20** Var(F) = $2^2\times5$ = **20**	B1 M1 A1	3	CAO $2^2\mathrm{V}(X_1)$ + 0 CAO
(iii)	D: Mean ≠ Variance	B1		Negative values possible
	F: Values < 10 impossible Odd values impossible	B1	2	$2X_1 = X_1+X_1$ is not sum of independent Po variables
(c)	B ~ Po(5) C ~ Po(2)			
	T = 24 × (5 + 2) ~ Po(**168**)	B1		CAO
	T ~ approx N(168, 168)	M1		Normal with $\mu=\sigma^2$
	$\mathrm{P}(T_{\mathrm{Po}}>175)\approx\mathrm{P}(T_{\mathrm{N}}>175.5)$	B1		175.5
	= $\mathrm{P}\left(Z>\frac{175.5-168}{\sqrt{168}}\right)$ = $\mathrm{P}(Z>0.58)$ =	M1		Standardising 174.5, 175 or 175.5 with $\mu=\sigma^2$
	$1-\mathrm{P}(Z<0.58)$ = **0.28(0) to 0.283**	m1 A1	6	Area change AWFW
	Total		**19**	
	TOTAL		**75**	