

General Certificate of Education  
June 2008  
Advanced Level Examination



**MATHEMATICS**  
**Unit Mechanics 2B**

**MM2B**

Friday 6 June 2008 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 A particle moves in a straight line and at time  $t$  seconds has velocity  $v \text{ m s}^{-1}$ , where

$$v = 6t^2 + 4t - 7, \quad t \geq 0$$

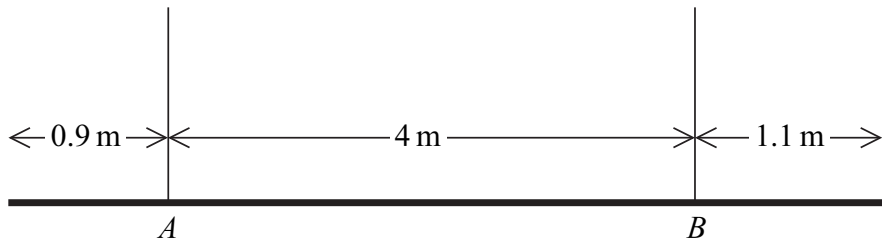
- (a) Find an expression for the acceleration of the particle at time  $t$ . (2 marks)
- (b) The mass of the particle is 3 kg.

Find the resultant force on the particle when  $t = 4$ . (2 marks)

- (c) When  $t = 0$ , the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time  $t$ . (4 marks)

- 2 A uniform plank, of length 6 metres, has mass 40 kg. The plank is held in equilibrium in a horizontal position by two vertical ropes attached to the plank at  $A$  and  $B$ , as shown in the diagram.



- (a) Draw a diagram to show the forces acting on the plank. (1 mark)
- (b) Show that the tension in the rope attached to the plank at  $B$  is  $21g \text{ N}$ . (3 marks)
- (c) Find the tension in the rope that is attached to the plank at  $A$ . (2 marks)
- (d) State where in your solution you have used the fact that the plank is uniform. (1 mark)

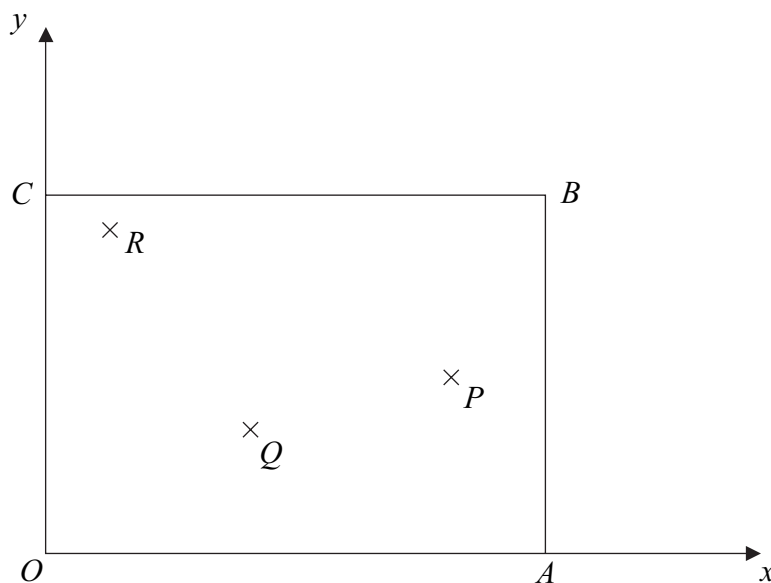
- 3 Three particles are attached to a light rectangular lamina  $OABC$ , which is fixed in a horizontal plane.

Take  $OA$  and  $OC$  as the  $x$ - and  $y$ -axes, as shown.

Particle  $P$  has mass 1 kg and is attached at the point  $(25, 10)$ .

Particle  $Q$  has mass 4 kg and is attached at the point  $(12, 7)$ .

Particle  $R$  has mass 5 kg and is attached at the point  $(4, 18)$ .



Find the coordinates of the centre of mass of the three particles.

(4 marks)

- 4 A van, of mass 1500 kg, has a maximum speed of  $50 \text{ m s}^{-1}$  on a straight horizontal road. When the van travels at a speed of  $v \text{ m s}^{-1}$ , it experiences a resistance force of magnitude  $40v$  newtons.

(a) Show that the maximum power of the van is 100 000 watts. (2 marks)

(b) The van is travelling along a straight horizontal road.

Find the maximum possible acceleration of the van when its speed is  $25 \text{ m s}^{-1}$ .

(3 marks)

(c) The van starts to climb a hill which is inclined at  $6^\circ$  to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill.

(6 marks)

Turn over ►

- 5 A particle moves on a horizontal plane in which the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are directed east and north respectively.

At time  $t$  seconds, the particle's position vector,  $\mathbf{r}$  metres, is given by

$$\mathbf{r} = 8\left(\cos\frac{1}{4}t\right)\mathbf{i} - 8\left(\sin\frac{1}{4}t\right)\mathbf{j}$$

- (a) Find an expression for the velocity of the particle at time  $t$ . (2 marks)
- (b) Show that the speed of the particle is a constant. (3 marks)
- (c) Prove that the particle is moving in a circle. (2 marks)
- (d) Find the angular speed of the particle. (2 marks)
- (e) Find an expression for the acceleration of the particle at time  $t$ . (2 marks)
- (f) State the magnitude of the acceleration of the particle. (1 mark)
- 6 A car, of mass  $m$ , is moving along a straight smooth horizontal road. At time  $t$ , the car has speed  $v$ . As the car moves, it experiences a resistance force of magnitude  $0.05mv$ . No other horizontal force acts on the car.

- (a) Show that

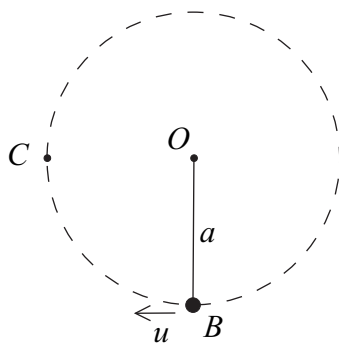
$$\frac{dv}{dt} = -0.05v \quad (1 \text{ mark})$$

- (b) When  $t = 0$ , the speed of the car is  $20 \text{ m s}^{-1}$ .

Show that  $v = 20e^{-0.05t}$ . (4 marks)

- (c) Find the time taken for the speed of the car to reduce to  $10 \text{ m s}^{-1}$ . (3 marks)

- 7 A small bead, of mass  $m$ , is suspended from a fixed point  $O$  by a light inextensible string, of length  $a$ . The bead is then set into circular motion with the string taut at  $B$ , where  $B$  is vertically below  $O$ , with a horizontal speed  $u$ .



- (a) Given that the string does not become slack, show that the least value of  $u$  required for the bead to make complete revolutions about  $O$  is  $\sqrt{5ag}$ . *(5 marks)*
- (b) In the case where  $u = \sqrt{5ag}$ , find, in terms of  $g$  and  $m$ , the tension in the string when the bead is at the point  $C$ , which is at the same horizontal level as  $O$ , as shown in the diagram. *(3 marks)*
- (c) State one modelling assumption that you have made in your solution. *(1 mark)*

**Turn over for the next question**

**Turn over ►**

- 8 (a) Hooke's law states that the tension in a stretched string of natural length  $l$  and modulus of elasticity  $\lambda$  is  $\frac{\lambda x}{l}$  when its extension is  $x$ .

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is  $e$  is  $\frac{\lambda e^2}{2l}$ . (3 marks)

- (b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point  $O$ .

- (i) Find the extension of the elastic string when the particle hangs in equilibrium directly below  $O$ . (2 marks)
- (ii) The particle is pulled down and held at the point  $P$ , which is 0.9 metres vertically below  $O$ .

Show that the elastic potential energy of the string when the particle is in this position is 11.25 J. (2 marks)

- (iii) The particle is released from rest at the point  $P$ . In the subsequent motion, the particle has speed  $v \text{ m s}^{-1}$  when it is  $x$  metres **above**  $P$ .

Show that, while the string is taut,

$$v^2 = 10.4x - 50x^2 \quad (7 \text{ marks})$$

- (iv) Find the value of  $x$  when the particle comes to rest for the first time after being released, given that the string is still taut. (2 marks)

**END OF QUESTIONS**

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