General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS
Unit Mechanics 2A

MM2A/W

ASSESSMENT and QUALIFICATIONS ALLIANCE

Friday 6 June 2008 1.30 pm to 2.45 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2A/W.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Mechanics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves in a straight line and at time t seconds has velocity $v\,\mathrm{m\,s^{-1}}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

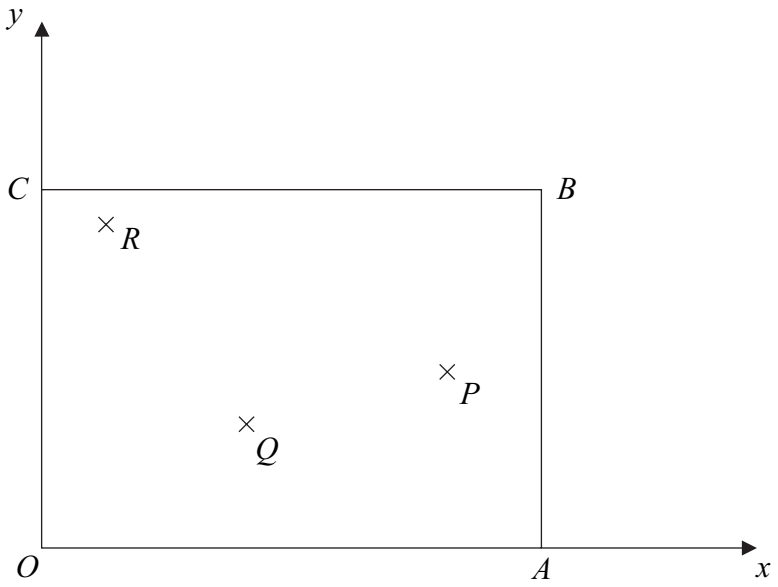
(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

2 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x- and y-axes, as shown.

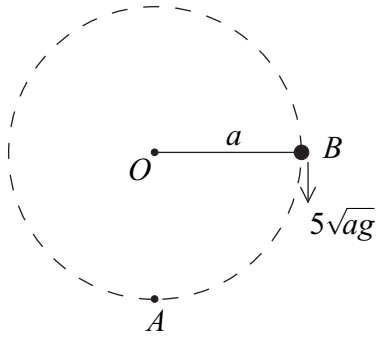
Particle P has mass 1 kg and is attached at the point $(25, 10)$.
Particle Q has mass 4 kg and is attached at the point $(12, 7)$.
Particle R has mass 5 kg and is attached at the point $(4, 18)$.



Find the coordinates of the centre of mass of the three particles. *(4 marks)*

3 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end of the string. The particle is set into vertical circular motion with radius a and centre O.

When the particle is at B, on the same horizontal level as O, the string is taut and the particle is moving vertically downwards with speed $5\sqrt{ag}$.



(a) Find, in terms of a and g, the speed of the particle at the lowest point, A, of its path. *(4 marks)*

(b) Find, in terms of m and g, the tension in the string when the particle is at A. *(3 marks)*

4 A particle moves on a horizontal plane in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the particle's position vector, **r** metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

5 A car, of mass m, is moving along a straight smooth horizontal road. At time t, the car has speed v. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

$$\frac{dv}{dt} = -0.05v \qquad \textit{(1 mark)}$$

(b) When $t = 0$, the speed of the car is $20\,\text{m s}^{-1}$.

Show that $v = 20e^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\text{m s}^{-1}$. *(3 marks)*

(d) Find, in terms of m, the work done by the force in slowing the car from $20\,\text{m s}^{-1}$ to $10\,\text{m s}^{-1}$. *(3 marks)*

6 A van, of mass 1500 kg, has a maximum speed of $50\,\text{m s}^{-1}$ on a straight horizontal road. When the van travels at a speed of $v\,\text{m s}^{-1}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the van is 100 000 watts. *(2 marks)*

(b) The van is travelling along a straight horizontal road.

Find the maximum possible acceleration of the van when its speed is $25\,\text{m s}^{-1}$. *(3 marks)*

(c) The van starts to climb a hill which is inclined at 6° to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill. *(6 marks)*

7 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\frac{\lambda x}{l}$ when its extension is x.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\frac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(ii) The particle is pulled down and held at the point P, which is 2 metres vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 245 J. *(2 marks)*

(iii) The particle is released from rest at the point P. Find the speed of the particle when it reaches O. *(4 marks)*

END OF QUESTIONS

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