

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Mechanics 4

MM04

Thursday 12 June 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM04.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

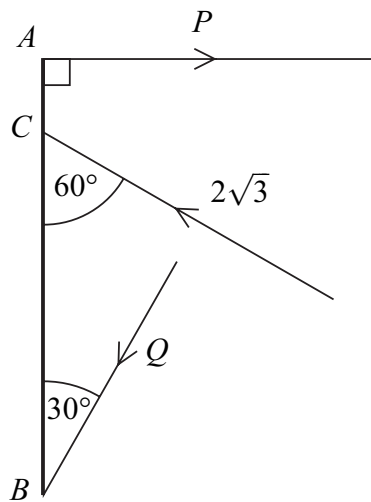
Answer **all** questions.

- 1 A light rod AB has length 5 metres and the point C on the rod is 1 metre from A . The rod is on a smooth horizontal table and is acted upon by three horizontal forces of magnitude P , Q and $2\sqrt{3}$ newtons.

The force of magnitude P acts at A , at right angles to the rod.

The force of magnitude $2\sqrt{3}$ acts at C , at an angle of 60° to the rod.

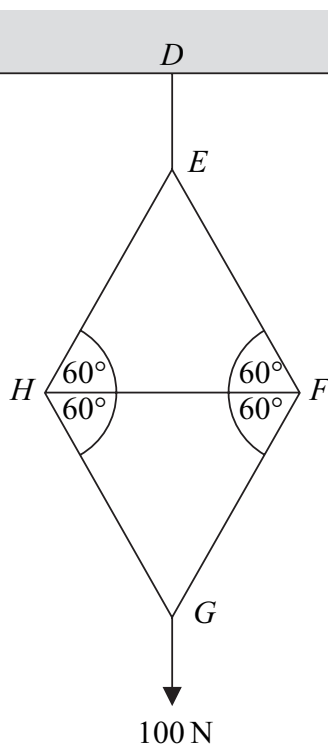
The force of magnitude Q acts at B , at an angle of 30° to the rod, as shown in the diagram.



The three forces are equivalent to a couple.

- (a) Show that $Q = 2$ and find the value of P . (5 marks)
- (b) Determine the magnitude of the couple. (3 marks)
- (c) State the sense of the couple. (1 mark)

- 2 A framework $EFGH$ consists of five identical light rods, EF , EH , FG , GH and FH , which are smoothly jointed at E , F , G and H . Each of the rods EF , EH , FG and GH makes an angle of 60° with the rod FH . The framework is suspended from a fixed point D by a string DE . The rod FH is horizontal, and G is vertically below D . A force of 100 N is applied vertically at G . The system, as shown in the diagram, is in equilibrium.



- (a) State the magnitude of the force in the string DE , giving a reason for your answer. (2 marks)
- (b) Explain why the forces in the rods EF , EH , FG and GH must be of equal magnitude. (2 marks)
- (c) Find the magnitude of the forces in each of the rods EF , EH , FG and GH . (2 marks)
- (d) Find the magnitude of the force in the rod FH . (3 marks)
- (e) State which of the five rods could be replaced by ropes, giving reasons for your answers. (2 marks)

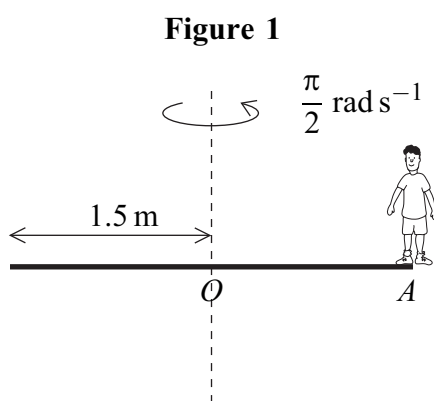
3 A light rod has its ends at the points $A(2, 3, 5)$ and $B(4, 6, -1)$. A force \mathbf{F} acts at B , where

$$\mathbf{F} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$$

- (a) Find \overrightarrow{AB} . *(1 mark)*
- (b) Find the moment of \mathbf{F} about the point A . *(3 marks)*
- (c) Show that the magnitude of this moment is $10\sqrt{5}$. *(2 marks)*
- (d) Hence, or otherwise, find the acute angle between \mathbf{F} and the rod, giving your answer to the nearest degree. *(4 marks)*

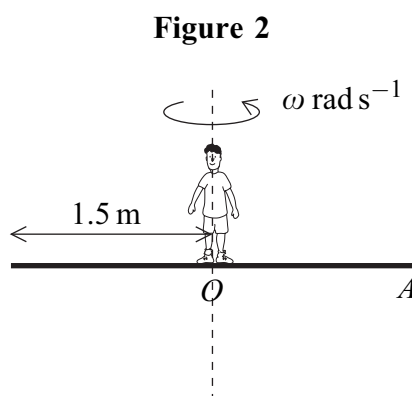
- 4 (a) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius r , about an axis through its centre and perpendicular to the plane of the disc is $\frac{1}{2}mr^2$. (5 marks)
- (b) A roundabout in a playground can be modelled as a uniform circular disc of mass 200 kg and radius 1.5 m. The roundabout can rotate freely in a horizontal plane about a vertical axis through its centre O .

The roundabout is rotating at $\frac{\pi}{2}$ radians per second, with Dominic, a child of mass 25 kg, standing at a point A on the edge, as shown in **Figure 1**.



Assume that Dominic can be modelled as a particle.

- (i) Show that the moment of inertia of the system about the vertical axis through O shown in **Figure 1** is 281.25 kg m^2 . (3 marks)
- (ii) Dominic then walks to the centre O , as shown in **Figure 2**. The angular speed of the roundabout changes from $\frac{\pi}{2}$ radians per second to ω radians per second.

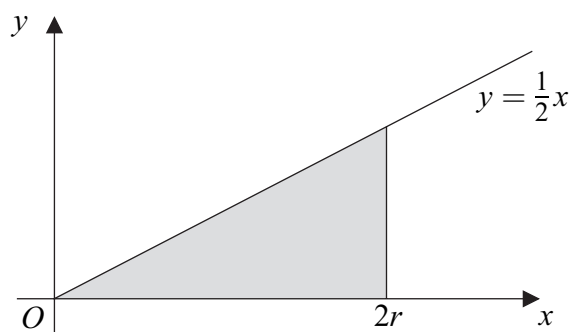


Explain why the total angular momentum of the system remains constant as Dominic walks from A to O . (1 mark)

- (iii) Find the value of ω . (4 marks)

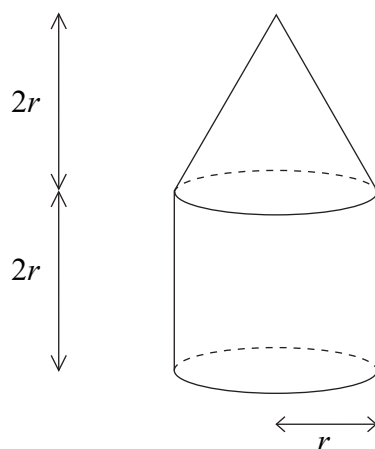
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- 5 The region bounded by the line $y = \frac{1}{2}x$, the x -axis and the line $x = 2r$ is shown in the diagram.



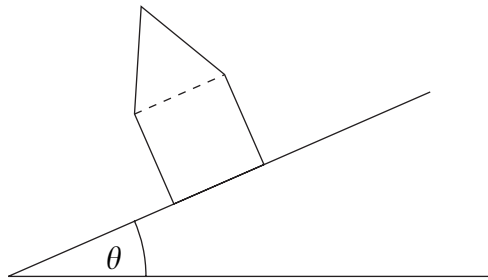
This region is rotated about the x -axis to form a uniform solid cone of height $2r$ and radius r .

- (a) Show, using integration, that the centre of mass of the cone is at a distance of $\frac{3r}{2}$ from the origin. (5 marks)
- (b) A rocket consists of two parts. The lower part of the rocket may be modelled as a uniform solid cylinder with radius r , height $2r$ and density ρ . The upper part of the rocket may be modelled as a uniform solid cone of radius r , height $2r$ and density $k\rho$, as shown in the diagram.



- (i) Show that the centre of mass of the rocket is at a distance of $\left(\frac{6+5k}{6+2k}\right)r$ from the base of the rocket. (5 marks)

- (ii) The rocket is now placed on a rough plane, which is inclined at an angle of θ to the horizontal, where $\tan \theta = \frac{2}{3}$.



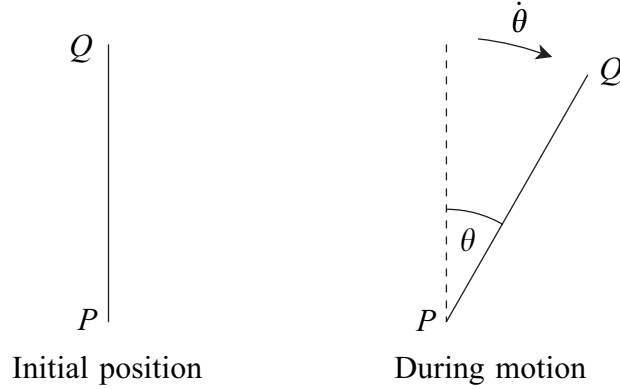
Given that the rocket does **not** slide and is just on the point of toppling, find the value of k . (5 marks)

Turn over for the next question

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- 6 A uniform rod PQ , of mass m and length $6a$, is free to rotate in a vertical plane about a fixed horizontal axis through P . Initially, the rod is at rest with Q vertically above P .

The rod is slightly disturbed from its initial position. In the subsequent motion, it makes an angle θ with the upward vertical at time t .



- (a) (i) Show that the moment of inertia of the rod about the axis through P is $12ma^2$.
(1 mark)
- (ii) Show that $\dot{\theta}^2 = \frac{g}{2a}(1 - \cos \theta)$.
(4 marks)
- (iii) Hence, or otherwise, determine an expression for $\ddot{\theta}$ in terms of a , g and θ .
(2 marks)
- (b) Find, in terms of m , g and θ , the force at P which the axis exerts on the rod:
- (i) in the direction PQ ;
(4 marks)
- (ii) perpendicular to PQ .
(3 marks)
- (c) Determine the magnitude of the force exerted by the axis on the rod when Q is vertically below P .
(3 marks)

END OF QUESTIONS