General Certificate of Education June 2007 Advanced Level Examination



MATHEMATICS Unit Further Pure 3

MFP3

Wednesday 20 June 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

P93926/Jun07/MFP3 6/6/ MFP3

Answer all questions.

1 (a) Find the value of the constant k for which kx^2e^{5x} is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 6e^{5x}$$
 (6 marks)

(b) Hence find the general solution of this differential equation. (4 marks)

2 The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = \sqrt{x^2 + y^2 + 3}$$

and

$$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with h = 0.1, to obtain an approximation to y(1.1), giving your answer to four decimal places. (3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and h = 0.1, to obtain an approximation to y(1.1), giving your answer to four decimal places. (6 marks)

3 By using an integrating factor, find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + (\tan x)y = \sec x$$

given that y = 3 when x = 0.

(8 marks)

- 4 (a) Show that $(\cos \theta + \sin \theta)^2 = 1 + \sin 2\theta$. (1 mark)
 - (b) A curve has cartesian equation

$$(x^2 + y^2)^3 = (x + y)^4$$

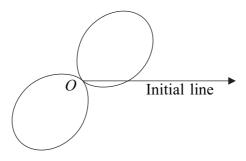
Given that $r \ge 0$, show that the polar equation of the curve is

$$r = 1 + \sin 2\theta \tag{4 marks}$$

(c) The curve with polar equation

$$r = 1 + \sin 2\theta, \quad -\pi \leqslant \theta \leqslant \pi$$

is shown in the diagram.



- (i) Find the two values of θ for which r = 0. (3 marks)
- (ii) Find the area of one of the loops. (6 marks)

Turn over for the next question

5 (a) A differential equation is given by

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x^2 + 1$$

Show that the substitution

$$u = \frac{\mathrm{d}y}{\mathrm{d}x} + x$$

transforms this differential equation into

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2xu}{x^2 - 1} \tag{4 marks}$$

(b) Find the general solution of

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2xu}{x^2 - 1}$$

giving your answer in the form u = f(x).

(5 marks)

(c) Hence find the general solution of the differential equation

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x^2 + 1$$

giving your answer in the form y = g(x).

(3 marks)

6 (a) The function f is defined by

$$f(x) = \ln(1 + e^x)$$

Use Maclaurin's theorem to show that when f(x) is expanded in ascending powers of x:

(i) the first three terms are

$$\ln 2 + \frac{1}{2}x + \frac{1}{8}x^2 \tag{6 marks}$$

- (ii) the coefficient of x^3 is zero. (3 marks)
- (b) Hence write down the first two non-zero terms in the expansion, in ascending powers of x, of $\ln\left(\frac{1+e^x}{2}\right)$. (1 mark)
- (c) Use the series expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

to write down the first three terms in the expansion, in ascending powers of x, of $\ln\left(1-\frac{x}{2}\right)$.

(d) Use your answers to parts (b) and (c) to find

$$\lim_{x \to 0} \left[\frac{\ln\left(\frac{1+e^x}{2}\right) + \ln\left(1-\frac{x}{2}\right)}{x - \sin x} \right] \tag{4 marks}$$

7 (a) Write down the value of

$$\lim_{x \to \infty} x e^{-x} \tag{1 mark}$$

- (b) Use the substitution $u = xe^{-x} + 1$ to find $\int \frac{e^{-x}(1-x)}{xe^{-x} + 1} dx$. (2 marks)
- (c) Hence evaluate $\int_{1}^{\infty} \frac{1-x}{x+e^x} dx$, showing the limiting process used. (4 marks)

END OF QUESTIONS

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