General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS
Unit Mechanics 2B

MM2B

Tuesday 16 January 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

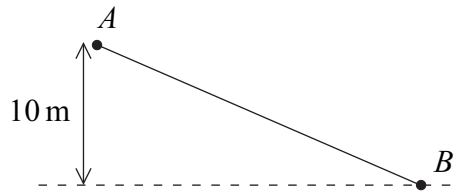
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A child, of mass 35 kg, slides down a slide in a water park. The child, starting from rest, slides from the point A to the point B, which is 10 metres vertically below the level of A, as shown in the diagram.



(a) In a simple model, all resistance forces are ignored.

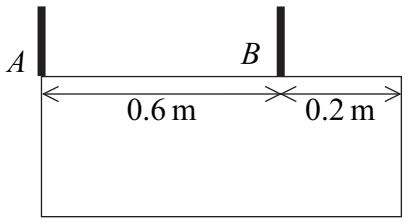
Use an energy method to find the speed of the child at B. *(3 marks)*

(b) State one resistance force that has been ignored in answering part (a). *(1 mark)*

(c) In fact, when the child slides down the slide, she reaches B with a speed of $12\,\text{m}\,\text{s}^{-1}$.

Given that the slide is 20 metres long and the sum of the resistance forces has a constant magnitude of F newtons, use an energy method to find the value of F. *(4 marks)*

2 A hotel sign consists of a uniform rectangular lamina of weight W. The sign is suspended in equilibrium in a vertical plane by two vertical light chains attached to the sign at the points A and B, as shown in the diagram. The edge containing A and B is horizontal.



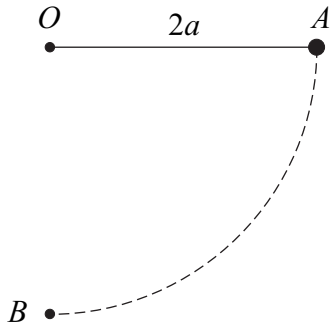
The tensions in the chains attached at A and B are T_A and T_B respectively.

(a) Draw a diagram to show the forces acting on the sign. *(1 mark)*

(b) Find T_A and T_B in terms of W. *(4 marks)*

(c) Explain how you have used the fact that the lamina is uniform in answering part (b). *(1 mark)*

3 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B, which is directly below O. The points O, A and B are as shown in the diagram.

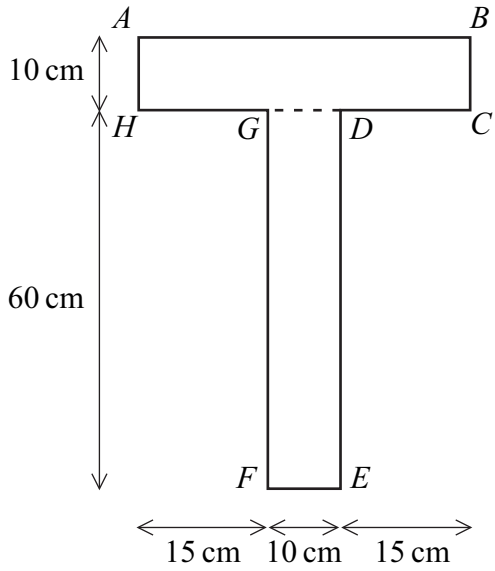


(a) Show that the speed of the particle at B is $2\sqrt{ag}$. *(3 marks)*

(b) Find the tension in the string as the particle passes through B. Give your answer in terms of m and g. *(3 marks)*

Turn over for the next question

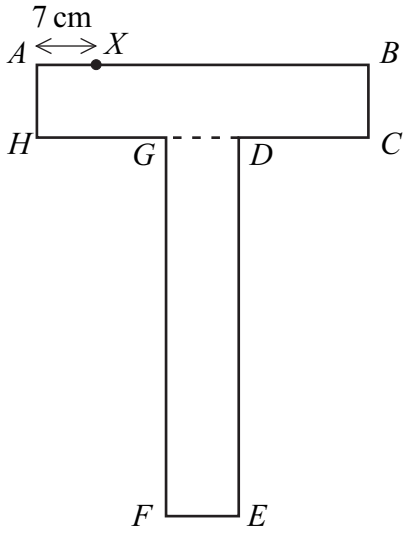
4 A uniform T-shaped lamina is formed by rigidly joining two rectangles $ABCH$ and $DEFG$, as shown in the diagram.



(a) Show that the centre of mass of the lamina is 26 cm from the edge AB. *(4 marks)*

(b) Explain why the centre of mass of the lamina is 5 cm from the edge GF. *(1 mark)*

(c) The point X is on the edge AB and is 7 cm from A, as shown in the diagram below.



The lamina is freely suspended from X and hangs in equilibrium.

Find the angle between the edge AB and the vertical, giving your answer to the nearest degree. *(4 marks)*

5 Tom is on a fairground ride.

Tom's position vector, **r** metres, at time t seconds is given by

$$\mathbf{r} = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (10 - 0.4t)\mathbf{k}$$

The perpendicular unit vectors **i** and **j** are in the horizontal plane and the unit vector **k** is directed vertically upwards.

(a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*

(ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*

(iii) Write down the first **two** values of t for which Tom is directly below his starting point. *(2 marks)*

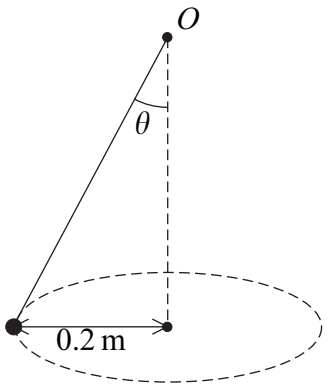
(b) Find an expression for Tom's velocity at time t. *(3 marks)*

(c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude. State the magnitude of the resultant force. *(5 marks)*

Turn over for the next question

6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O. The angle between the string and the vertical is θ, as shown in the diagram.



(a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) The radius of the circle is 0.2 metres.

Find, in terms of π, the magnitude of the acceleration of the particle. *(2 marks)*

(c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T\cos\theta = mg$. *(1 mark)*

(iii) Find the value of θ, giving your answer to the nearest degree. *(5 marks)*

7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V\,\text{m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is $60\,\text{m s}^{-1}$.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at $20\,\text{m s}^{-1}$, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v\,\text{m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

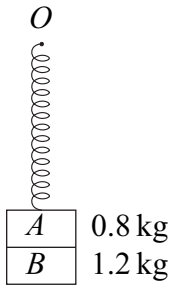
(i) Show that $\dfrac{\text{d}v}{\text{d}t} = -\dfrac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from $20\,\text{m s}^{-1}$ to $10\,\text{m s}^{-1}$. *(6 marks)*

Turn over for the next question

8 Two small blocks, A and B, of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N. One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O.

(a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. *(3 marks)*

(b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J. *(2 marks)*

(c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

(i) Show that x satisfies the equation

$$x^2 + 0.16x - 0.008 = 0$$ *(5 marks)*

(ii) Find the value of x. *(2 marks)*

END OF QUESTIONS