General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Wednesday 25 January 2006 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the **blue** AQA booklet of formulae and statistical tables You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. *(1 mark)*

(ii) Show that $f(-2) = 0$. *(1 mark)*

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

where a and b are integers. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of d. *(3 marks)*

2 A curve is defined by the parametric equations

$$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of t. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. *(4 marks)*

(c) Verify that the cartesian equation of the curve can be written as

$$(x-3)(y-1) + 8 = 0$$ *(3 marks)*

3 It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of R. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of θ at which this maximum value occurs. *(3 marks)*

4 On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £V, of the sculpture is modelled by the formula $V = A k^t$, where t is the time in years since 1 January 1900 and A and k are constants.

(a) Write down the value of A. *(1 mark)*

(b) Show that $k \approx 1.07664$. *(3 marks)*

(c) Use this model to:

(i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; *(2 marks)*

(ii) find the year in which the value of the sculpture will first exceed £800 000. *(3 marks)*

5 (a) (i) Obtain the binomial expansion of $(1-x)^{-1}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence, or otherwise, show that

$$\frac{1}{3-2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1-x)^2}$ up to and including the term in x^2. *(2 marks)*

(c) Given that $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ can be written in the form $\dfrac{A}{(3-2x)} + \dfrac{B}{(1-x)} + \dfrac{C}{(1-x)^2}$,

find the values of A, B and C. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ up to and including the term in x^2. *(3 marks)*

Turn over for the next question

6 (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where a and b are constants. *(2 marks)*

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, \mathrm{d}x = \frac{\pi}{a}$, where a is an integer. *(5 marks)*

7 The quadrilateral $ABCD$ has vertices $A\,(2, 1, 3)$, $B\,(6, 5, 3)$, $C\,(6, 1, -1)$ and $D\,(2, -3, -1)$.

The line l_1 has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that the line AB is parallel to l_1. *(1 mark)*

(iii) Verify that D lies on l_1. *(2 marks)*

(b) The line l_2 passes through $D\,(2, -3, -1)$ and $M\,(4, 1, 1)$.

(i) Find the vector equation of l_2. *(2 marks)*

(ii) Find the angle between l_2 and AC. *(3 marks)*

8 (a) Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2(x-6)^{\frac{1}{2}}$$

to find t in terms of x, given that $x = 70$ when $t = 0$. *(6 marks)*

(b) Liquid fuel is stored in a tank. At time t minutes, the depth of fuel in the tank is x cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2(x-6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. *(1 mark)*

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. *(2 marks)*

END OF QUESTIONS