Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
TOTAL	

AQA

Free-Standing Mathematics Qualification
Advanced Level
June 2014

Modelling with Calculus 6992/2

Unit 12

Friday 16 May 2014 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should normally be given to three significant figures, unless stated otherwise.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.

Advice
- You do not necessarily need to use all the space provided.

Section A

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Javelin** *on page 2 of the Data Sheet.*

1 Susie throws a javelin.

The vertical height of the tip, h **centimetres**, above the point O can be modelled by the equation

$$h = 81x - 1.5x^2$$

where x **metres** is the horizontal distance from point O.

Use this model and calculus to answer the following questions.

(a) Find the height of the tip above O when $x = 4$.

[1 mark]

(b) Find $\frac{\mathrm{d}h}{\mathrm{d}x}$.

[2 marks]

(c) Find x when $\frac{\mathrm{d}h}{\mathrm{d}x} = 0$.

[2 marks]

(d) Hence predict the maximum vertical height of the javelin above O.

[2 marks]

(e) (i) Find $\frac{\mathrm{d}^2h}{\mathrm{d}x^2}$.

[1 mark]

(ii) State how this value confirms that the answer to part **(d)** is the maximum height and not the minimum height.

[1 mark]

(f) Susie lets go of the javelin when the tip is 4 metres above the level of the horizontal ground.

Find the horizontal distance which the tip travels before hitting the ground.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

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Answer space for question 1

Section B

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Temperature** *on page 2 of the Data Sheet.*

2 The temperature, $T\,^{\circ}\mathrm{C}$, in Sarajevo, on 4 May 2012, h hours after midnight, may be modelled by the function

$$T = 0.036(940 - 288h + 33h^2 - h^3)$$

for values of h from 5 to 22 .

Use this model and calculus to answer the following questions.

(a) Find the values of h at the turning points of T.

[5 marks]

(b) Find $\dfrac{\mathrm{d}^2T}{\mathrm{d}h^2}$.

[2 marks]

(c) At what time on 4 May 2012 was the temperature in Sarajevo a maximum, and what is this maximum?

Confirm that your value is a maximum.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

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Answer space for question 2

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Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

3 (a) (i) Use the trapezium rule with four strips to find an estimate for

$$\int_6^{18} (940 - 288h + 33h^2 - h^3)\,\mathrm{d}h$$

[5 marks]

(ii) Explain how a more accurate value could be found.

[1 mark]

(b) Use integration to find the value of

$$\int_6^{18} (940 - 288h + 33h^2 - h^3)\,\mathrm{d}h$$

[5 marks]

(c) The temperature, $T\,^{\circ}\mathrm{C}$, in Sarajevo, on 4 May 2012, h hours after midnight, may be modelled by the function

$$T = 0.036(940 - 288h + 33h^2 - h^3)$$

for values of h from 5 to 22 .

Hence find the average temperature, between 06.00 and 18.00, in Sarajevo on 4 May 2012.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 3

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Answer space for question 3

Section C

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Radioactive isotopes** *on page 3 of the Data Sheet.*

4 The mass, m grams, of a radioactive isotope decreases with time, t hours. The rate of change of the mass is directly proportional to the mass at that time.

This can be expressed by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -km$$

where k is a positive constant.

(a) Find the general solution of this differential equation.

[4 marks]

(b) When $t = 0$, the mass of the radioactive isotope is 10 grams.

The half-life of the radioactive isotope is 6 hours.

(i) Show that $m = 10\mathrm{e}^{-kt}$.

[2 marks]

(ii) Find k.

Give your answer to three significant figures.

[3 marks]

(iii) Find the mass of the radioactive isotope when $t = 18$.

[2 marks]

(iv) Find the value of t when the mass is 1 gram.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

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Section D

Answer **all** questions.

Answer each question in the space provided for that question.

Use **BMX track** *on page 4 of the Data Sheet.*

5 The height, h metres, of the track above the point O may be modelled by the function

$$h = 6\sin\left(\frac{\pi}{20}x\right)$$

where x metres is the horizontal distance from O, the starting point of the dirt jumps section.

(a) Find the height of the track predicted by the model when Evan is:

(i) a horizontal distance of 20 metres from O ;

(ii) a horizontal distance of 50 metres from O .

[2 marks]

(b) Find an expression for $\frac{\mathrm{d}h}{\mathrm{d}x}$.

[2 marks]

(c) Find the maximum value of $\frac{\mathrm{d}h}{\mathrm{d}x}$.

Give your answer to three significant figures.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

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Answer space for question 5

END OF QUESTIONS