FSMQ

MATHEMATICS

6992/2 – Modelling with Calculus
Mark scheme

6992
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Version/Stage: Final v1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Question	Solution	Marks	Total	Comments
1(a)	when $x = 4$, $h = 324 - 24$ height is 300 cm or 3 m	B1	1	penalise no units
(b)	$\dfrac{\mathrm{d}h}{\mathrm{d}x} = 81 - 3x$	M1A1	2	
(c)	$\dfrac{\mathrm{d}h}{\mathrm{d}x} = 0 \Rightarrow$ $81 - 3x = 0$ $x = 27$	M1 A1	2	
(d)	When $x = 27$ $h = 81 \times 27 - 1.5 \times (27)^2$ $\quad = 1093.5$ maximum height is 1090 cm or 10.9 m	M1 A1	2	condone
(e)(i)	$\dfrac{\mathrm{d}^2 h}{\mathrm{d}x^2} = -3$	B1		Ft (from (b))
(ii)	this is negative; $\rightarrow$ a maximum height	E1	2	
(f)	tip hits ground when $h = -400$ $-400 = 81x - 1.5x^2$ $3x^2 - 162x - 800 = 0$ $x = \dfrac{162 \pm \sqrt{162^2 + 4.3.800}}{6}$ $= 58.6$ and -4.55 distance is 58.6 m	B1 M1 M1 A1	4	If h = +4 used, possible mark M1M1 Needs correct algebra SC 3 marks if $h = -4$ used
	Total		**13**	

Question	Solution	Marks	Total	Comments
2(a)	$T = 0.036(940 - 288h + 33h^2 - h^3)$			
	$\dfrac{\mathrm{d}T}{\mathrm{d}h} = 0.036(-288 + 66h - 3h^2)$	M1A1		M1 2 correct terms
	$\dfrac{\mathrm{d}T}{\mathrm{d}h} = 0 \Rightarrow$	M1		If delete 0.036 term; M1 M2 A1 possible
	$3h^2 - 66h + 288 = 0$			
	$h^2 - 22h + 96 = 0$			
	$(h - 16)(h - 6) = 0$	M1		$h = \dfrac{22 \pm \sqrt{484 - 384}}{2}$
	$h = 6$ or 16	A1	5	Both answers required
(b)	$\dfrac{\mathrm{d}^2 T}{\mathrm{d}h^2} = 0.036(66 - 6h)$	M1A1	2	M1 either term correct
				Condone deletion of 0.036 if penalised in (a)
(c)	maximum when $h = 16$, time is 4 pm or 1600	B1		
	$T = 0.036(940 - 4608 + 8448 - 4096)$	M1		
	$T = 24.6$ maximum temperature is $24.6°$C	A1		
	$\dfrac{\mathrm{d}^2 T}{\mathrm{d}h^2} = 0.036(66 - 6h)$			
	$= 0.036 \times -30$	B1		
	this is negative, hence answer is a maximum	E1	5	
	Total		**12**	

Question	Solution	Marks	Total	Comments
3(a)(i)	four strips	M1		
	$\Rightarrow$ values of h are 6, 9, 12, 15 and 18			
	when $h = 6$, $T = 184$	B2		B1 for 3 correct
	$h = 9$, $T = 292$			
	$h = 12$, $T = 508$			
	$h = 15$, $T = 670$			If 3 strips used
	$h = 18$, $T = 616$			B2 for 184,616&360&632
	area			(B1 for 3)
	$\approx \dfrac{1}{2} \times 3\{184 + 616 + 2(292 + 508 + 670)\}$	M1		M1
	$= \dfrac{3}{2}(800 + 2 \times 1470)$			
	$= 5610$	A1	5	
(ii)	use more strips	E1	1	
(b)	$\displaystyle\int_{6}^{18} (940 - 288h + 33h^2 - h^3)\,\mathrm{d}h$			
	$= \left[940h - 144h^2 + 11h^3 - \dfrac{1}{4}h^4 \right]_{6}^{18}$	B1B1		B1 for 2 correct
	$= (16920 - 46656 + 64152 - 28244) -$			
	$5640 - 5184 + 2376 - 324)$	M1		
	$= 8172 - 2508$	A1		
	$= 5664$	A1	5	
(c)	$\therefore$ average temperature is $\dfrac{5664 \times 0.036}{12}$	M1M1		M1 for 0.036; M1 for division by 12
	$= 16.992$			Average 13 hourly temperatures SC2
	$= 17.0°C$	A1	3	Answer 16.8 SC1 for above 13 hour but incorrect
	Total		**14**	

Question	Solution	Marks	Total	Comments
4(a)	$\dfrac{\mathrm{d}m}{\mathrm{d}t}=-km$ $\int\dfrac{\mathrm{d}m}{m}=-\int k\,\mathrm{d}t$ $\ln m = -kt+c$ $m = C\mathrm{e}^{-kt}$	M1 A1,A1, B1	4	B1 for $+\,c$ Last mark could be seen in b (1)
(b)(i)	when $t=0$, $m=10$, $\therefore \mathrm{C}=10$ $m=10\mathrm{e}^{-kt}$	M1 A1	2	0 for quote c is m_0
(ii)	when $t=6$, $5=10\mathrm{e}^{-6k}$ $\mathrm{e}^{-6k}=0.5$ $k=\dfrac{1}{6}\ln 2$ $=0.116$	M1 A1 A1	3	
(iii)	when $t=18$, three halving periods 1.25 grams	M1 A1	2	or $m=10\mathrm{e}^{-0.116\times 18}$ SC1 for 1.24
(iv)	when $m=1$, $1=10\mathrm{e}^{-kt}$ $10=\mathrm{e}^{0.115t}$ **or** $10=\mathrm{e}^{0.116t}$ $0.115t=\ln 10$ $0.116t=\ln 10$ $t=19.9358$ $t=19.8498$ $t=19.9$ $t=19.8$	B1 B1 B1	3	
	Total		**14**	
5(a)(i)	when $x=20$, $h=6\sin\pi$ $=0$	B1		
(ii)	when $x=50$, $h=6\sin 2.5\pi=6$	B1	2	
(b)	$\dfrac{\mathrm{d}h}{\mathrm{d}x}=6.\dfrac{\pi}{20}\cos\dfrac{\pi}{20}x$ $=\dfrac{3\pi}{10}\cos\dfrac{\pi}{20}x$	B2	2	B1 for $\dfrac{\pi}{20}$ Accept $=\dfrac{6\pi}{20}\cos\dfrac{\pi}{20}x$
(c)	maximum value of $\dfrac{\mathrm{d}h}{\mathrm{d}x}=\dfrac{3\pi}{10}$ $=0.942$	B1 B1 B1	3	use of max of cos is $+1$
	Total		**7**	
	TOTAL		**60**	