Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
Notation	
Argument	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2012

Use of Mathematics UOM4/2

Applying Mathematics Paper 2

Thursday 24 May 2012 9.00 am to 10.30 am

For this paper you must have:
- a graphics calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You will be awarded up to 3 marks for your ability to present information accurately using correct notation **and** up to 3 marks for mathematical arguments presented clearly and logically.

Advice
- You do not necessarily need to use all the space provided.

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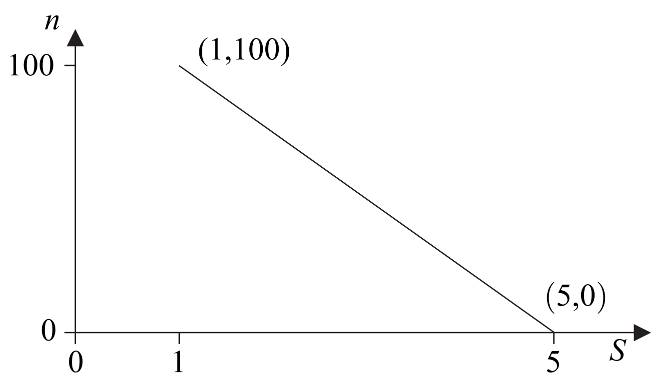
Answer **all** questions.

Answer each question in the space provided for that question.

1 Jack develops a mobile phone app and has to decide how much to sell it for. He considers selling it for at least £ 1 . After doing some research, he finds that if he sells it for £ 1 he can expect to sell 100 000 and if he sells it for £ 5 he can expect to sell zero. He assumes that he can model sales as linear between these two extremes and that a graph of the number sold (in 1000s), n, against selling price, £ S, is as shown below, for $1 \leqslant S \leqslant 5$.



(a) Show that n can be modelled as a function of S using

$$n = -25S + 125$$ *(3 marks)*

(b) The costs of producing this app are £ 50 000 . The profit, P (in £ 1000s), is found by multiplying the number of apps he sells by the selling price and subtracting the production costs.

(i) Express P in terms of n, S and the production costs. *(2 marks)*

(ii) Using $n = -25S + 125$, show clearly that $P = -25S^2 + 125S - 50$. *(2 marks)*

(iii) Calculate the profit when the selling price is £ 1.60 . *(2 marks)*

(c) Solve $-25S^2 + 125S - 50 = 0$ to find the value of S which leads to zero profit. *(4 marks)*

(d) Show clearly that $S^2 - 5S + 2$ can be written as $(S - 2.5)^2 - 4.25$. *(2 marks)*

(e) The expression for P can be written as $P = -25(S^2 - 5S + 2)$.

Using part **(d)** or otherwise, find:

(i) the selling price that gives maximum profit; *(2 marks)*

(ii) the maximum value of the profit that Jack can make. *(2 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 1

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2 The radioactive substance Carbon-14 decays exponentially so that the amount, m, after time, t years, can be modelled by the function $m = m_0 e^{-\lambda t}$, where m_0 is the original amount of Carbon-14. The amount, m, as a proportion of the original amount is given by $\frac{m}{m_0} = e^{-\lambda t}$.

(a) Carbon-14 has a half-life of 5715 years. That is, when $t = 5715$,

$$\frac{m}{m_0} = \frac{1}{2}$$

Show that $\lambda = 0.000121$. *(3 marks)*

(b) After how many years will the amount of Carbon-14 in an object be $\frac{1}{8}$ of its original amount? *(2 marks)*

(c) The remains of the Egyptian Pharaoh, Tutankhamun, are thought to be 3335 years old. Calculate the proportion of the original amount of Carbon-14 in these remains. *(2 marks)*

(d) Pre-historic cave paintings in France were found to have 15% of their original amount of Carbon-14. Estimate the age of the paintings. *(3 marks)*

(e) (i) Sketch a graph of $\frac{m}{m_0}$ against t, showing clearly any distinctive features. *(3 marks)*

(ii) Interpret the graph in terms of the situation that it models. *(2 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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3 Sara finishes her university studies with a student loan of £ 12 000.
After she has worked for n years, her salary is £ S_n. Each year, her salary increases by £ 1500, so $S_n = S_{n-1} + 1500$.
When she starts work, her salary is £ 14 500, so $S_0 = 14\,500$.

(a) Complete the second column in the table opposite which shows Sara's salary. *(2 marks)*

(b) Sara assumes that:

- her outstanding loan is £ L_n, so that $L_0 = 12\,000$
- her loan repayments, £ R_n, will be 15 % of everything she earns in the year above £ 15 000 so that $R_0 = 0$
- the interest rate remains constant at 2 % throughout the entire loan period and interest is added to the outstanding loan at the start of each year.

Explain why the recurrence relations for R_n and L_n are given by:

(i) $R_n = 0.15(S_n - 15\,000)$; *(2 marks)*

(ii) $L_n = 1.02(L_{n-1} - R_{n-1})$. *(2 marks)*

(c) Show clearly calculations that confirm that:

(i) $R_1 = 150$; *(2 marks)*

(ii) $L_1 = 12\,240$. That is, at the start of her second year of working, Sara owes more than she did initially. *(2 marks)*

(d) Complete as much of the opposite table as necessary to find the value of n for which Sara first owes less than her original outstanding loan of £ 12 000. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

n	S_n	R_n	L_n
0	14 500	0	12 000.00
1			
2			
3			
4			
5			

QUESTION PART REFERENCE

Answer space for question 3

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QUESTION PART REFERENCE

Answer space for question 3

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4 At a toll bridge, a driver may spend time queuing at a pay station in addition to the time spent actually paying. To investigate the likely total time taken by drivers, the manager runs a simulation.

The table below shows how she assigns randomly generated integers between 0 and 9 inclusive to simulate how long it takes drivers to pay. The times vary considerably because some drivers pay the exact amount using coins, others require change and others use credit cards.

Time taken to pay	Random integer
30 seconds	0, 1, 2, 3
1 minute	4, 5, 6
1 minute 30 seconds	7, 8
2 minutes	9

(a) (i) Write down the probability that a driver takes 30 seconds to pay. *(1 mark)*

(ii) Explain how you deduced your answer. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

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(b) To run her simulation, the manager assumes that a car arrives every 30 seconds. She also assumes that each car will leave the pay station immediately after the driver has paid.

Complete the table below to show a simulation of how cars queue when **only one pay station** is open.

Time of arrival	**Car arriving**	**Random integer**	**Time taken to pay**	**Pay station 1**
0	A	2	30 seconds	A
30 seconds	B	4	1 minute	B
1 minute	C	7	1 minute 30 seconds	B C
1 minute 30 seconds	D	1		C D
2 minutes	E	5		C D E
2 minutes 30 seconds	F	2		C D E F
3 minutes	G	9		
3 minutes 30 seconds	H	4		
4 minutes	I	3		
4 minutes 30 seconds	J	0		
5 minutes	K	2		

(5 marks)

QUESTION PART REFERENCE

Answer space for question 4

Question 4 continues on page 16

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(c) The manager investigates the effect of opening a second pay station. She further assumes that:

- each driver joins the shorter queue
- if the length of the queue is the same at both pay stations the driver joins the queue at Pay station 1, unless a car is on the point of leaving Pay station 2, then the driver joins the queue at that pay station

Complete the table below to show how cars queue when **two pay stations** are open.

Time of arrival	**Car arriving**	**Random integer**	**Time taken to pay**	**Pay station 1**	**Pay station 2**
0	A	1	30 seconds	A	–
30 seconds	B	3	30 seconds	–	B
1 minute	C	7			
1 minute 30 seconds	D	3			
2 minutes	E	4			
2 minutes 30 seconds	F	0			
3 minutes	G	8			
3 minutes 30 seconds	H	7			
4 minutes	I	2			
4 minutes 30 seconds	J	5			
5 minutes	K	8			

The table showing how the manager assigns randomly generated integers to simulate how long it takes to pay is repeated below.

Time taken to pay	**Random integer**
30 seconds	0, 1, 2, 3
1 minute	4, 5, 6
1 minute 30 seconds	7, 8
2 minutes	9

(5 marks)

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QUESTION PART REFERENCE

Answer space for question 4

Question 4 continues on page 18

Turn over ▶

(d) Would you advise the manager to open a third pay station? Explain your answer. *(2 marks)*

(e) Give two ways in which this simulation could be improved. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 4

END OF QUESTIONS

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