Version

Free-Standing Mathematics Qualification
June 2012

Mathematics Advanced Level 6992

(Specification 6992)

Modelling with Calculus

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Free-Standing Mathematics Qualification
Advanced Level – Modelling with Calculus (6992)
Answers and Marking Scheme - June 2012

Q	Solution	Marks	Total	Comments
1(a)(i)	$\frac{dp}{dt} = 31 - 10t$	M1A1		
	$\frac{dp}{dt} = 0 \rightarrow$			
	$31 - 10t = 0$	M1		
	$t = \frac{31}{10}$ or 3.1	A1		
	when $t = 3.1$			
	$p = 31 \times 3.1 - 5 \times (3.1)^2$	M1		
	= 48.05			(48.05 M3 A2)
	= 48 050	A1	6	Accept 48 000 or 48 100
(ii)	quadratic shape with maximum point	B1		
	passes through the origin and goes lower as $t \rightarrow 6$	B1	2	
(iii)	the model is not appropriate for values of $t > 6$ **or** the model does not have a second maximum	E1	1	p will become negative
(b)	four strips $\rightarrow$ values of t are 0, 1, 2, 3 and 4			
	when $t = 0, p = 0$	B2		B1 for any 2 correct
	$t = 1, p = 26$			B1 only if all correct but more values included
	$t = 2, p = 42$			
	$t = 3, p = 48$			
	$t = 4, p = 44$			
	area			
	$\approx \frac{1}{2} \times 1(0 + 44 + 2(26 + 42 + 48))$	M1A1		M1A1ft if at least 4 values correct above
	$= \frac{1}{2}(44 + 2 \times 116)$			
	= 138			
	number of people is 138 000	A1	5	condone 138
(c)	all the edges of the trapezia are underneath the curve	E1	1	the curve is convex
	Total		**15**	

2(a)	$\frac{dp}{dt} = 3t^2 - 100t + 625$	M1 A1		
	$\frac{dp}{dt} = 0 \Rightarrow 3t^2 - 100t + 625 = 0$	M1		
	$t = \frac{100 \pm \sqrt{10000 - 7500}}{6}$	M1		$(3t - 25)(t - 25) = 0$
	$= \frac{50}{6}$ or 25			
	= 8.33 or 25	A1	5	$t = \frac{25}{3}$, 25
(b)	$\frac{d^2p}{dt^2} = 6t - 100$	M1 A1	2	
(c)	when $t = \frac{25}{3}$,	M1		
	$p = \left(\frac{25}{3}\right)^3 - 50 \times \left(\frac{25}{3}\right)^2 + 625 \times \left(\frac{25}{3}\right)$			
	= 2314.814			
	$L = 0.0012 \times 2314.8 + 22.4$ maximum length of time is 25.2 or 25.17…	A1		
	∴ best year to retire is 2010 or 2011.	A1		
	when $t = \frac{25}{3}$,	B1		
	$\frac{d^2p}{dt^2} = -50$			
	this is negative, hence answer is a maximum	E1	5	
	Total		**12**	
3(a)	$\int_0^{30} (t^3 - 50t^2 + 625t)dt$			
	$= \left[\frac{1}{4}t^4 - \frac{50}{3}t^3 + \frac{625}{2}t^2\right]_0^{30}$	B1B1		B1 for 2 correct
	= (202 500 – 450 000 + 281 250) – 0	M1		
	= 33 750	A1	4	
(b)	∴ average value of p is $\frac{33750}{30}$	M1		
	= 1125			
	average length of time is $0.0012 \times 1125 + 22.4$	M1		
	= 23.75	A1	3	
	Total		**7**	

4(a)	$\frac{dv}{dt} = \lambda v$			
	$\int \frac{dv}{v} = \int \lambda dt$	M1		
	$\ln v = \lambda t + c$	A1 A1 B1	4	B1 for + c
	$v = C\,e^{\lambda t}$			
(b)	when t = 0, $v = 3000$, $\therefore C = 3000$	M1		
	$v = 3000\ e^{\lambda t}$	A1	2	
(c)	when $t = 1$, $3150 = 3000\ e^{\lambda}$	M1		
	$e^{\lambda} = 1.05$	A1		
	$\ln 1.05 = \lambda$	A1	3	
(d)	$v = 3000e^{\lambda t} = 3000\left(e^{\lambda}\right)^t$	M1		need $e^{\lambda t} = \left(e^{\lambda}\right)^t$
	hence $v = 3000\ (1.05)^t$	A1	2	
(e)	when $t = 8$, $v = 3000\ (1.05)^8$	B1		no marks when using $1.05t$
	$= 3000 \times 1.477455$	B1		
	= £4432.366			
	= £4432.37	B1	3	accept £4432.36, £4430, £4432
(f)	when $v = 5000$, $5000 = 3000\ (1.05)^t$	M1		
	$\ln 1.666667 = t \ln 1.05$	A1		
	$t = 10.4698$			
	$t = 10.47$ years or 10.5 years	A1	3	accept 11 years
	Total		**17**	
5(a)	When $t = \frac{2}{3}$, $s = 2.1 + 0.2 \cos \pi$			
	$= 2.1 + 0.2 \times -1$	B1		B1 for $\cos \pi = -1$
	Distance below O is 1.9 m	B1	2	
(b)(i)	when $t = 2$, $\cos 3\pi = -1$	E1		
	which is a minimum value			
	hence Tim is at the highest point of his swing	E1	2	accept 1.9
(ii)	the model predicts another high point when $\cos\frac{3\pi}{2}t = -1$	M1		
	the next high point is when $t = \frac{10}{3}$	A1	2	
(c)	$\frac{ds}{dt} = -0.2\frac{3\pi}{2}\sin\frac{3\pi}{2}t$	B1 B1		$\frac{3\pi}{2}$ $\sin\frac{3\pi}{2}t$
	$= -\frac{3\pi}{10}\sin\frac{3\pi}{2}t$	B1	3	all correct and simplified
	Total		**9**	
	TOTAL		**60**	