Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
TOTAL	

Free-Standing Mathematics Qualification
Advanced Level
June 2012

Modelling with Calculus 6992/2

Unit 12

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should normally be given to three significant figures, unless stated otherwise.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.

Advice
- You do not necessarily need to use all the space provided.

Section A

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Mall shoppers** *on page 2 of the Data Sheet.*

1 The number of people, p, in thousands per hour, entering the mall can be modelled by the equation

$$p = 31t - 5t^2$$

for values of t from 0 to 5, where t is the time in hours after 9 am.

(a) (i) **Use this model and calculus** to predict the maximum number of people per hour who were entering the mall. *(6 marks)*

(ii) Sketch a graph of the equation $p = 31t - 5t^2$. *(2 marks)*

(iii) State one problem with using this model. *(1 mark)*

(b) Use the trapezium rule with four strips to find an estimate for the total number of people who entered the mall in the first four hours that it was open. *(5 marks)*

(c) Explain how you would know that the trapezium rule, in this case, would give an underestimate of the total. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

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Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

Section B

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Length of retirement** *on page 3 of the Data Sheet.*

2 The life expectancy of a woman, L years, when she reaches state pension age in the UK, t years after the year 2002, may be modelled by the function

$$L = 0.0012\,p + 22.4$$

where $p = t^3 - 50t^2 + 625t$

for values of t from 0 to 30.

Use this model and calculus to answer the following questions.

(a) Find the values of t at the turning points of p. *(5 marks)*

(b) Find $\dfrac{\mathrm{d}^2p}{\mathrm{d}t^2}$. *(2 marks)*

(c) In which year will a woman reaching state pension age have the maximum life expectancy?

Calculate this maximum life expectancy, confirming that your value is a maximum. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

3 (a) Find

$$\int_0^{30} (t^3 - 50t^2 + 625t)\mathrm{d}t \qquad \textit{(4 marks)}$$

(b) Hence find the average life expectancy of a woman when she reaches the state pension age in the years 2002–2032. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

Section C

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Investment** *on page 4 of the Data Sheet.*

4 The value, £ v, of the investment increases with time, t years.
The rate of increase in the value of the investment is directly proportional to the value of the investment at the start of that year.

This can be expressed by the differential equation

$$\frac{dv}{dt} = \lambda v$$

where λ is a positive constant.

(a) Show that the general solution for v of this differential equation is

$$v = Ce^{\lambda t}$$ *(4 marks)*

(b) When $t = 0$, the value of the investment was £ 3000.

Show that $v = 3000e^{\lambda t}$. *(2 marks)*

(c) Ruth expects the value of her investment to increase at 5 % per year:
ie when $t = 1$, the value of her investment will be £ 3150.

Show that $\lambda = \ln 1.05$ *(3 marks)*

(d) Hence show that $v = 3000(1.05)^t$. *(2 marks)*

(e) Find the value of the investment after 8 years. *(3 marks)*

(f) Find the value of t when the value of the investment is £ 5000. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

Section D

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Swing** *on page 4 of the Data Sheet.*

5 Tim is swinging slowly on a garden swing.

The distance, s metres, of Tim below the point, O, from where the swing is suspended can be modelled by the equation

$$s = 2.1 + 0.2\cos\frac{3\pi}{2}t$$

where t is the time in seconds after Tim is first at his lowest point.

(a) Find the distance below O predicted by the model when $t = \frac{2}{3}$. *(2 marks)*

(b) (i) Show that the model predicts that Tim is at his highest point when $t = 2$. *(2 marks)*

(ii) Find when the model predicts the next highest point. *(2 marks)*

(c) Find an expression for $\frac{ds}{dt}$. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

END OF QUESTIONS