Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
Notation	
Argument	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2011

Use of Mathematics UOM4/2

Applying Mathematics Paper 2

Thursday 26 May 2011 9.00 am to 10.30 am

For this paper you must have:
- a graphics calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You will be awarded up to 3 marks for your ability to present information accurately using correct notation **and** up to 3 marks for mathematical arguments presented clearly and logically.

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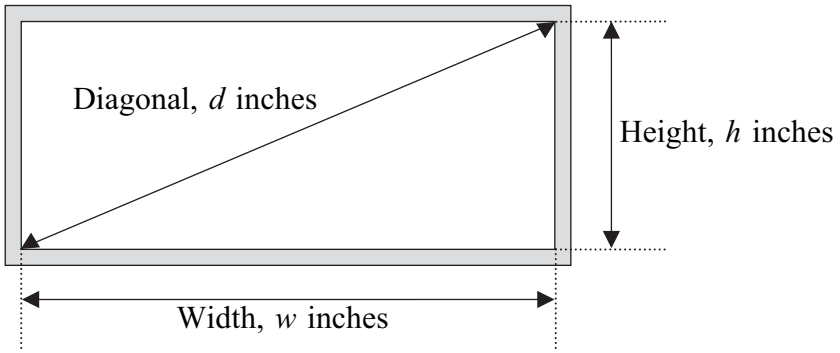
Answer **all** questions in the spaces provided.

1 TVs are measured by the length of the diagonal of the screen.

Not to scale



A widescreen TV with a diagonal of d inches, a width of w inches and a height of h inches is shown in the diagram.

The width : height ratio for a widescreen TV is $16 : 9$.

Therefore, if the height of the widescreen TV is h inches then its width, w inches, is given by $w = \frac{16}{9}h$.

In this case, $d^2 = h^2 + \left(\frac{16}{9}h\right)^2$.

(a) What size, d inches, would be given for a widescreen TV with a height of 24 inches? Give your answer to the nearest inch. *(2 marks)*

(b) For a 32-inch widescreen TV, $32^2 = h^2 + \left(\frac{16}{9}h\right)^2$.

Show that this TV has a screen height of 15.7 inches and a width of 27.9 inches. *(4 marks)*

(c) For the best view of the picture on a widescreen TV, it is advised that you sit a certain distance away, x inches, from the widescreen TV, given by the formula $x = 3h$.

How far away should you sit from a 32-inch widescreen TV for the best view? *(1 mark)*

Question 1 continues on page 4

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QUESTION PART REFERENCE

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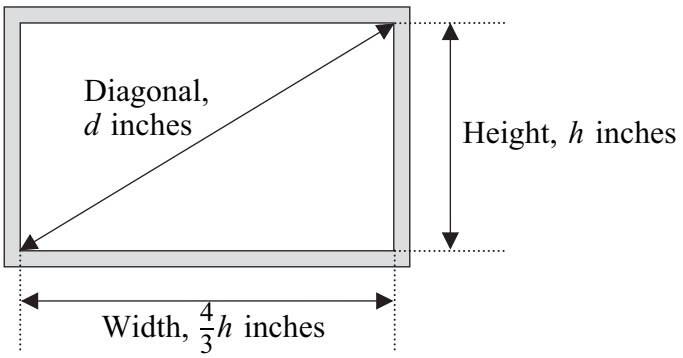
1 (d) Non-widescreen TVs have a width : height ratio of 4 : 3, leading to the situation in the diagram below. In this case, d is given in terms of h by $d^2 = h^2 + \left(\frac{4}{3}h\right)^2$.

Not to scale



For the best view of the picture on a non-widescreen TV, it is advised that you sit a certain distance away, x inches, from the non-widescreen TV, given by the formula $x = 6h$.

How far away should you sit from a 32-inch non-widescreen TV for the best view?

(4 marks)

(e) For a non-widescreen TV, show that x can be expressed in terms of d as $x = 3.6d$.

(4 marks)

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2 Zipf's Law suggests that the rth largest city in a country has population, P, given by $P = kr^{-n} = \frac{k}{r^n}$, where k and n are positive constants.

(a) Sketch, using the axes below, a graph of $P = \frac{k}{r^n}$, for $n = 1$, showing all significant features clearly. *(2 marks)*

QUESTION PART REFERENCE

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2 (b) (i) For $P = \dfrac{k}{r^n}$, show that $\ln P = \ln k - n \ln r$. *(3 marks)*

(ii) Sketch, using the axes below, a graph of $\ln P = \ln k - n \ln r$, showing all significant features clearly. *(2 marks)*

Question 2 continues on the next page

QUESTION PART REFERENCE

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2 (c) The table shows, in order, the size, P, of the ten cities in Belgium that have the largest populations.

(i) Complete the columns in the table giving $\ln r$ and $\ln P$. *(4 marks)*

QUESTION PART REFERENCE

r	City	Population, P	$\ln r$	$\ln P$
1	Antwerp	470 000	0.000	13.060
2	Gent	231 000	0.693	12.350
3	Charleroi	207 000	1.099	12.240
4	Liège	197 000	1.386	12.191
5	Brussels	136 000	1.609	11.820
6	Brugge	117 000	1.792	11.670
7	Namur	104 000		
8	Mons	92 000		
9	La Louvière	76 000		
10	Kortrijk	75 000		

2 (c) (ii) Use your answers to **(c)(i)** to plot, on the graph below, $\ln P$ against $\ln r$ for the four remaining cities. *(2 marks)*

(iii) On the graph, draw, by eye, a line of best fit and use this to find k and n and hence express P in terms of r. *(5 marks)*

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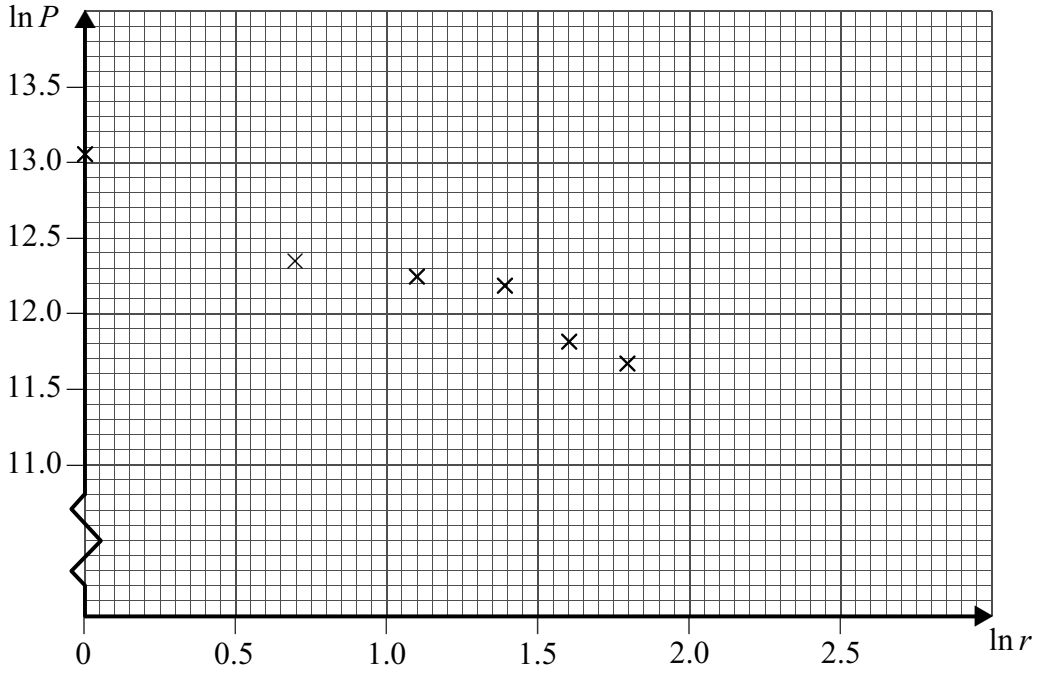

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3 The table below shows the depth of water, d metres, at low tide in the morning at a point on the River Tyne on the first 20 days in October.

day, n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
depth, metres	1.12	0.93	0.79	0.73	0.75	0.87	1.07	1.36	1.69	2.00
day, n	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
depth, metres	2.15	2.03	1.75	1.45	1.19	0.98	0.87	0.84	0.92	1.28

These data can be modelled by the function $d = 1.4 - 0.7 \sin 25n°$, where n is the day number.

(a) Find, to two decimal places, the depth of water predicted by the function at low tide on the morning of 1 October. *(2 marks)*

(b) State:

(i) the maximum depth of the low tide predicted by the model; *(1 mark)*

(ii) the minimum depth of the low tide predicted by the model; *(1 mark)*

(iii) the mean depth of the low tide predicted by the model. *(1 mark)*

(c) The model suggests that the low tide depth follows a cyclic pattern.

What is the period of this cycle? *(2 marks)*

Question 3 continues on page 12

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3 (d) On the axes below, sketch a graph of the function $d = 1.4 - 0.7\sin 25n°$, showing all important features clearly. *(3 marks)*

(e) A boat needs a minimum depth of 1.5 metres to be able to pass this point on the river. So, according to the model, $1.4 - 0.7\sin 25n° \geqslant 1.5$.

Use this model to find the first date in October on which the boat can safely pass this point on the river at low tide.

You must show all your working. *(5 marks)*

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4 A company director decides that, to encourage workers not to drive to work, a reward scheme will be introduced. She suggests that workers who do not drive will receive one of three types of token randomly with their monthly payslip. They will receive either a Red, White or Yellow token. If in any three consecutive months they get one of each type of token in any order, they will win a £50 shopping voucher. Each token can only be used to claim one voucher.

In an attempt to determine how much the scheme will cost over a year, the company director carries out a simulation. She assigns randomly generated integers between 0 and 9 inclusive to simulate receiving each type of token, as shown in the table below.

Token	Random integer assigned
Red (R)	0, 1, 2, 3, 4
White (W)	5, 6, 7
Yellow (Y)	8, 9

(a) Write down the probability that a worker will receive a yellow token.

Give an explanation for your answer. *(2 marks)*

(b) Complete the table of values opposite, to show the tokens that the remaining workers (H – J) receive throughout the year and the amount that each worker will win in vouchers.

You may assume that all workers from A – J do **not** drive to work. *(3 marks)*

(c) What is the average reward per worker? *(2 marks)*

Question 4 continues on page 16

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		Month												
Worker		**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**	**Won**
A	Random number	1	0	2	5	6	3	5	7	5	2	8	8	£ 50
	Token	R	R	R	W	W	R	W	W	W	R	Y	Y	
B	Random number	1	4	7	5	9	8	4	8	8	3	2	6	£ 0
	Token	R	R	W	W	Y	Y	R	Y	Y	R	R	W	
C	Random number	7	3	8	7	8	3	4	6	0	2	8	8	£ 100
	Token	W	R	Y	W	Y	R	R	W	R	R	Y	Y	
D	Random number	8	6	1	8	8	1	7	3	7	6	9	7	£ 100
	Token	Y	W	R	Y	Y	R	W	R	W	W	Y	W	
E	Random number	0	6	0	5	6	4	0	2	4	2	1	0	£ 0
	Token	R	W	R	W	W	R	R	R	R	R	R	R	
F	Random number	8	9	1	0	3	1	3	8	8	8	6	9	£ 0
	Token	Y	Y	R	R	R	R	R	Y	Y	Y	W	Y	
G	Random number	7	0	5	5	1	5	8	7	7	8	1	0	£ 100
	Token	W	R	W	W	R	W	Y	W	W	Y	R	R	
H	Random number	6	9	5	3	7	9	1	3	7	0	5	8	£
	Token													
I	Random number	8	8	8	8	4	8	8	7	8	0	7	2	£
	Token													
J	Random number	2	5	5	6	4	5	8	4	7	4	2	9	£
	Token													

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4 (d) In an attempt to make the simulation less costly, the company director changes the random integer assignment and also adds in an additional coloured token.

The details for the new simulation are given below.

Token	Random integer assigned
Purple (P)	0, 1
Red (R)	2, 3, 4, 5, 6
White (W)	7, 8
Yellow (Y)	9

To win a voucher, a worker must still receive a red, white and yellow token in three consecutive months in any order.

Explain how this might prove less costly for the company. *(2 marks)*

(e) Complete the table of values opposite, to show the tokens that the remaining workers (H – J) receive throughout the year and the amount that each worker will win in vouchers.

You may assume that all workers from A – J do **not** drive to work. *(3 marks)*

Question 4 continues on page 18

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		Month												
Worker		**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**	**Won**
A	Random number	2	5	9	5	4	7	0	1	3	7	0	3	£ 0
	Token	R	R	Y	R	R	W	P	P	R	W	P	R	
B	Random number	4	3	7	5	1	9	4	7	7	9	4	2	£ 100
	Token	R	R	W	R	P	Y	R	W	W	Y	R	R	
C	Random number	4	6	2	9	1	4	6	4	0	9	2	8	£ 50
	Token	R	R	R	Y	P	R	R	R	P	Y	R	W	
D	Random number	6	3	7	5	8	6	7	4	2	3	8	2	£ 0
	Token	R	R	W	R	W	R	W	R	R	R	W	R	
E	Random number	0	1	1	0	3	6	0	6	4	5	1	4	£ 0
	Token	P	P	P	P	R	R	P	R	R	R	P	R	
F	Random number	8	4	0	1	0	5	0	7	0	3	6	8	£ 0
	Token	W	R	P	P	P	R	P	W	P	R	R	W	
G	Random number	8	9	2	2	1	3	3	9	9	5	5	2	£ 50
	Token	W	Y	R	R	P	R	R	Y	Y	R	R	R	
H	Random number	3	7	3	7	7	6	3	9	6	8	5	7	£
	Token													
I	Random number	3	3	4	3	5	5	1	6	9	6	9	1	£
	Token													
J	Random number	9	7	7	0	2	6	4	3	1	7	3	3	£
	Token													

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4 (f) For this new simulation, what is the average reward per worker? *(2 marks)*

(g) Give one way in which the simulations could be improved. *(2 marks)*

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END OF QUESTIONS

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