General Certificate of Education
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Use of Mathematics UOM4/1PM

Applying Mathematics Paper 1

Preliminary Material

Data Sheet

To be opened and issued to candidates between
Thursday 12 May 2011 and Thursday 19 May 2011

REMINDER TO CANDIDATES

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A CLEAN COPY WILL BE MADE AVAILABLE.

Diet + exercise = weight loss (or gain)!

There are many newspaper headlines about obesity. However, there seems to be less detailed advice about how to behave differently so that you can lose weight. At a very simple level, it is important to consider the amount of energy that you take in each day through eating and the amount that you lose by exercising. You can investigate this mathematically to give you some idea of whether you can expect to lose or gain weight depending on how much you currently eat and how active you are. By adjusting either or both of these two factors, you should then be able to gain or lose weight.

As you have probably noticed, packaged foods give lots of nutritional information about the product in the package. One crucial factor is their calorific value, as this is the unit used to measure the energy they supply. **Figure 1** shows how this information is typically presented on a packet of breakfast cereal. The energy supplied by a typical serving of 40 grams of cereal with 125 millilitres of semi-skimmed milk is 202 kilocalories. Often this is quoted as just 202 calories. The factor of 1000 is often ignored when discussing dietary matters.

To understand how we should control our energy intake from food, we need to know, first of all, what might be the typical energy needs for different people. There are a number of different ways of calculating this, some more complicated than others. One simple method is given in **Figure 2**.

Figure 1 Nutritional information for a typical breakfast cereal

Ingredients

Wholewheat (68%), Dried Fruit (28%) {Raisins, Coconut, Banana (Flavouring), Apple, Hazelnuts}, Sugar, Salt, Barley Malt Flavouring, Niacin, Iron, Vitamin B_6, Riboflavin (B_2), Thiamin (B_1), Folic Acid, Vitamin B_{12}.

Allergy information

Contains Nuts, Wheat & Barley

Nutritional information

	Typical value per 100 g		40 g serving with 125 ml of semi-skimmed milk	
Energy	1514 kJ	358 kcal	857 kJ	202 kcal
Protein	8 g		8 g	
Carbohydrates	68 g		33 g	
Fat	6 g		4.5 g	
of which saturates	3.5 g		3 g	
Fibre	10 g		4 g	
Sodium	0.55 g		0.3 g	
Salt	1.4 g		0.7 g	
Vitamins:		(% RDA)		(% RDA)
Thiamin (B_1)	0.9 mg	(63)	0.4 mg	(29)
Riboflavin (B_2)	1 mg	(63)	0.7 mg	(45)
Niacin	11.3 mg	(63)	4.7 mg	(26)
Vitamin B_6	1.3 mg	(63)	0.6 mg	(29)
Folic acid	250 μg	(125)	108 μg	(54)
Vitamin B_{12}	0.63 μg	(63)	0.77 μg	(77)
Minerals:				
Iron	8.8 mg	(63)	3.5 mg	(25)

Figure 2 Simple method for calculating daily calorie needs

Daily calorie needs: Simple method for weight in kg
Male's calorie needs: $C = \text{weight} \times 24 \times \text{Activity Factor}$ Female's calorie needs: $C = 0.9 \times \text{weight} \times 24 \times \text{Activity Factor}$ **Activity Factor (*AF*):**

Sedentary	1	No exercise and desk job or at home most of day
Light active	1.2	Regular exercise 3 times/week and desk job or at home most of day
Active	1.4	Daily exercise and work on feet most of day
Very active	1.5	Daily intense exercise and construction work most of day

The formulae above allow you to calculate a typical target daily calorie intake. If you have a relatively constant weight from day to day, week to week and month to month you can assume that your calorie intake is balanced by the amount of calories you use up each day due to the exercise you do. However, if you are taking in, or burning up, more calories than you need you will either gain or lose weight, and nutritionists suggest that ±7800 calories is equivalent to a weight gain or loss of about 1 kg.

A recurrence relation can be used to model a person's weight from day to day:

$$\text{weight today} = \text{weight yesterday} + \text{weight change}$$

Therefore, based on the simple method of calculating daily calorie needs, if a person has a daily calorie intake I, for weight w_n kg on day n, this gives,

for a male, $w_n = w_{n-1} + \dfrac{I - 24w_{n-1} \times AF}{7800}$,

and for a female $w_n = w_{n-1} + \dfrac{I - 0.9 \times 24w_{n-1} \times AF}{7800}$.

To see how such recurrence relations can be used, consider Ben as an example. Assuming that he has a total intake of 2500 calories and a sedentary Activity Factor, the appropriate recurrence relation reduces to

$$w_n = 0.3205 + 0.9969w_{n-1}$$

Suppose Ben has an initial weight of 80 kg. The table in **Figure 3**, on the next page, shows that in just fourteen days Ben will gain almost a kilogram in weight.

Figure 3 Table showing a recurrence relation model of Ben's weight over 14 days

	Activity Factor = 1
day, n	w_n
0	80.00
1	80.07
2	80.14
3	80.22
4	80.29
5	80.36
6	80.43
7	80.50
8	80.57
9	80.64
10	80.71
11	80.79
12	80.86
13	80.93
14	80.99

Figure 4 Exercising in a gym

It is clear that Ben must either reduce his calorie intake or increase the amount of exercise he does. If he changes his life-style so that he is in the 'light active' category, he will still put on weight, although more slowly. In this case, the recurrence relation that you can use to calculate his weight each day is $w_n = 0.3205 + 0.9963w_{n-1}$.

It would help Ben to lose weight if he undertook some regular daily exercise. The table in **Figure 5** gives the calories burned per kilogram per hour for various types of exercise. So, if Ben starts skateboarding for one hour every day, on the first day he will burn up $5 \times 80 = 400$ calories.

Figure 5 Calories burned per kilogram per hour during selected exercise — note this is exercise in addition to that associated with the activity factor that indicates a general lifestyle

Exercise	**Calories burned per kg / hour**	**Exercise**	**Calories burned per kg / hour**
Running (10 mph)	16.0	Skateboarding	5.0
Track running	10.0	Tennis (general)	5.0
Soccer (competitive)	10.0	Walking (very brisk)	4.0
Swimming (laps)	10.0	Volleyball (competitive)	4.0
Cycling (moderate)	8.0	Swimming (leisurely)	4.0
Weight lifting/body building	6.0	Walking (moderate)	3.5

To find, for Ben, what types of exercise might result in a weight loss, we need to consider a more general form of recurrence relation that introduces a factor E for the number of calories burned per kilogram per hour for the exercise that he does, assuming that the exercise is for h hours per day.

This then gives $w_n = w_{n-1} + \dfrac{I - 24w_{n-1} - E \times h \times w_{n-1}}{7800}$ (assuming that, in general, Ben has a sedentary lifestyle).

For weight loss, $\dfrac{I - 24w_{n-1} - E \times h \times w_{n-1}}{7800} \leqslant 0$.

Therefore for Ben, taking $I = 2500$, and $w_0 = 80$, $E \times h \geqslant 7.25$. This suggests that, if Ben started running, for example, for half an hour per day rather than skateboarding for an hour per day, he would slowly start to lose weight.

The simple method considered here does not take into account factors such as a person's height and age. A more complex method that takes these factors into account is therefore given in **Figure 6**.

Figure 6 Complex method for calculating daily calorie needs

Daily calorie needs: Complex method for weight in kg, height in cm, age in years

Male's calorie needs:
$C = (66.47 + 13.75 \times \text{weight} + 5 \times \text{height} - 6.76 \times \text{age}) \times AF$
Female's calorie intake:
$C = (655.1 + 9.65 \times \text{weight} + 1.84 \times \text{height} - 4.68 \times \text{age}) \times AF$

Activity Factor (*AF*):

	Male	**Female**	
Resting	1.0	1.0	Sleeping, reclining
Sedentary	1.3	1.3	Minimal moving, mainly sitting, lying
Light active	1.6	1.5	Light activity involving walking
Moderate	1.7	1.6	Light manual labour
Very active	2.1	1.9	Agricultural work, military duty, building
Extremely active	2.4	2.2	Athletes with daily strenuous training, construction workers, miners

To return to the original situation considered above, where Ben has a sedentary Activity Factor, with an intake of 2500 calories each day and doing no additional exercise, Ben's weight, w kilograms, can also be modelled by an exponential function:

$$w = \frac{2500 - 580e^{-0.0031t}}{24}$$

where t is measured in days.

As the table showing the two models, in **Figure 7** on the next page, shows, both recurrence relation $w_n = 0.3205 + 0.9969w_{n-1}$ and function $w = \dfrac{2500 - 580e^{-0.0031t}}{24}$ predict reasonably close values for Ben's weight each day over this period.

Figure 7 Table showing both recurrence relation and exponential function models of Ben's weight over 12 days

day, n	**recurrence relation**	**exponential function**
0	80.00	80.00
1	80.07	80.07
2	80.14	80.15
3	80.22	80.22
4	80.29	80.30
5	80.36	80.37
6	80.43	80.45
7	80.50	80.52
8	80.57	80.59
9	80.64	80.66
10	80.71	80.74
11	80.79	80.81
12	80.86	80.88

Of course, the modelling of weight discussed here is very much simplified compared with reality and, most importantly, it looks at only one aspect of a person's requirements. For example, as the data for a breakfast cereal shown in **Figure 1** demonstrate, if you are to consider dietary needs you also need to consider a range of other factors such as vitamins and minerals.

END OF DATA SHEET

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