General Certificate of Education
Advanced Subsidiary Examination
June 2010

Use of Mathematics UOM4/1PM

Applying Mathematics Paper 1

Preliminary Material

Data Sheet

To be opened and issued to candidates between
Thursday 13 May 2010 and Thursday 20 May 2010

REMINDER TO CANDIDATES

YOU MUST **NOT** BRING THIS DATA SHEET
WITH YOU WHEN YOU SIT THE EXAMINATION.
A CLEAN COPY WILL BE MADE AVAILABLE.

It's a gas

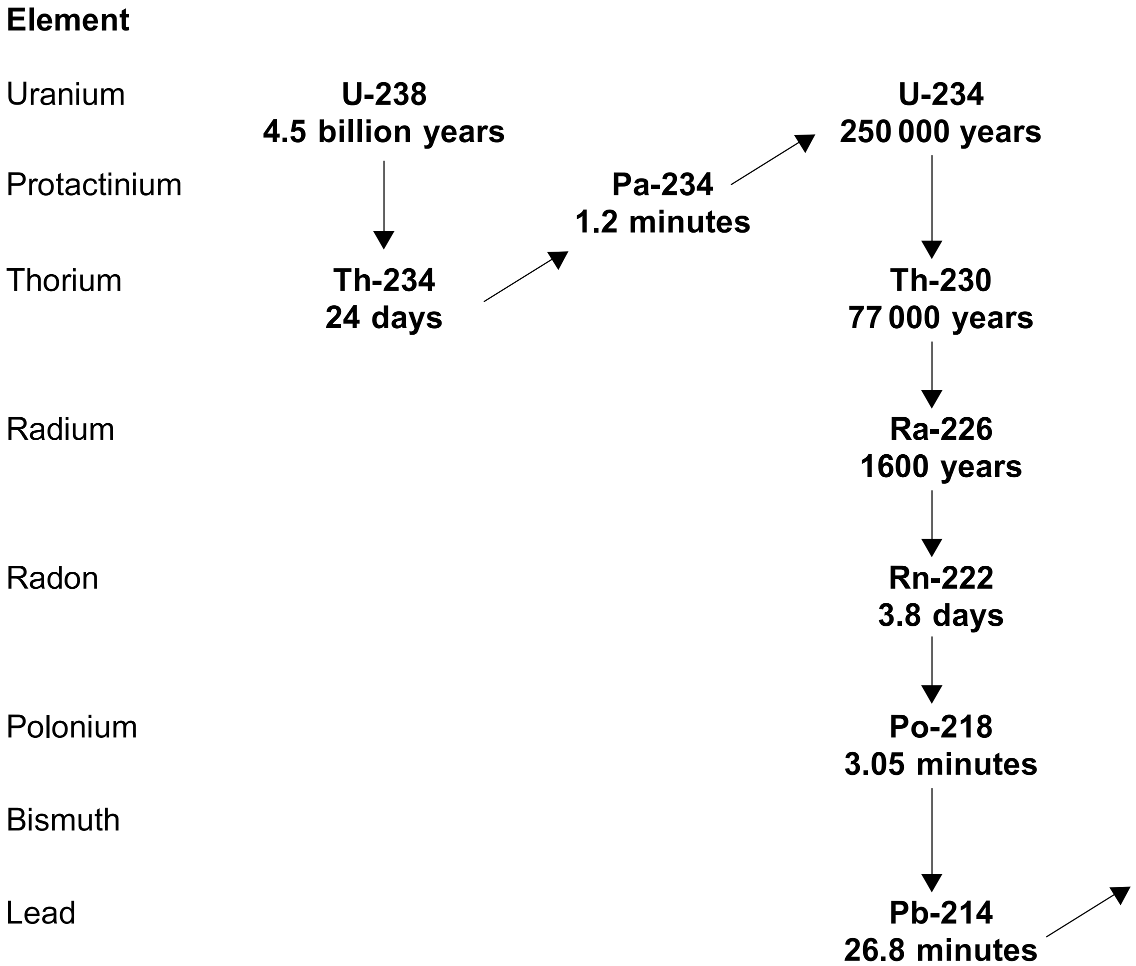
As we go about our daily lives, radioactivity is usually of negligible concern. However, there is one radioactive substance that can become a problem for some in their homes: radon-222. This is particularly the case in certain areas, such as Cornwall, Devon and Derbyshire. Part of the problem is that radon-222 is a gas. Although it occurs naturally and is found in rocks and soils, it may escape into the atmosphere and build up into higher-than-healthy concentrations in some indoor spaces, including homes.

Figure 1 Stone house in Derbyshire

A radioactive substance, like radon-222, changes into a different substance by emitting radioactive particles. The change is called decay. The time for half the substance to decay is called its half-life. Radon-222 has a relatively short half-life of only 3.8 days. For radon-222, therefore, every 3.8 days the amount of radioactive gas remaining is half of that previously present, with radioactive particles being emitted every time an atom decays.

Radon is formed during a chain of radioactive decay starting with uranium-238, which is widely distributed in rocks and soils throughout the Earth's crust. Part of the chain is shown in **Figure 2**, which gives the half-life of each substance.

Figure 2 Radioactive decay series that includes radon-222

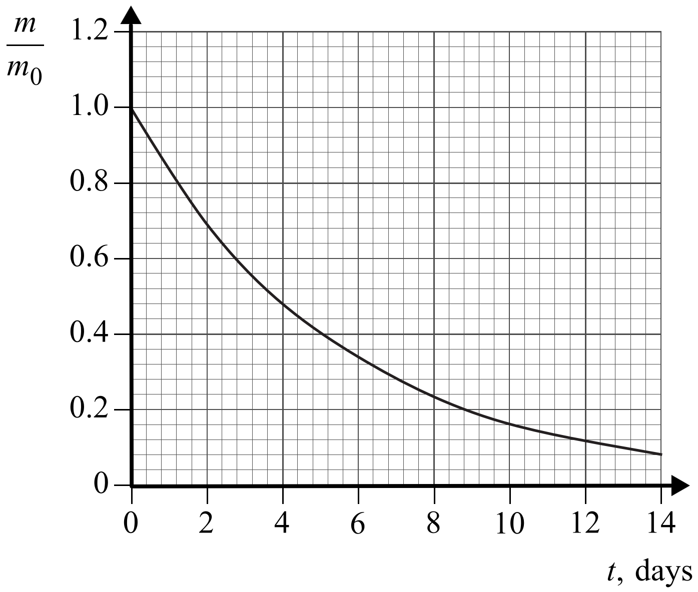


Radioactive decay can be modelled mathematically using the exponential function. The mass, m, of radioactive substance at time t can be modelled by the function $m = m_0 e^{-\lambda t}$, where m_0 is the mass of substance present at time $t = 0$, and λ is a parameter which depends on the half-life of the substance.

For radon-222, which has a half-life of 3.8 days, we can determine λ. This is done by considering that $m = \frac{m_0}{2}$ when $t = 3.8$. So $\frac{m_0}{2} = m_0 e^{-\lambda \times 3.8}$, leading to $\lambda = 0.182$. Hence, the function $m = m_0 e^{-0.182t}$, where t is measured in days, can be used to model the decay of radon-222.

A graph of $\frac{m}{m_0}$ plotted against t days is shown in **Figure 3**. You can see that, after 3.8 days, $\frac{m}{m_0} = \frac{1}{2}$; after a further 3.8 days, $(t = 7.6)$, $\frac{m}{m_0} = \frac{1}{4}$, and so on.

Figure 3 Graph showing the model of the decay of radon-222, $\frac{m}{m_0} = e^{-0.182t}$



Radioactive decay can be modelled by exponential functions such as the one we have found for radon-222. Whether a particular atom decays or not is a random process; after its formation, it may decay immediately or it may be many days or even years later that it does so.

Given that the probability that a single atom of a radioactive substance will **not** decay in one day is p, the half-life of the substance is given by $p^{\text{half-life}} = \frac{1}{2}$. For a substance for which $p = \frac{1}{2}$, the half-life is one day, since $\left(\frac{1}{2}\right)^1 = \frac{1}{2}$. A graph for such a substance is shown plotted in **Figure 4** on the next page.

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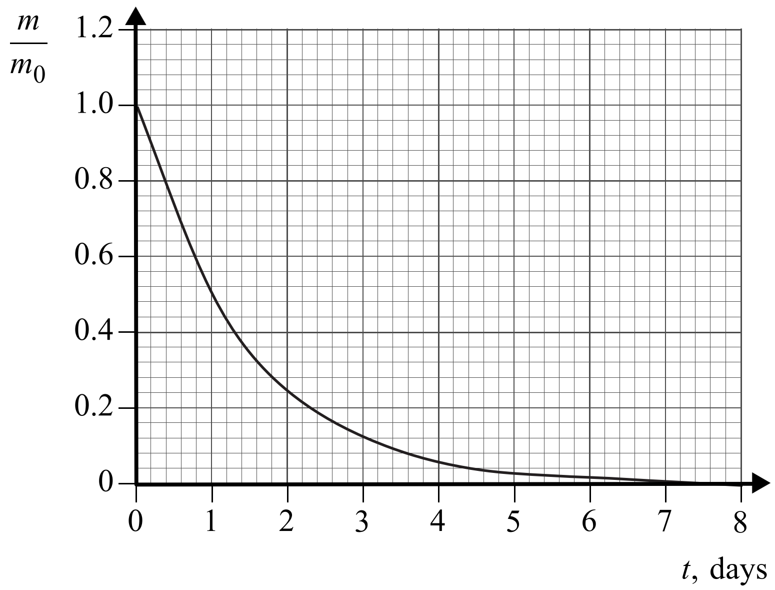
Figure 4 Graph showing the model of the decay of a radioactive substance with a half-life of one day



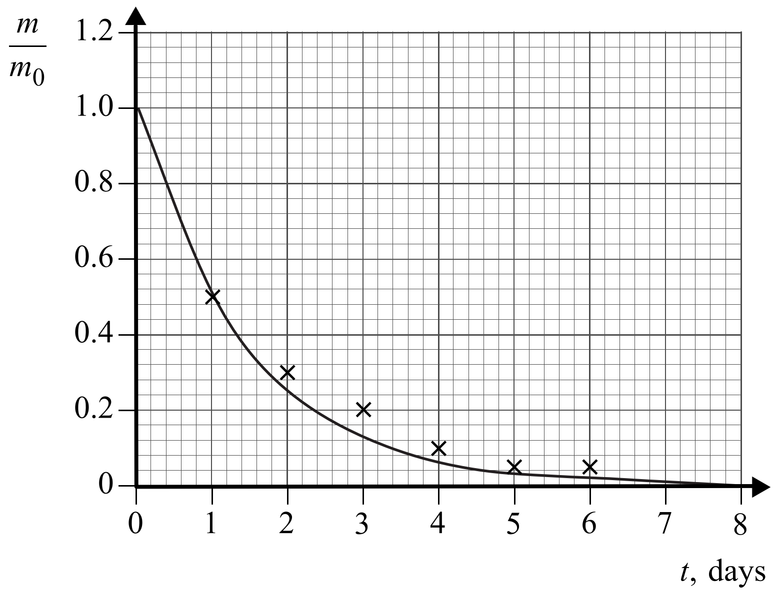
An interesting way to simulate this decay is for, say, twenty people to consider themselves as atoms and for each to spin a coin, with those getting a 'head' to consider themselves as having decayed in the first day. Those 'atoms' that remain spin their coins again to simulate the events of the second day, and, again, those that get a head are considered to have decayed, and so on. The spreadsheet shown in **Figure 5** demonstrates such a situation, with a 0 in each of columns C to F representing a head and a 1 in these columns representing a tail. The total of 1s in each column, therefore, gives the number of atoms remaining after each time period; row 23 shows this total expressed as a fraction of the original number of atoms, and this is shown plotted together with the exponential function in **Figure 6**.

Figure 5 Simulating the decay of a radioactive substance with a half-life of one day

	A	B	C	D	E	F	G	H
1	atom	t=0	t=1	t=2	t=3	t=4	t=5	t=6
2	A	1	0	0	0	0	0	0
3	B	1	0	0	0	0	0	0
4	C	1	0	0	0	0	0	0
5	D	1	0	0	0	0	0	0
6	E	1	1	0	0	0	0	0
7	F	1	1	0	0	0	0	0
8	G	1	1	1	0	0	0	0
9	H	1	1	1	1	1	1	1
10	I	1	0	0	0	0	0	0
11	J	1	0	0	0	0	0	0
12	K	1	1	0	0	0	0	0
13	L	1	1	0	0	0	0	0
14	M	1	1	1	0	0	0	0
15	N	1	0	0	0	0	0	0
16	O	1	0	0	0	0	0	0
17	P	1	0	0	0	0	0	0
18	Q	1	0	0	0	0	0	0
19	R	1	1	1	1	1	0	0
20	S	1	1	1	1	0	0	0
21	T	1	1	1	1	0	0	0
22	total	20	10	6	4	2	1	1
23	N/N0	1	0.5	0.3	0.2	0.1	0.05	0.05

Chart1 Sheet3 Sheet2 Sheet1

Figure 6 Graph showing the simulation of the decay of a radioactive substance with half-life of one day, together with an exponential model for this



You can see that the simulation matches the exponential model reasonably well. As you might imagine, the more atoms you simulate, the closer the resulting data will be to the curve.

Finally, a word of reassurance about radon-222. Whilst radon-222 may build up in enclosed spaces, it is usually quite easy to improve ventilation so that it disperses and therefore does not cause any problems to human health.

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