Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
TOTAL	

Free-Standing Mathematics Qualification
Advanced Level
June 2010

Working with Algebraic and Graphical Techniques 6991/2

Unit 11

Wednesday 26 May 2010 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a ruler
- a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.

Section A

Answer **all** questions in the spaces provided.

Use **Income** *on page 2 of the Data Sheet.*

1 The demand, d, for an item is given by the equation $d = 150 - 5x$, where £x is the selling price of the item.

(a) Explain why this equation cannot be used for $x > 30$. *(1 mark)*

(b) The income, £I, is given by the equation $I = x(150 - 5x)$.

On the grid opposite, draw the graph of $I = x(150 - 5x)$ for $0 \leqslant x \leqslant 30$. *(4 marks)*

(c) Use your graph to state:

(i) the maximum income; *(1 mark)*

(ii) the value of x that gives the maximum income. *(1 mark)*

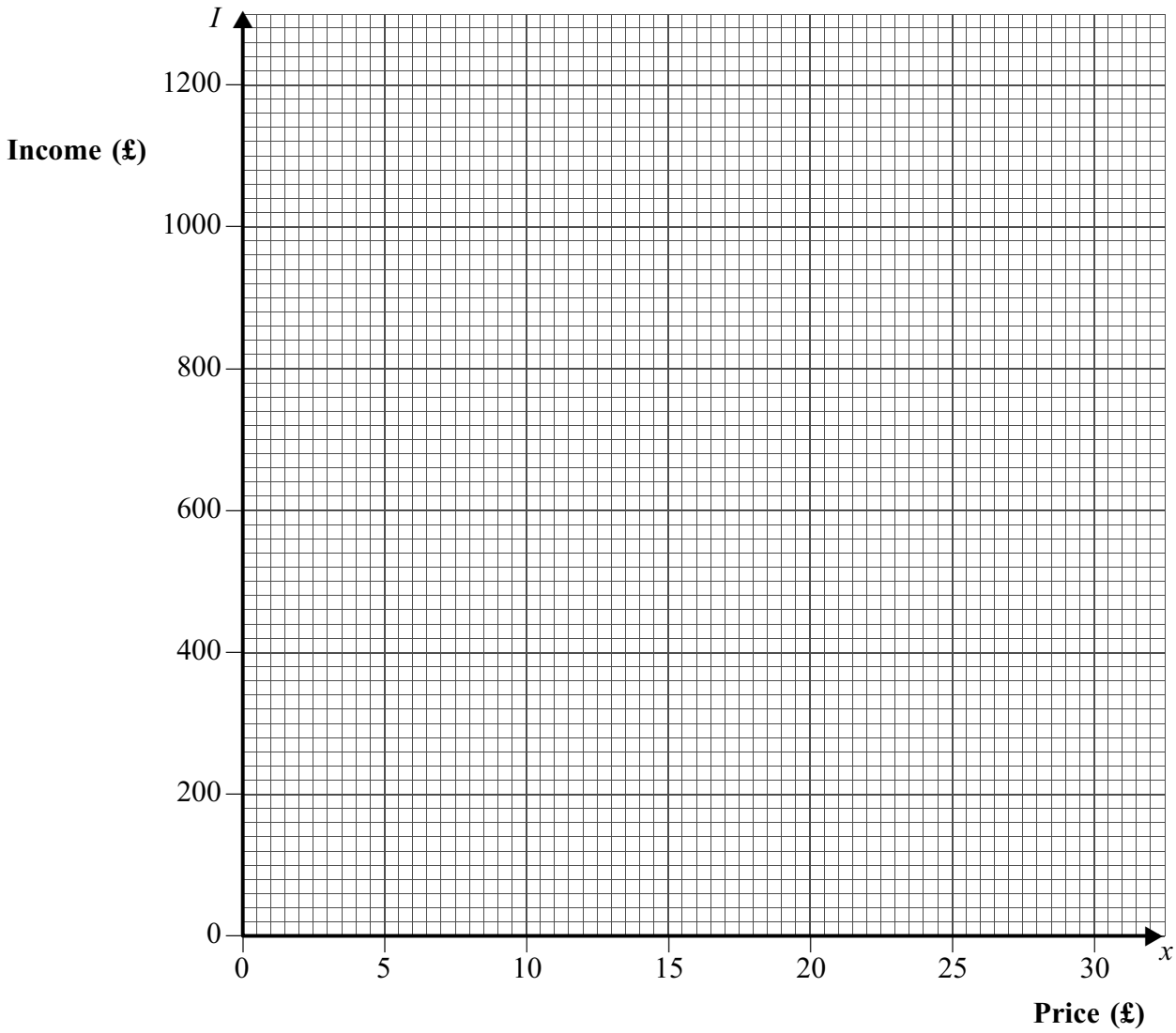
(d) Rearrange $150x - 5x^2$ in the form $p - 5(x - q)^2$. *(3 marks)*

(e) State how the values of p and q are connected to your answers to part **(c)(i)** and part **(c)(ii)**. *(2 marks)*

(f) When the income is £800, find the values of x to three significant figures.

The solutions of $ax^2 + bx + c = 0$ are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$. *(3 marks)*

QUESTION PART REFERENCE



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Section B

Answer **all** questions in the spaces provided.

Use **Disease** *on page 2 of the Data Sheet.*

2 A model for the number of people, N, who have been infected is given by the equation

$$N = 400 - 300\,e^{-0.1t}$$

where t is the number of days since records of the disease were started.

(a) (i) Using the model, complete the table of values **opposite**. *(2 marks)*

(ii) **On the grid opposite**, draw the graph of the equation

$$N = 400 - 300\,e^{-0.1t} \quad \text{for} \quad 0 \leqslant t \leqslant 40$$ *(3 marks)*

(b) When does the model predict that the value of N will be 398? *(2 marks)*

(c) The actual number of people who have been infected by the 25th day is 360.

Calculate the percentage error in using the model to predict the number of people who have been infected by the 25th day. *(3 marks)*

(d) What is the maximum value of N predicted by this model? *(1 mark)*

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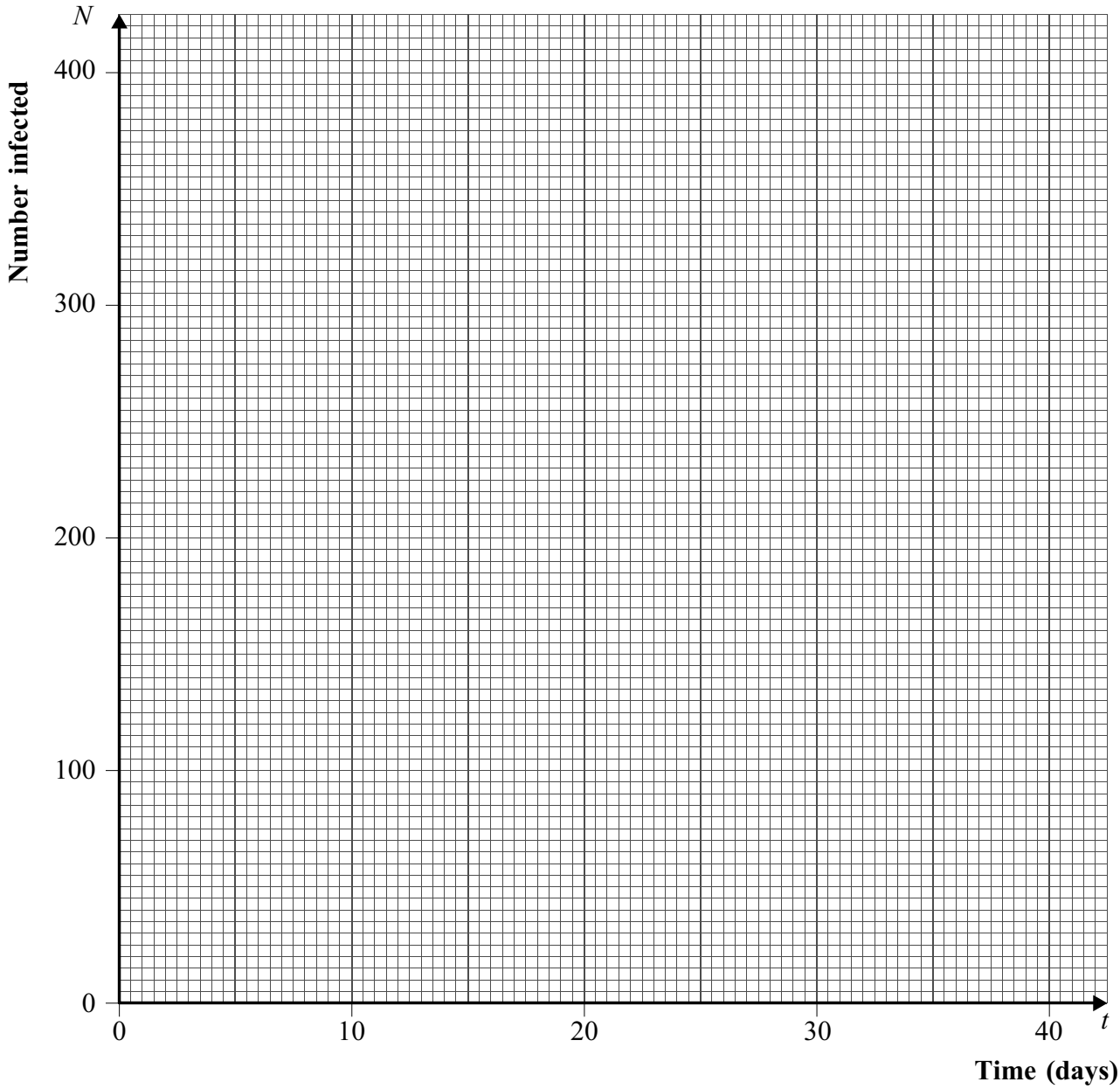
t	0	10	20	30	40
N	100				395



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3 Another model for the data is given by the equation $N = a + b\sqrt{t}$.

(a) Using the data values when $t = 0$ and $t = 40$, set up and solve two simultaneous equations to find the values of a and b. *(4 marks)*

(b) Use your model to predict the value of N when $t = 30$. *(2 marks)*

(c) What does this model predict when the value of t is very large? *(1 mark)*

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Section C

Answer **all** questions in the spaces provided.

Use **Nuclear reactors** *on page 3 of the Data Sheet.*

4 The number of nuclear reactors, R, that are operating in the world can be modelled by the equation

$$R = 450 \sin\left(\frac{90t}{50}\right)^\circ$$

where t is the number of years since 1950.

(a) **On the grid opposite**, draw the graph of $R = 450 \sin\left(\frac{90t}{50}\right)^\circ$ for $0 \leqslant t \leqslant 60$. *(4 marks)*

(b) (i) What is the maximum number of nuclear reactors predicted by this model? *(1 mark)*

(ii) In what year does this maximum occur? *(1 mark)*

(c) In what year does the model predict that there will be no nuclear reactors operating in the world? *(2 marks)*

(d) Describe fully the transformations that map the graph of the function $R = \sin(t)^\circ$ onto the graph of $R = 450 \sin\left(\frac{90t}{50}\right)^\circ$. *(3 marks)*

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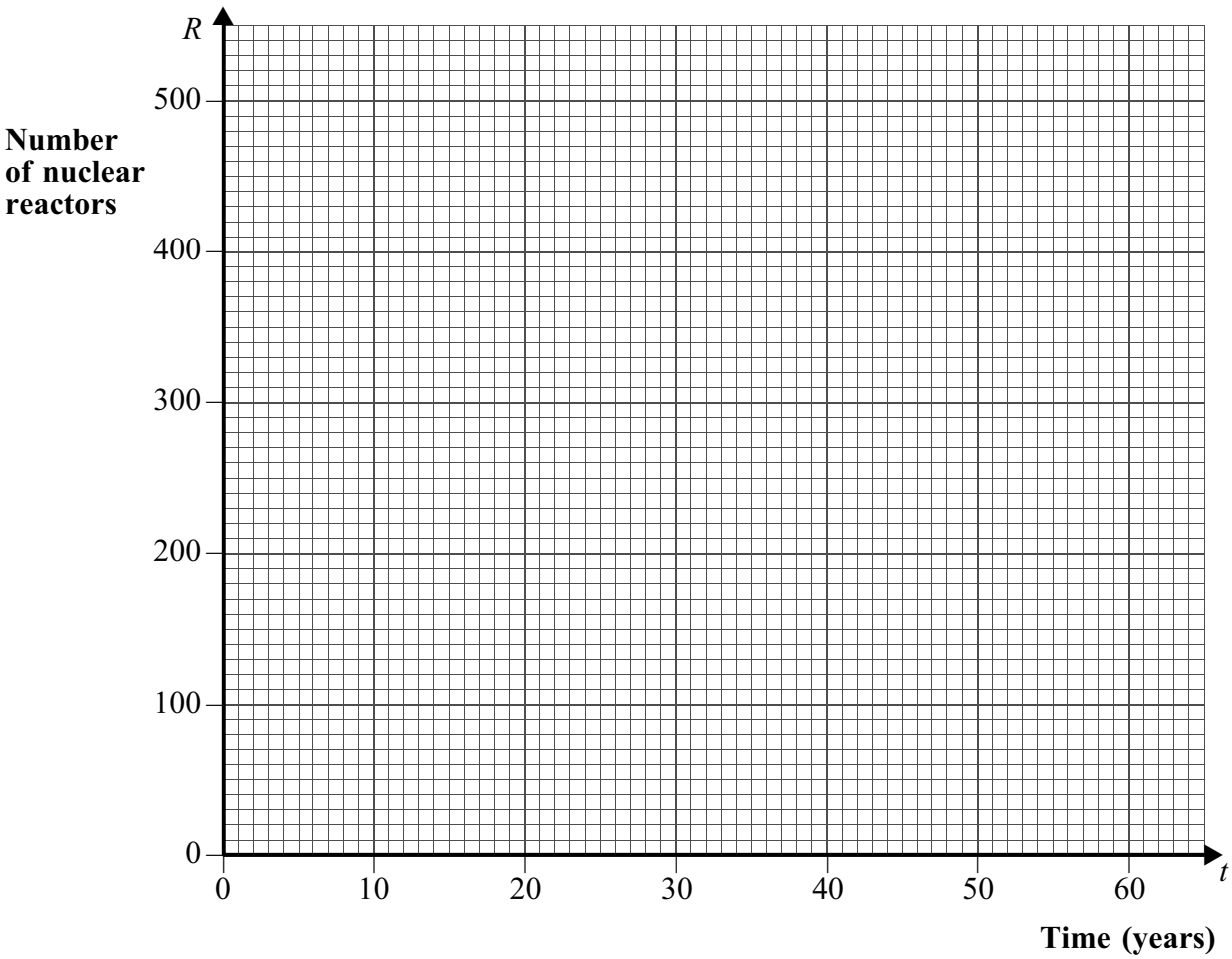

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Section D

Answer **all** questions in the spaces provided.

Use **Cinema admissions** *on page 3 of the Data Sheet.*

5 The number of cinema admissions, N millions, can be modelled by the equation

$$N = kt^c$$

where t is the number of years since 1954, and k and c are constants.

(a) For this model, show that $\ln N = \ln k + c \ln t$. *(1 mark)*

(b) Complete the table of values **opposite**, giving the values of $\ln N$ and $\ln t$ to three significant figures. *(2 marks)*

(c) **On the grid opposite**, plot $\ln N$ against $\ln t$.

Draw a line of best fit on your graph. *(2 marks)*

(d) Find the equation for the line of best fit in terms of $\ln N$ and $\ln t$. *(3 marks)*

(e) Hence express N in terms of t. *(2 marks)*

(f) **The graph printed on page 19** shows the number of cinema admissions from 1930 to 2005.

(i) Explain why the model used in part **(a)** to part **(e)** above is **not** suitable to describe the number of cinema admissions between 1930 and 1955. *(1 mark)*

(ii) Explain why the model used in part **(a)** to part **(e)** above is **not** suitable to describe the number of cinema admissions after 1985. *(1 mark)*

(g) (i) **Using the graph printed on page 19**, find the gradient of the graph in 1943. *(2 marks)*

(ii) State the units of the gradient. *(1 mark)*

(iii) What information does the gradient give? *(1 mark)*

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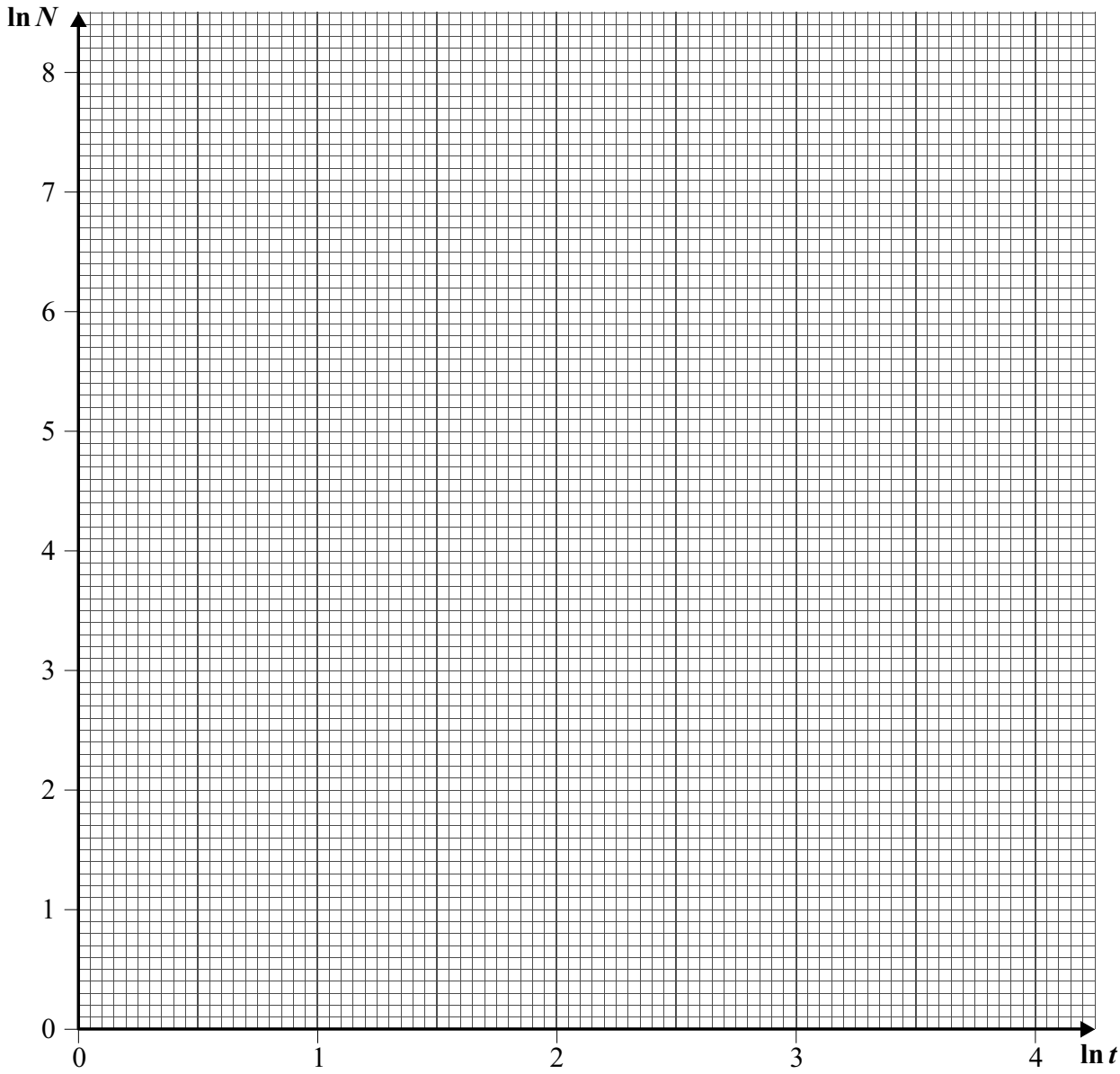
t	1	6	11	16	21	26	31
$\ln t$			2.40		3.04		
N	1182	280	167	120	99	81	72
$\ln N$			5.12		4.60		



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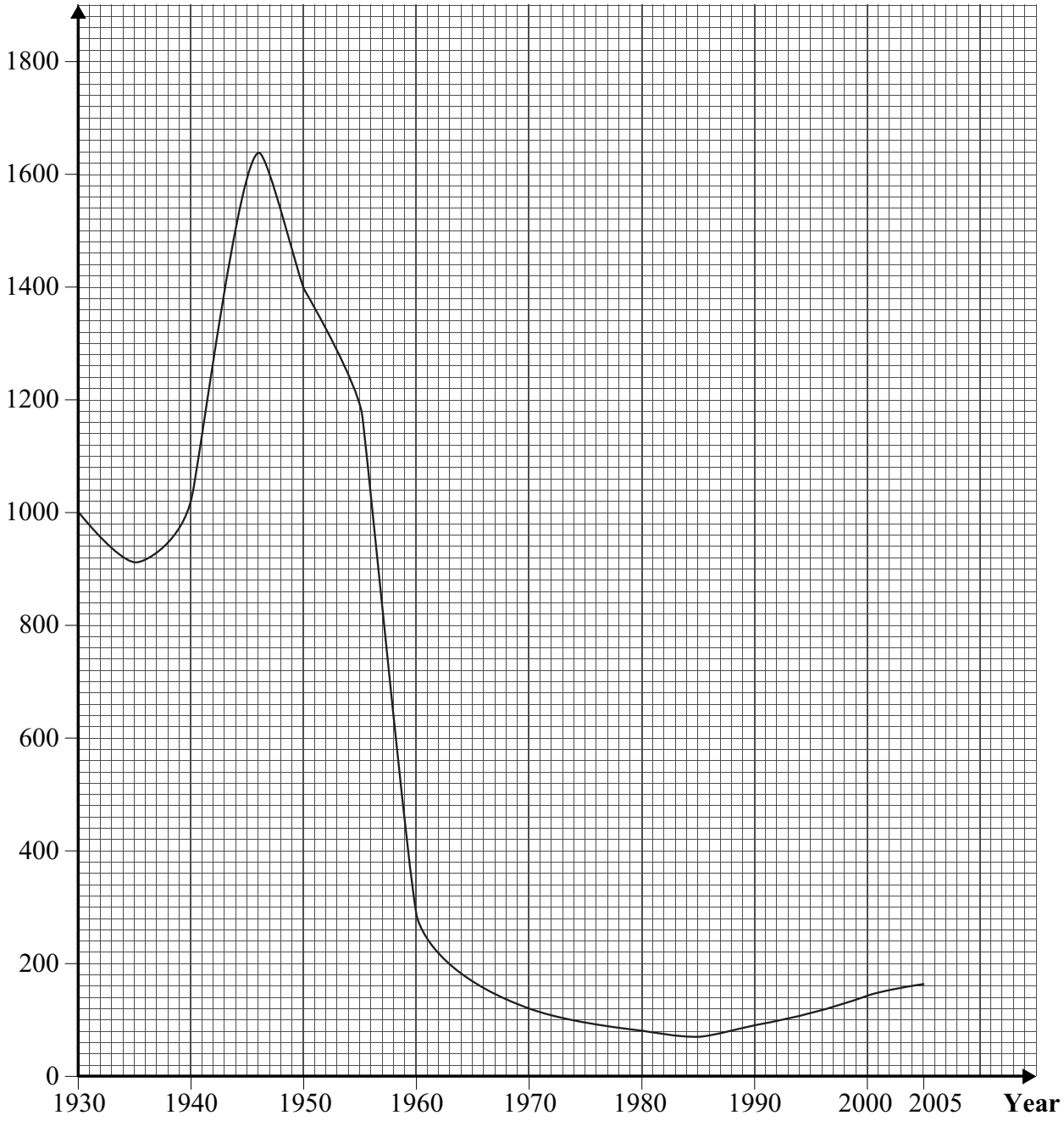

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END OF QUESTIONS