General Certificate of Education
June 2007
Advanced Subsidiary Examination

APPLYING MATHEMATICS **UOM4/2**
Paper 2

Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- an answer sheet for Questions 1, 2, 3 and 4 (enclosed)
- a ruler
- a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book **and** on the top of the answer sheet for Questions 1, 2, 3 and 4. The *Examining Body* for this paper is AQA. The *Paper Reference* is UOM4/2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of a calculator should normally be given to three significant figures.
- At the end of the examination, remember to hand in both your answer book **and** the answer sheet for Questions 1, 2, 3 and 4.

Information

- The maximum mark for this paper is 70.
- The marks for questions are shown in brackets.
- You will be awarded up to 3 marks for your ability to present information accurately using correct notation **and** up to 3 marks for mathematical arguments presented clearly and logically.

SECTION A

Answer **all** questions.

1 Route planning software that can be used on a computer classifies roads into three different types. It calculates the time taken for journeys by assuming that a motorist will travel at a different average speed on each type of road:

average speed on motorways: 60 miles per hour;
average speed on dual carriageways: 50 miles per hour;
average speed on single carriageway roads: 40 miles per hour.

A motorist plans to travel from home to London. The software gives two different possible routes:

route A: 160 miles on single carriageway roads;

route B: 105 miles on motorways followed by 75 miles on dual carriageways.

(a) Calculate how long the software would calculate that the motorist would take using:

(i) Route A;

(ii) Route B. *(4 marks)*

(b) On the grid on the answer sheet, plot distance travelled from home against time for a motorist who plans to arrive in London at 4 pm, using:

(i) Route A;

(ii) Route B. *(5 marks)*

(c) The motorway section of Route B can be represented by an equation $d_B = a + bt$ where d_B is the distance travelled from home in miles in terms of time t, the number of hours after 12 noon.

(i) Find numerical values for parameters a and b. *(3 marks)*

(ii) Interpret the physical significance of parameter b. *(2 marks)*

SECTION B

Answer **all** questions.

2 The function $m = m_0 e^{kt}$ can be used to model the mass, m grams, of radioactive substance remaining in an organism t years after it died. When it died, the mass of radioactive substance was m_0.

For the radioactive substance carbon-14, $k = -0.000121$.

An organism contained 1 gram of carbon-14 when it died. Therefore, the mass, m grams, of carbon-14, t years after it died, will be given by $m = e^{-0.000121\,t}$.

(a) Show that it takes approximately 5730 years for the amount of carbon-14 in this organism to decay to half its starting value, that is 0.5 grams. *(2 marks)*

(b) Find the mass of carbon-14 remaining in this organism after a further 5730 years. *(2 marks)*

(c) For this decay, sketch a graph of m against t on the grid given on the answer sheet. *(3 marks)*

(d) Compare the gradient of your graph when $t = 0$ with its gradient when $t = 20\,000$. Interpret your answer. *(2 marks)*

(e) Find the number of years after which the mass of carbon-14 remaining in the organism will be 0.75 grams by solving $0.75 = e^{-0.000121\,t}$. *(4 marks)*

(f) For a different radioactive substance with an initial mass of 1 gram, $m = e^{-0.0005\,t}$.

Write a brief description of how the decay of this substance would be different from that of carbon-14. *(2 marks)*

Turn over for the next question

SECTION C

Answer **all** questions.

3 Maria has completed a university course and has a student loan of £10 000 to repay. Each year, Maria has to pay back 9% of anything she earns over £15 000 during that year.

Throughout this question, you may assume that interest is not charged on a student loan.

Maria earns £25 000 in the first year and gets an increase in her earnings of £2000 per year in each subsequent year.

The following recurrence relations are used to model this situation until the loan is repaid:

- $E_n = E_{n-1} + 2000$ gives Maria's earnings each year $(E_1 = 25\,000)$;
- $R_n = 0.09(E_n - 15\,000)$ gives the amount Maria repays each year;
- $L_n = L_{n-1} - R_n$ gives the amount of Maria's loan that remains at the end of each year $(L_0 = 10\,000)$.

(a) Given that $E_1 = 25\,000$ and $L_0 = 10\,000$, show that:

(i) $R_1 = 900$;

(ii) $L_1 = 9100$. *(4 marks)*

(b) Use the recurrence relations to complete the table on the answer sheet. *(6 marks)*

(c) How long will it take Maria to pay back her student loan? *(1 mark)*

(d) Complete the graph of loan remaining, L_n, plotted against year, n, on the grid on the answer sheet. *(2 marks)*

(e) Explain why the points on your graph do not lie on a straight line. *(2 marks)*

SECTION D

Answer **all** questions.

4 A bank manager carries out a simulation to find out whether a single queue should replace the multiple queues that customers currently form in his branch when waiting to see one of three cashiers.

The bank manager carries out a survey and uses this to set up the simulation. The table below shows how integers generated randomly between 0 and 9 inclusive are assigned by the manager to simulate how long customers take to complete their transactions.

Time to complete transaction in minutes	Random integer assigned
3	0, 1
4	2, 3, 4
5	5, 6, 7, 8
6	9

(a) (i) Write down the probability that a customer takes 5 minutes to complete their transaction. *(1 mark)*

(ii) Explain how you deduced your answer. *(1 mark)*

(iii) Complete **Table 1** on the answer sheet, using the given random numbers, to show how long each customer takes to complete their transaction at the bank. *(2 marks)*

Question 4 continues on the next page

(b) First of all, the manager simulates the multiple queuing system in which customers join one of three queues as they arrive at the bank.

To run the simulation, the manager assumes that:

- a single customer arrives at the bank every minute;
- customers immediately join the shortest queue available;
- if there is more than one queue of the same shortest length, the customer joins the queue to see the lowest numbered cashier;
- no time is used between a customer finishing a transaction and the next starting;
- no customer changes queue after having joined one.

Table 2 on the answer sheet has been completed for the first six customers, A–F, showing where each of them is from when they arrive at the bank until they have completed their transactions and leave.

(i) Customer D finds this system of queuing annoying. Explain why this is the case. *(2 marks)*

(ii) Use information from **Table 1** to complete **Table 2** for the next eight customers that arrive at the bank. *(4 marks)*

(c) For the alternative simulation, there is a single queue which each customer joins when they enter the bank if one of the cashiers is not immediately available.

Table 3 on the answer sheet has been completed for the first six customers, A–F, showing where each of them is from when they arrive at the bank until they have completed their transactions using the alternative simulation.

(i) Does this queuing system improve the situation for customer D? Explain your answer. *(2 marks)*

(ii) Use information from **Table 1** to complete **Table 3** for the next eight customers that arrive at the bank. *(4 marks)*

(d) Would you advise the bank manager to introduce the single queuing system? Use evidence from the simulations to support your answer. *(2 marks)*

(e) Give **two** ways in which the simulations could be improved. *(2 marks)*

END OF QUESTIONS

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