General Certificate of Education June 2004 Advanced Subsidiary Examination



# APPLYING MATHEMATICS Paper 2

**UOM4/2** 

Monday 24 May 2004 Morning session

#### In addition to this paper you will require:

- an 8-page answer book;
- an answer sheet for Questions 4 and 5 (enclosed);
- · a graphics calculator;
- a ruler.

Time allowed: 1 hour 30 minutes

#### **Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book **and** on the top of the answer sheet for Ouestions 4 and 5.
- The Examining Body for this paper is AQA. The Paper Reference is UOM4/2.
- Answer all questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of a calculator should normally be given to three significant figures.
- At the end of the examination, remember to hand in both your answer book **and** the answer sheet for Questions 4 and 5.

#### **Information**

- The maximum mark for this paper is 70.
- Mark allocations are shown in brackets.

#### SECTION A

#### **Body Mass Index**

Answer all questions.

- 1 Body Mass Index, BMI, is used to indicate whether people are under- or over-weight. The BMI of a person of height h metres and mass m kilograms is  $\frac{m}{h^2}$ .
  - (a) For people of height 1.8 metres, the BMI is  $\frac{m}{1.8^2}$ .
    - (i) For such people, **sketch** a graph of BMI against mass.

(2 marks)

- (ii) On your graph add a line for people of height 2 metres. Indicate clearly which line belongs to which group of people. (2 marks)
- (b) For people of mass 75 kilograms the BMI is  $\frac{75}{h^2}$ .
  - (i) For such people, **sketch** a graph of BMI against height.

(2 marks)

- (ii) On your graph add a curve for people of mass 90 kilograms. Indicate clearly which curve belongs to which group of people. (2 marks)
- (c) For people of BMI 20, **sketch** a graph of *m* against *h*.

(2 marks)

(d) For a person of height  $h_1$  centimetres and mass m kilograms, show that his or her BMI is  $\frac{10000 \, m}{h_1^2}$ .

(2 marks)

#### **SECTION B**

#### **Traffic accidents**

Answer all questions.

2 When a driver applies a car's brakes in an emergency, the car may skid. By measuring the length, d metres, of the skid marks, traffic accident investigators can estimate the speed, v metres per second, of the car.

The investigators use the formula  $v^2 = 20\mu d$ , where  $\mu$  has a value that depends on the friction between the car's tyres and the road.

In an accident in an area with a 30 mph speed limit, a car leaves skid marks of length 28 metres. Investigators do an experiment with this car and find that, when it makes an emergency stop from 10 metres per second, it leaves skid marks of length 17.5 metres.

(a) Use this experimental data to find a value of  $\mu$ .

(3 marks)

- (b) Find the speed that the motorist was travelling at when she applied the brakes in the accident.

  (3 marks)
- (c) Was the motorist breaking the speed limit? You should use 1 mph = 0.447 metres per second. Explain your reasoning. (4 marks)
- (d) A traffic investigator suggests that a mistake has been made in measuring the length of the **experimental** skid marks, so that the value of d should, in fact, have been greater than 17.5.

What effect might this mistake have had on your answers to parts (b) and (c)? Explain your reasoning carefully. (4 marks)

TURN OVER FOR THE NEXT QUESTION

#### **SECTION C**

#### Cooling cup of tea

Answer all questions.

3	The temperature,	$T^{\circ}C$ ,	of a cup	of tea,	t	minutes	after	it is	made,	may	be	modelled	by	the	function
	$T = 78e^{-0.02t} + 18$	8.													

(a) What is the temperature of the cup of tea when it is first made? (2 marks)

(b) What will be the temperature of the cup of tea after a long time? (2 marks)

(c) Sketch a graph of the function  $T = 78e^{-0.02t} + 18$ . (3 marks)

- (d) Explain carefully the geometrical transformations you would need to apply to a graph of  $T = e^{-0.02t}$  to give the graph of  $T = 78e^{-0.02t} + 18$ .
- (e) More generally, the temperature of a cooling cup of tea can be modelled by the function  $T = Be^{-0.02t} + A$ .

Interpret what is represented by each of the parameters:

(i) *A*;

(ii) B. (4 marks)

#### SECTION D

#### **Olympics**

Answer all questions.

4 A chocolate manufacturer sells three types of chocolate bar *A*, *B* and *C*. In an "Olympics" promotion, customers are given either a Gold, Silver or Bronze token with each bar they buy. When a customer has collected a Gold token, a Silver token and a Bronze token, he or she can claim an "Olympic" medal.

To find out how many bars customers will have to buy to be able to claim their medals, the manufacturer runs a simulation. In this, it is assumed that customers buy chocolate bars of types A, B and C randomly

with the following probabilities:  $A:\frac{2}{10}$ ,  $B:\frac{3}{10}$ , and  $C:\frac{5}{10}$ .

- (a) Describe how you would assign integers randomly generated between 0 and 9 (inclusive) to simulate which bar a customer buys. (2 marks)
- (b) Gold, Silver and Bronze tokens are distributed randomly in each chocolate bar of **type** *A* in the following proportions 1:2:7.

For type A chocolate bars, describe how you would assign integers randomly generated between 0 and 9 (inclusive) to simulate which token a customer receives. (2 marks)

(c) In **type B** chocolate bars, Gold, Silver and Bronze tokens are distributed randomly in fixed proportions. Randomly generated integers between 0 and 9 (inclusive) are assigned to simulate the type of token a customer receives in bars of type **B**. Here are some results from such a simulation:

0 = Gold 9 = Bronze 9 = Bronze 5 = Bronze 4 = Bronze 1 = Gold 3 = Silver 7 = Bronze 2 = Silver 2 = Silver 2 = Silver 5 = Bronze 5 = Bronze 2 = Silver 8 = Bronze 5 = Bronze 5 = Bronze 5 = Bronze 6 = Bronze 7 = Bronze 7 = Bronze 8 =

State the proportions of Gold, Silver and Bronze tokens in bars of type B. (2 marks)

(d) In **type** *C* chocolate bars, Gold, Silver and Bronze tokens are distributed randomly in the ratio 1:3:6. Randomly generated integers between 0 and 9 (inclusive) are assigned to simulate the type of token a customer receives in bars of type *C*. Here are some results from such a simulation:

3 = Bronze 1 = Bronze 9 = Gold 0 = Bronze 7 = Silver 6 = Silver 4 = Bronze 2 = Bronze 0 = Bronze 4 = Bronze 5 = Bronze

What token does 8 represent?

(2 marks)

(e) In a full simulation, a pair of randomly generated integers (x, y) are used, where x simulates the type of chocolate bar (A, B, C) and y simulates the type of token (Gold, Silver, Bronze).

Use your rules from parts (a) and (b) and the rules given in parts (c) and (d) to complete the table on the answer sheet using the given randomly generated pairs of integers. (4 marks)

(f) The manufacturer wants to make sure that customers will, on average, have to buy fewer bars to collect sufficient tokens to claim a medal. Suggest how this might be achieved. (2 marks)

Turn over

#### **SECTION E**

#### Tree growth

Answer all questions.

- 5 A tree grows so that each year its cross-sectional area increases. After n years, its cross-sectional area is  $A_n \text{cm}^2$ . This growth may be modelled assuming that  $A_{n+1} = A_n + A_1$ .
  - (a) (i) Show that  $A_2 = 2 A_1$ . (2 marks)
    - (ii) Find  $A_3$  and  $A_4$  in terms of  $A_1$ . Hence write  $A_n$  in terms of  $A_1$ . (3 marks)
  - (b) Assuming that the cross-section of the tree is circular with diameter  $d_n$  cm after n years, then  $A_n = \frac{1}{4}\pi d_n^2.$ 
    - (i) Show that  $d_2^2 = 2d_1^2$ . (2 marks)
    - (ii) Show clearly how the formula  $d_{n+1} = \sqrt{d_n^2 + d_1^2}$  can be derived from  $A_{n+1} = A_n + A_1$ . (2 marks)
    - (iii) For a tree for which  $d_1 = 8$  cm, complete the table on the answer sheet. Give all the values correct to **two** decimal places. (3 marks)
    - (iv) Suggest **two** reasons why in reality the growth of the diameter of a tree trunk might not be as described above. (3 marks)

#### **END OF QUESTIONS**

# THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

# THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

Surname	Other N	Names						
Centre Number	(	Candidate						
Candidate Signature								

General Certificate of Education June 2004 Advanced Subsidiary Examination

# ASSESSMENT and QUALIFICATIONS ALLIANCE

# APPLYING MATHEMATICS Paper 2

UOM4/2AS

This answer sheet is to be used when answering Questions 4 and 5, as indicated.

Fasten this sheet securely to your answer book.

## This table is to be used when answering Question 4.

(e)

Random pair (x, y)	Type of chocolate bar	Token
(0,9)		
(5,0)		
(6,7)		
(7,1)		
(4,1)		

## This table is to be used when answering Question 5.

# (b)(iii)

n	$d_n$ cm
1	8.00
2	
3	
4	
5	17.89
6	19.60