

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education

AS GCE STATISTICS (MEI)

Unit Z1

G241

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

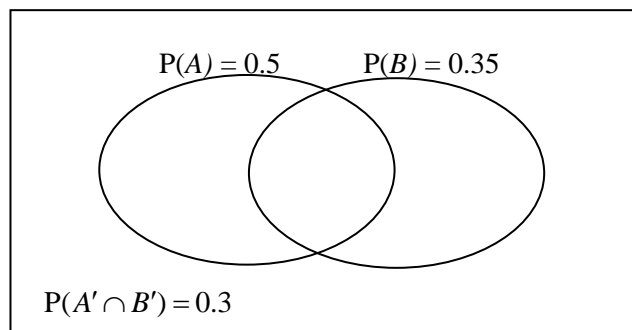
- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

- 1 The diagram illustrates the occurrence of two events A and B .



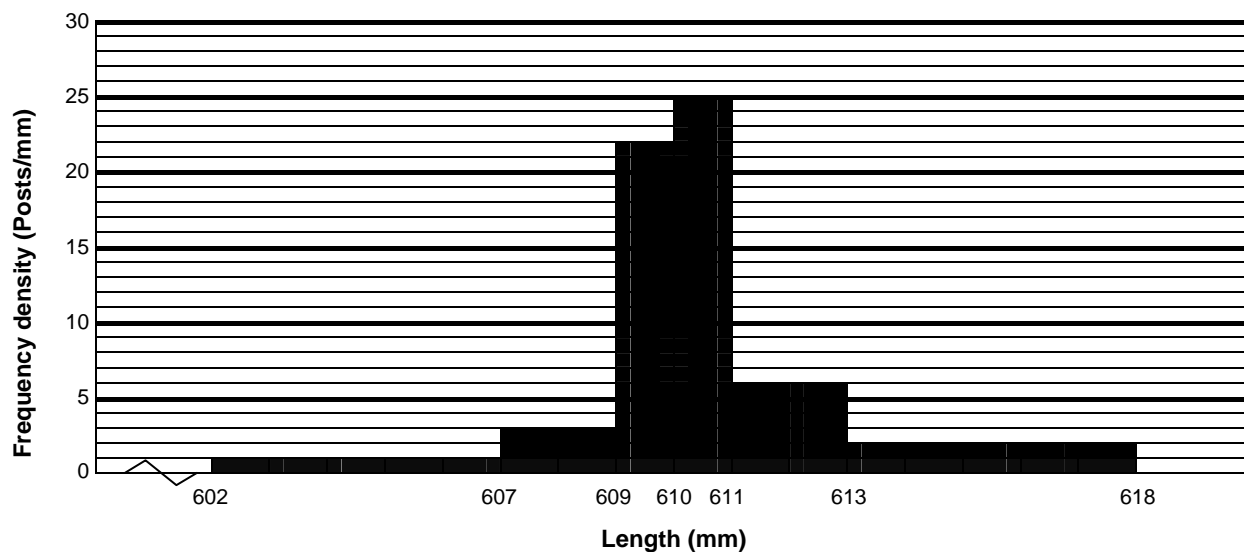
You are given these probabilities:

that A occurs	0.5,
that B occurs	0.35,
that neither A nor B occurs	0.3.

Find the probability that both A and B occur.

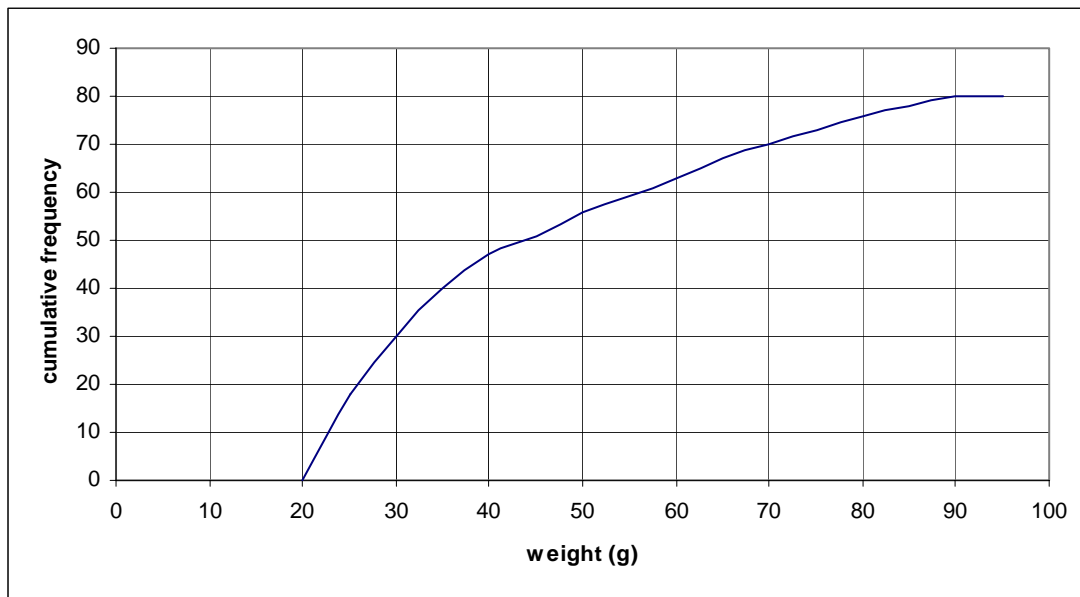
[3]

- 2 A sawmill cuts wooden posts which should be 610 mm long. They measure the lengths of a sample of 80 posts. Their lengths are illustrated in the histogram below.



- (i) State the number of posts in each of the classes used in the histogram. [3]
- (ii) What can you say about the range of the lengths of the posts in the sample? [1]
- (iii) Without doing any further calculations, explain why an estimate of the mean will be greater than 610 mm. [2]
- 3 In a group of 36 blood donors, 16 are male and 20 are female. Four of these people are chosen at random for an interview.
- (i) In how many ways can they be chosen? [2]
- (ii) Find the probability that they are all of the same sex. [3]

- 4 As part of a survey of fish stocks in a river, 80 specimens of a particular type of fish are trapped and weighed.
The results are shown on the cumulative frequency graph below.



- (i) Find the median and quartiles of the distribution. [2]
- (ii) Draw a box and whisker plot to illustrate the distribution. [2]
- (iii) Comment on the shape of the distribution and draw a rough sketch of it. [3]

- 5 A train operating company does a survey of the time-keeping of a particular train over the working days in two weeks.
The results for this sample are shown in Table 5.1 below.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	0	2	3	0	5
Week 2	6	1 early	32	0	3

Table 5.1: Minutes late

- (i) Calculate:
- (A) the mean; [1]
- (B) the root mean square deviation; [2]
- (C) the standard deviation. [1]

of these data.

- (ii) Use your results from part (i) to justify classifying the figure for Week 2 Wednesday as an outlier. [2]
- (iii) The delay on Week 2 Wednesday was caused by a security alert.
The train operating company says this was not their fault and so removes the outlier from the data set.
What effect does this have on the mean and standard deviation? [2]

- 6 The number, X , of occupants of cars coming into a city centre is modelled by the probability distribution $P(X = r) = \frac{k}{r}$ for $r = 1, 2, 3, 4$.

- (i) Tabulate the probability distribution and determine the value of k . [3]
- (ii) Calculate $E(X)$ and $\text{Var}(X)$. [4]

Section B (36 marks)

- 7 Wendy is an amateur weather forecaster.
She classifies the weather on a day as either wet or fine.
From past records she suggests that:
- if a day is wet then the probability that the next day is wet is 0.5,
 - if a day is fine then the probability that the next day is fine is 0.8.
- In a particular week, it is wet on Monday.
- (i) Draw a probability tree diagram for wet or fine days on Tuesday, Wednesday and Thursday. [4]
- (ii) Find the probability that Tuesday, Wednesday and Thursday all have the same weather. [3]
- (iii) Find the probability that the weather is wet on Thursday. [4]
- (iv) Find the probability that it is fine on Tuesday and wet on Thursday. [3]
- (v) Given that it is wet on Thursday, find the conditional probability that it was fine on Tuesday. [3]

8 A police road-safety team examines the tyres of a large number of commercial vehicles. They find that 17% of lorries and 20% of vans have defective tyres.

(i) Six lorries are stopped at random by the road-safety team. Find the probability that:

- (A)** none of the lorries has defective tyres, [2]
- (B)** exactly two lorries have defective tyres, [3]
- (C)** more than two lorries have defective tyres. [3]

Following a road-safety campaign to reduce the proportion of vehicles with defective tyres, 18 vans are stopped at random and their tyres are inspected.

Just one of the vans has defective tyres.

You are to carry out a suitable hypothesis test to examine whether the campaign appears to have been successful.

- (ii)** State your hypotheses clearly, justifying the form of the alternative hypothesis. [3]
- (iii)** Carry out the test at the 5% significance level, stating your conclusions clearly. [4]
- (iv)** State, with a reason, the critical value for the test. [2]
- (v)** Give a level of significance such that you would come to the opposite conclusion for your test. Explain your reasoning. [2]

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**Advanced Subsidiary General Certificate of Education
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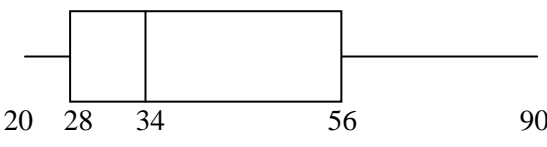
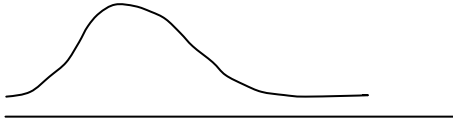
AS GCE STATISTICS (MEI)

Unit Z1

G241

MARK SCHEME

Qu	Answer	Mark	Comment																
Section A																			
1	$P(A \cup B) = 1 - 0.3 = 0.7$ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= 0.5 + 0.35 - 0.7$ $= 0.15$	B1 M1 A1 [3]																	
2(i)	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;"><u>Length</u></th> <th style="text-align: right;"><u>Frequency</u></th> </tr> </thead> <tbody> <tr> <td>602 to 607</td> <td style="text-align: right;">5</td> </tr> <tr> <td>607 to 609</td> <td style="text-align: right;">6</td> </tr> <tr> <td>609 to 610</td> <td style="text-align: right;">22</td> </tr> <tr> <td>610 to 611</td> <td style="text-align: right;">25</td> </tr> <tr> <td>611 to 613</td> <td style="text-align: right;">12</td> </tr> <tr> <td>613 to 618</td> <td style="text-align: right;"><u>10</u></td> </tr> <tr> <td>Total</td> <td style="text-align: right;">80</td> </tr> </tbody> </table>	<u>Length</u>	<u>Frequency</u>	602 to 607	5	607 to 609	6	609 to 610	22	610 to 611	25	611 to 613	12	613 to 618	<u>10</u>	Total	80	B1 B1 B1 [3]	For 5 and 10 For 6 and 12 For figures with total 80
<u>Length</u>	<u>Frequency</u>																		
602 to 607	5																		
607 to 609	6																		
609 to 610	22																		
610 to 611	25																		
611 to 613	12																		
613 to 618	<u>10</u>																		
Total	80																		
2(ii)	The range lies between 6 and 16	B1 [1]																	
2(iii)	Mean is estimated as $\frac{\sum (\text{Mid-point} \times \text{Frequency})}{\text{Total}}$ The intervals are symmetrically placed either side of 610 but in each case the frequency on the right is greater	B1 B1 [2]	Allow 1 mark for each of two Sensible statements																
3(i)	Number of ways 4 may be chosen from 36 $= {}^{36}C_4 = 58905$	M1 A1 [2]	${}^{36}C_4$ term																
3(ii)	$P(\text{All of same sex}) = P(\text{All male}) + P(\text{All female})$ $= \frac{16}{36} \times \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} + \frac{20}{36} \times \frac{19}{35} \times \frac{18}{34} \times \frac{17}{33}$ $= 0.113$ (3 s.f.)	M1 M1 A1 [3]	Attempt at correct numbers cao																

Qu	Answer	Mark	Comment
Section A (continued)			
4(i)	Median = 34 Upper quartile = 56 Lower quartile = 26	B1 B1 [2]	Median Quartiles
4(ii)		B1 B1 [2]	Box Whiskers
4(iii)	Positive skew 	B1 B1 B1 [3]	1 mark for skew 1 mark for positive Sketch
5(i)(A)	$\bar{x} = \frac{50}{10} = 5$	B1	
5(i)(B)	$\sum (x - \bar{x})^2 = 858 \Rightarrow rmsd = \sqrt{\frac{858}{10}} = 9.26$	B1 B1	For 858 seen cao
5(i)(C)	$s = \sqrt{\frac{858}{9}} = 9.76$	B1 [4]	For division by 9
5(ii)	$\bar{x} + 2s = 5 + 2 \times 9.76 = 24.52$ Since $32 > 24.52$, 32 may be classified as an outlier	M1 E1 [2]	
5(iii)	Without the 32, $\bar{x} = \frac{18}{9} = 2$, $s = \sqrt{\frac{48}{8}} = 2.45$ Both the mean and standard deviation are much reduced	B1 B1 [2]	One mark both

Qu	Answer	Mark	Comment										
Section A (continued)													
6(i)	<table border="1"> <tr> <td>r</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X=r)$</td> <td>k</td> <td>$\frac{1}{2}k$</td> <td>$\frac{1}{3}k$</td> <td>$\frac{1}{4}k$</td> </tr> </table>	r	1	2	3	4	$P(X=r)$	k	$\frac{1}{2}k$	$\frac{1}{3}k$	$\frac{1}{4}k$	M1	Tabulation (SOI)
	r	1	2	3	4								
$P(X=r)$	k	$\frac{1}{2}k$	$\frac{1}{3}k$	$\frac{1}{4}k$									
	<p>Now $k + \frac{1}{2}k + \frac{1}{3}k + \frac{1}{4}k = 1$</p> $\Rightarrow k = \frac{12}{25} = 0.48$	M1 A1 [3]	Value of k										
6(ii)	$E(X) = 1 \times 0.48 + 2 \times 0.24 + 3 \times 0.16 + 4 \times 0.12$ $= 1.92$	B1	$E(X)$ (provided $\sum p = 1$)										
	$E(X^2) = 1 \times 0.48 + 4 \times 0.24 + 9 \times 0.16 + 16 \times 0.12$	M1	$E(X^2)$ ($\sum p = 1$)										
	<p>Hence $\text{Var}(X) = E(X^2) - [E(X)]^2$</p> $= 4.8 - 1.92^2$ $= 4.8 - 3.6864$ $= 1.1136 \text{ or } 1.11 \text{ (to 3 s.f.)}$	M1 A1 [4]	Positive variance cao										
			Section A Total: 36										

Qu	Answer	Mark	Comment
Section B			
7(i)		B1 B1 B1 B1 [4]	Overall shape 1 st pair branches 2 nd set branches 3 rd set branches
7(ii)	P(same weather on Tuesday, Wednesday, and Thursday) $= 0.5^3 + 0.5 \times 0.8^2 = 0.445$	M1 M1 A1 [3]	2 triple products Sum of products cao
7(iii)	P(wet Thursday) $= 0.5^3 + 0.5^2 \times 0.2 + 0.5^2 \times 0.2 + 0.5 \times 0.8 \times 0.2$ $= 0.305$	M1 A1 M1 A1 [4]	4 triples Correct triples Sum of products cao
7(iv)	P(fine Tuesday and wet Thursday) $= 0.5 \times 0.2 \times 0.5 + 0.5 \times 0.8 \times 0.2$ $= 0.13$	M1 A1 A1 [3]	2 triples
7(v)	P(fine Tuesday wet Thursday) Use of $P(A B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{0.13}{0.305}$ $= 0.426$ (3 s.f.) or $\frac{26}{61}$	M1 A1 A1 [3]	Numerator and denominator cao

Qu	Answer	Mark	Comment
Section B (continued)			
8(i)(A)	P(no lorries have defective tyres) = $0.83^6 = 0.327$ (3 s.f.) = 0.33 (2 s.f.)	M1 A1 [2]	cao
8(i)(B)	P(exactly 2 lorries have defective tyres) = ${}^6C_2 \times 0.17^2 \times 0.83^4$ = 0.206 (to 3 s.f.) = 0.21 (2 s.f.)	M1 M1 A1 [3]	For $0.17^2 \times 0.83^4$ For ${}^6C_2 \times \dots$ cao
8(i)(C)	P(1 lorry has defective tyres) = ${}^6C_1 \times 0.17 \times 0.83^5$ = 0.402 (to 3 s.f.) P(more than 2 lorries have defective tyres) = $1 - (0.327 + 0.402 + 0.206)$ = $0.065(5)$	B1 M1 A1 [3]	
8(ii)	$H_0 : p = 0.2$ $H_1 : p < 0.2$ H_1 takes this form because we are looking for a <i>reduction</i> in the proportion of defective tyres	B1 B1 E1 [3]	Null hypothesis Alternative hyp. Explanations
8(iii)	Let $X \sim B(18, 0.2)$ $P(X \leq 1) = 0.0991$ Since $0.0991 > 0.05$, do not reject H_0 (or accept H_0) There is not enough evidence to suggest that there has been a (significant) reduction in the proportion of defective tyres or 'campaign appears to have been successful'	M1 B1 M1 A1 [4]	Tail probability Comparison Conclusion in words
8(iv)	The critical value for the test is 0, since $P(X \leq 0) [= 0.018] < 0.05$	B1 B1 [2]	Critical value Reason
8(v)	The opposite conclusion would be reached provided the significance level was above 9.91%, e.g. 10%	B1 E1 [2]	Suitable percentage Explicit comparison with 9.91%
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number							
			1	2	3	4	5	6	7	8
1	14-22	19	1	1	2	2	1	4	4	4
2	14-22	18	1	2	1	3	1	3	4	3
3	18-26	21	-	-	2	-	2	-	8	9
4	7-15	8	-	3	-	2	2	-	-	1
5	3-11	6	1	-	-	-	2	-	1	2
Totals		72	3	6	5	7	8	7	17	19

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AS GCE STATISTICS (MEI)

Unit Z2

G242

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

- 1 The concentration of a particular chemical in a healthy person's blood varies from person to person. It is Normally distributed with mean 24 and standard deviation 2.6 (in suitable units of concentration).

Medical researchers are investigating whether consumption of a certain food increases the mean concentration of this chemical.

They have already established that it does not change the standard deviation and that the distribution remains Normal.

Ten healthy volunteers go on a specific diet including this food and undergo tests.

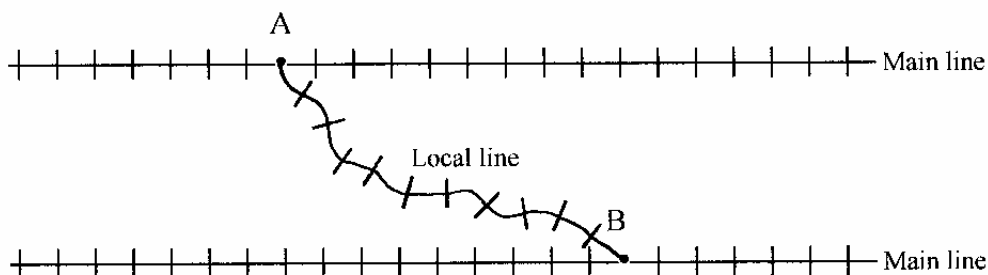
They may be taken to form a random sample.

Their blood concentrations of the chemical are as follows (measured in the same unit as above).

23.8 25.2 24.7 26.1 23.4 28.2 22.1 27.9 29.6 25.5

- (i) Examine, at the 5% level of significance, whether the data provide evidence that the mean concentration for people on this diet exceeds 24, stating clearly your null and alternative hypotheses. **[10]**
- (ii) Provide a two-sided 95% confidence interval for the mean concentration for people on this diet. **[4]**

- 2 The diagram below illustrates two main railway lines and a local line which connects station A on one main line to station B on the other.



A local train leaves station A punctually at 10:07 hours.

A main-line train is scheduled to arrive at A at 10:00 hours, but in fact its arrival time is Normally distributed about the scheduled time with standard deviation 5 minutes.

Transfer of passengers between the trains may be regarded as instantaneous.

- (i) Find the probability that the connection is made. [4]
- (ii) Find the probability that, in a 5-day working week, there is at least one day on which the connection is **not** made. [3]

The local train travels to station B, its journey time being Normally distributed with mean 30 minutes and standard deviation 2 minutes.

A main-line train departs from this station punctually at 10:42 hours.

Transfer of connecting passengers here may also be regarded as instantaneous.

- (iii) Show that the probability of making the connection at B is 0.9938. [2]
- (iv) The company running the local train consider changing its departure to a new time. They want the probability of making the connection at A to be at least 0.99. Find the earliest suitable scheduled departure time of the local train (which must still be an exact minute, e.g. 10:08, 10:09 hours). [4]

The local line between A and B is managed by a small train operating company which is unable to influence the timetables for the main-line trains through A and B.

- (v) Discuss briefly, and without doing any further calculations, the problems this company has in maintaining the connections between its local train and the two main-line trains. [You may use the additional information that if the departure time from A is changed to 10:12, the probability of making the connection at B is 0.5.] [2]

- 3 A researcher is studying the distribution of a particular species of tree over a certain area of countryside. She divides the area into 144 sub-regions of equal size, and counts the number of trees in each of them. The results are given in the following frequency distribution.

Number, x, of trees in a sub-region	0	1	2	3	4	5	6	7	>7
Frequency	66	26	7	22	10	10	1	2	0

The researcher believes the distribution of the underlying random variable X can be modelled by a Poisson distribution.

- (i) State the conditions under which the Poisson distribution is appropriate. [2]
- (ii) Discuss briefly, without reference to any statistical tests or procedures, whether you think the data in this question have been drawn from a Poisson distribution. [2]
- (iii) Verify that the sample mean, \bar{x} , is 1.5 and find the sample variance of the data in the table above.
Do the results of these calculations give you any reason to doubt the appropriateness of the Poisson distribution as a model? [4]
- (iv) Take the mean you have calculated in part (iii) as the *true* mean of the probability distribution in part (i).
Use the appropriate cumulative tables to find the probabilities that $X = x$ for $x = 0, 1, 2, 3, 4, 5, 6, 7$ and for $x > 7$.
Hence calculate the expected frequencies corresponding to the observed frequencies in the table above. [5]
- (v) Without carrying out any formal test, discuss briefly whether the expected frequencies are a sufficiently close match to the observed frequencies to suggest that the model is reasonable. State the name of the statistical test that could be used to examine this formally. [3]

- 4 A store has recently introduced a loyalty card for customers under which regular purchasers obtain small discounts.
To see what effect, if any, the card has had, the manager interviews a random sample of 100 customers.

They are asked whether they:

- have a loyalty card;
- know that the card exists but do not have one;
- have never heard of it.

They are also asked whether they shop around for their purchases or mainly use this store.
The results are as follows.

		Shop around	Mainly use this store
Loyalty card	Have one	8	32
	Know about it, but don't have one	20	17
	Never heard of it	6	17

- (i) Examine, at the 5% level of significance, whether these data provide any evidence of an association between these classification factors.
State clearly your null and alternative hypotheses. [12]

- (ii) Discuss your conclusions. [2]

- 5 A type of shellfish is regarded as an indicator species for water pollution.
In pure water, the mean weight of full-grown specimens is 10.5 grams.
In polluted water it is less.

A random sample of 12 full-grown specimens is collected from a particular part of a river.
Their weights, in grams, are as follows.

10.6 10.1 10.7 10.4 9.8 9.6 9.4 10.8 9.3 10.0 10.2 9.7

- (i) Use these data to estimate the population mean and standard deviation. [2]

- (ii) Examine at the 1% significance level whether the mean weight is less than 10.5 g.
State your null and alternative hypotheses clearly.
State also your conclusion about whether the water is polluted. [9]

- (iii) State an assumption underlying your test.
Suggest another test that could be used to examine whether, on the whole, full-grown specimens weigh less than 10.5 grams, if it is believed that this assumption is not justified. [2]



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AS GCE STATISTICS (MEI)

Unit Z2

G242

MARK SCHEME

Qu	Answer	Mark
1(i)	<p>$H_0: \mu = 24$ where μ is the population mean</p> <p>$H_1: \mu > 24$</p> <p>The sample mean, $\bar{x} = 25.65$</p> <p>The test statistic is $\frac{25.65 - 24}{\frac{2.6}{\sqrt{10}}} = 2.007$</p> <p>The critical value is found from the Normal distribution tables for $\Phi(z)$</p> <p>For a 1-tail test at the 5% significance level, it is 1.645</p> <p>Since $2.007 > 1.645$, the test is significant</p> <p>The evidence supports the hypothesis that the mean is greater than 24</p>	<p>B1 B1</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>E1</p> <p>[10]</p>
1(ii)	<p>The confidence interval is given by</p> $25.65 \pm 1.96 \times \frac{2.6}{\sqrt{10}} = 25.65 \pm 1.6115\dots$ $= (24.04, 27.26)$	<p>M1 B1 M1</p> <p>A1</p> <p>[4]</p>
2(i)	<p>The time of arrival, X, of the main-line train after 10:00 is Normally distributed with Mean = 0, Standard deviation 5</p> $P(X < 7) = P\left(Z < \frac{7-0}{5}\right)$ $= P(Z < 1.4)$ $= 0.9192$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>
2(ii)	<p>For 5 days, the probability that the connection is made every day is $(0.9192)^5 = 0.6562\dots$</p> <p>The probability that it is not made on at least one day is $1 - 0.6562\dots = 0.3438\dots$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>
2(iii)	<p>The time of arrival at B after 10:00 is Normally distributed with Mean = 37, Standard deviation 2</p> $P(\text{Arrival before } 42) = P\left(Z < \frac{42-37}{2}\right)$ $= P(Z < 2.5)$ $= 0.9938$	<p>M1</p> <p>A1</p> <p>[2]</p>
2(iv)	<p>For probability of 0.99, the value of z is $\Phi^{-1}(0.99) = 2.326$</p> <p>2.326 standard deviations are $2.326 \times 5 = 11.63$ minutes</p> <p>So the local train should be scheduled to depart at 10:11 $\frac{63}{100}$.</p> <p>This must be rounded up to a whole minute, i.e. 10:12</p>	<p>M1 A1</p> <p>A1</p> <p>E1</p> <p>[4]</p>

Qu	Answer	Mark																														
2(v)	Increasing the probability of making the connection at A reduces that of making it at B, and vice-versa. It is not possible to improve both simultaneously. To find the best compromise, probability calculations are needed. Or other suitable points	E1 E1 [2]																														
3(i)	Occurrences occur at random The average rate of occurrence is constant	B1 B1 [2]																														
3(ii)	There might be some evidence of clumping (see the high values for 3, 4, 5) so perhaps No	B1 B1 [2]																														
3(iii)	$\bar{x} = 1.5$, $s^2 = 3.30$ These are not very close, casting doubt on the Poisson model	B1 B1 E1 E1 [4]																														
3(iv)	Use of cumulative Poisson table for $\lambda = 1.5$ Attempt at differencing in the table	M1 M1																														
	<table border="1"> <thead> <tr> <th>x</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>>7</th> </tr> </thead> <tbody> <tr> <td>$P(x)$</td> <td>0.2231</td> <td>0.3347</td> <td>0.2510</td> <td>0.1256</td> <td>0.0470</td> <td>0.0141</td> <td>0.0036</td> <td>0.0007</td> <td>0.0002</td> </tr> <tr> <td>f_e</td> <td>32.13</td> <td>48.20</td> <td>36.14</td> <td>18.09</td> <td>6.77</td> <td>2.03</td> <td>0.52</td> <td>0.10</td> <td>0.03</td> </tr> </tbody> </table>	x	0	1	2	3	4	5	6	7	>7	$P(x)$	0.2231	0.3347	0.2510	0.1256	0.0470	0.0141	0.0036	0.0007	0.0002	f_e	32.13	48.20	36.14	18.09	6.77	2.03	0.52	0.10	0.03	A1 M1 A1 [5]
x	0	1	2	3	4	5	6	7	>7																							
$P(x)$	0.2231	0.3347	0.2510	0.1256	0.0470	0.0141	0.0036	0.0007	0.0002																							
f_e	32.13	48.20	36.14	18.09	6.77	2.03	0.52	0.10	0.03																							
3(v)	Not a good model The shape is very different (e.g. mode 0) from Poisson (1.5) χ^2 goodness of fit test	E1 E1 B1 [3]																														

Qu	Answer	Mark		
4(i)	H ₀ : No association H ₁ : Association	B1 B1		
	Expected frequencies			
		Shop around	Mainly use this store	
	Loyalty card	Have one	13.60	26.40
		Know about it, but don't have one	12.58	24.42
		Never heard of it	7.82	15.18
			M1 A1	
	Contributions to X^2	Shop around	Mainly use this store	
	Loyalty card	Have one	2.3059	1.1879
		Know about it, but don't have one	4.3765	2.2546
Never heard of it		0.4236	0.2182	
		M1 A1		
$X^2 = 10.767$ Refer to χ^2_2 Critical value for 5% significance level is 5.991 Since $10.767 > 5.991$ the test is significant The claim of association is justified at this level		M1A1 M1 A1 E1 E1 [12]		
4(ii)	People with the card are more likely than expected mainly to use the store, whereas people who know about it but don't have one are more likely to shop around	E1 E1 [2]		
5(i)	$\bar{x} = 10.05$, $s = 0.505\dots$	B1 B1 [2]		
5(ii)	H ₀ : $\mu = 10.5$ where μ is the population mean H ₁ : $\mu < 10.5$	B1 B1 B1		
	Test statistic is $t = \frac{10.05 - 10.5}{\frac{0.505\dots}{\sqrt{12}}} = -3.08\dots$	M1 A1		
	The lower-tail critical value for t_{11} for 1-tail test at 1% significance level is -2.718	M1 A1		
	Since $-3.08 < -2.718$ the test is significant (Note: Must be correct comparison of lower tail with -2.718, or be clear that working with absolute values.)	E1		
	The evidence supports the idea that the water in the river is polluted	E1 [9]		
5(iii)	The assumption is that the population is Normally distributed If not, the Wilcoxon single sample test could be used	B1 B1 [2]		
		Total: 72		

AO	Range	Total	Question Number				
			1	2	3	4	5
1	8-14	11	2	1	2	3	3
2	8-14	14	2	4	4	3	1
3	22-28	24	6	4	5	4	5
4	15-21	16	2	4	4	3	3
5	4-10	7	2	2	1	1	1
Totals		72	14	15	16	14	13

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education

AS GCE STATISTICS (MEI)

Unit Z3

G243

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (47 marks)

- 1** A computer manager is investigating two computer systems in respect of their performances in handling the jobs that commonly occur.
 Ten typical jobs are selected at random and run on each system.
 On each occasion, the performance of the system is measured by a score.
 The table below shows the scores for each job on each system.

Job	1	2	3	4	5	6	7	8	9	10
System A	102	66	142	136	84	96	83	123	106	152
System B	94	71	144	125	72	90	95	117	98	146

- (i) Explain why it is appropriate to design this investigation as a paired-comparison experiment. [3]
- (ii) Use a t test to analyse the data in the table to investigate whether the underlying populations have the same mean.
 Use a 5% significance level. [12]
- (iii) State an assumption about the underlying distribution which is required for this test. [1]
- 2** A geologist is studying the composition of gravel in two similar regions in the south of England. She takes 34 samples of gravel, 19 from the first region and 15 from the second. She divides the stones in the gravel into two types, flint and non-flint, and finds the percentage of the flint type in each sample. The results are as follows, listed in ascending order within each region.

Region 1	27	28	33	44	53	55	56	58	64	68	77	80	81	83	88	91	92	93	95
Region 2	25	26	41	42	43	45	47	50	52	54	61	75	76	78	90				

- (i) Using class limits of 24, 36, 48, 60, 72, 84, 96 sketch a histogram for the data from *either Region 1 or Region 2*.
 By considering this histogram, explain why she would be unwise to assume an underlying Normal distribution. [4]
- (ii) Use a suitable statistical procedure to examine, at the 5% level of significance, whether the true median percentages in the two regions are the same. [11]

- 3 People applying to a flying school are given an aptitude test. Those who are accepted take a progress test three months after the course has begun. The data below give the aptitude test scores, x , and the progress test scores, y , for a random sample of 10 students, A to J .

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>
Aptitude, x	27	35	36	37	38	40	41	42	43	49
Progress, y	6	22	15	9	20	13	11	18	12	38

For these data

$$\sum x = 388, \quad \sum y = 164, \quad \sum x^2 = 15358, \quad \sum y^2 = 3428, \quad \sum xy = 6670.$$

The flying school's management wishes to test whether there is a positive association between the results of students' aptitude tests and their progress tests.

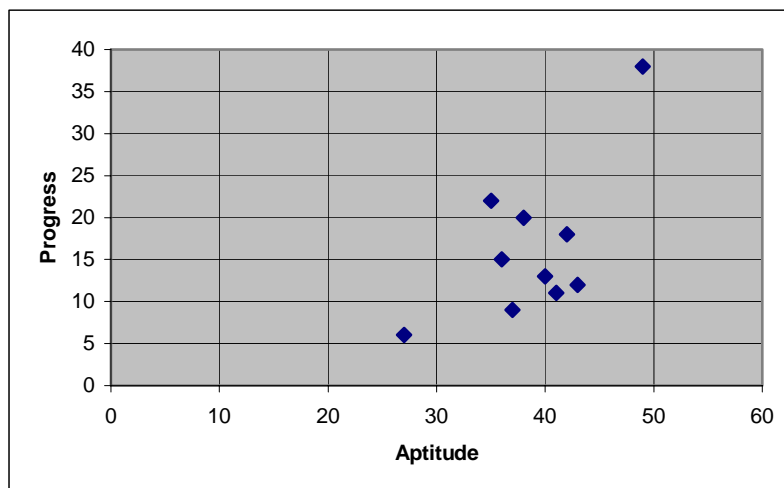
They calculate the value of Pearson's product moment correlation coefficient (pmcc) for the data.

- (i) Show that the value of the pmcc is 0.6480 (correct to 4 significant figures). [2]
- (ii) State appropriate null and alternative hypotheses for a test based on the pmcc. [3]
- (iii) Carry out the hypothesis test at the 5% significance level. State clearly the conclusion reached. [3]

One of the students plots the data on a scatter diagram, as shown below.

She tells the management that the test using the pmcc is not appropriate.

The student suggests using Spearman's rank correlation coefficient instead.



- (iv) Show that the value of Spearman's rank correlation coefficient is 0.3091 (correct to 4 significant figures). Use it to carry out the hypothesis test at the same significance level. State the conclusion reached. [6]
- (v) Identify one feature of the scatter diagram that supports the student's statement that the test using the pmcc is not appropriate. Explain briefly why this feature has less influence in the test based on Spearman's coefficient. [2]

Section B (25 marks)

- 4 The scientists in a pharmaceutical company believe that people become more forgetful after the age of 50.

They also believe that a new drug they are developing will help the condition.

The company decides to conduct trials.

The company is located in a village of about 300 people, many of whom are its own employees.

About thirty miles away there is a large town of 500 000 people.

There are two questions on which the company seeks information.

Question A Do people over 50 indeed become more forgetful?

Question B If so, will the drug provide an effective treatment?

There is debate within the company about how to research **Question A**.

Some people say “We should give a sample of 40 year olds a memory test then go back to them when they are 55 years old and give them a repeat test and see if they do any worse”.

Another group say “We should take a sample of 40 year olds and a different sample of 55 year olds *now* and compare their results”.

There is agreement that for **Question B** the best approach is to find a group of volunteers over 50 who can be given memory tests *before* and *after* a course of treatment with the drug.

- (i) The company decides to take any samples it may use from the town rather than the village. Give two reasons why it might make this decision. [2]
- (ii) In order to answer **Question A**, the company decides to select two different samples: one of people who are 40-45 now and another of people who are 55-60 now. Give two reasons why it might make this decision, rather than studying a sample of people who are 40-45 now and then studying the same people when they are 55-60. [2]
- (iii) The company decides that the samples will be of size 100. Give one reason *in favour* of a large sample size and one reason *against* it. [2]

The company considers a number of ways of finding the samples for the trials associated with **Question A**:

- choosing random telephone numbers from the town and asking about ages of people at that address, until there are enough of the right ages;
- selecting people with suitable dates of birth at random from the electoral register;
- employing students to catch people coming out of the town’s supermarkets.

- (iv) Make one comment on the merits, or otherwise, of *each* of these suggestions using technical language where appropriate. [3]

Question A is investigated by giving a memory test to the people in the two samples. The memory test gives rise to a score for each individual, giving a measure of that person's ability in respect of memory; higher scores indicate better memories.

- (v) State the null and alternative hypotheses that are appropriate for a test for **Question A**. [2]

Unfortunately only some of the data from the samples proves to be usable. The usable scores are summarised below:

Sample 1 (40-45 year olds): $n = 88$, $\sum x = 2730$, $\sum x^2 = 88768$

Sample 2 (55-60 year olds): $n = 80$, $\sum x = 2252$, $\sum x^2 = 64968$

- (vi) Using these summary data, carry out a suitable test at the 5% significance level for **Question A**. [11]

The company knows that full clinical trials will be needed before it can be licensed to sell the drug as a treatment for memory loss.

Before going to such expense, it conducts a pilot experiment on 10 subjects, women in the 55-60 age group.

The subjects take a memory test, then they are given the drug for a week and then they take the same memory test again.

The results are given in the table below.

Subject	A	B	C	D	E	F	G	H	I	J
Before treatment	25	29	22	34	34	23	27	26	28	23
After treatment	27	30	23	37	39	24	29	26	29	26

- (vii) By looking at the data, comment on the apparent effect of the treatment. Give an alternative possible explanation for the data. Explain briefly why the experiment is flawed. [3]

Oxford Cambridge and RSA Examinations

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

AS GCE STATISTICS (MEI)

Unit Z3

G243

MARK SCHEME

Qu	Answer	Mark																						
Section A																								
1(i)	The jobs are inherently variable So it is not sensible merely to compare one small sample with another But it is sensible to compare differences	B1 B1 B1 [3]																						
1(ii)	<p>$H_0: \mu_d = 0$ where μ_d is the population mean of the differences</p> <p>$H_1: \mu_d \neq 0$</p> <p>2-tail test</p> <p>t test</p> <table border="1"> <thead> <tr> <th>Job</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>8</th> <th>9</th> <th>10</th> </tr> </thead> <tbody> <tr> <td>Diff, d</td> <td>8</td> <td>-5</td> <td>-2</td> <td>11</td> <td>12</td> <td>6</td> <td>-12</td> <td>6</td> <td>8</td> <td>6</td> </tr> </tbody> </table> <p>For these values of d, $\sum d = 38 \Rightarrow \bar{d} = 3.8$</p> $S_{dd} = 674 - \frac{1}{10} \times 38^2 \Rightarrow s = \sqrt{\frac{529.6}{10-1}} = 7.67$ <p>[Accept write-down of \bar{d} and s]</p> <p>The test statistic is $\frac{3.8-0}{\frac{7.67}{\sqrt{10}}} = 1.567$</p> <p>The critical value for t_9 is 2.262</p> <p>Since $1.567 < 2.262$, the test is not significant</p> <p>There is insufficient evidence to suggest that the means are not the same</p>	Job	1	2	3	4	5	6	7	8	9	10	Diff, d	8	-5	-2	11	12	6	-12	6	8	6	B1 B1 B1 M1 B1 B1 M1 A1 M1 A1 M1 E1 [12]
Job	1	2	3	4	5	6	7	8	9	10														
Diff, d	8	-5	-2	11	12	6	-12	6	8	6														
1(iii)	Assumption: the differences are Normally distributed	B1 [1]																						
2(i)	<table border="1"> <thead> <tr> <th></th> <th>$24 \leq x < 36$</th> <th>$36 \leq x < 48$</th> <th>$48 \leq x < 60$</th> <th>$60 \leq x < 72$</th> <th>$72 \leq x < 84$</th> <th>$84 \leq x < 96$</th> </tr> </thead> <tbody> <tr> <td>Reg 1</td> <td>3</td> <td>1</td> <td>4</td> <td>2</td> <td>4</td> <td>5</td> </tr> <tr> <td>Reg 2</td> <td>2</td> <td>5</td> <td>3</td> <td>1</td> <td>3</td> <td>1</td> </tr> </tbody> </table> <p>One histogram drawn</p> <p>It does not look like Normal</p> <p>No central mode, or skewed or other reason</p>		$24 \leq x < 36$	$36 \leq x < 48$	$48 \leq x < 60$	$60 \leq x < 72$	$72 \leq x < 84$	$84 \leq x < 96$	Reg 1	3	1	4	2	4	5	Reg 2	2	5	3	1	3	1	B1(one) B1 B1 B1 [4]	
	$24 \leq x < 36$	$36 \leq x < 48$	$48 \leq x < 60$	$60 \leq x < 72$	$72 \leq x < 84$	$84 \leq x < 96$																		
Reg 1	3	1	4	2	4	5																		
Reg 2	2	5	3	1	3	1																		

Qu	Answer	Mark																																																
Section A (continued)																																																		
2(ii)	Use the Wilcoxon Rank Sum test H_0 : The medians of the two populations are the same H_1 : The medians of the two populations are different 2-tail test Data are ranked Rank sum from smaller sample = $1 + 2 + 6 + 7 + \dots + 30 = 204$ Refer to (15,19) table The lower $2\frac{1}{2}\%$ point is required The critical value is 205 Since $204 < 205$ the result is significant The evidence supports the alternative hypothesis, that the medians are not the same	B1 B1 B1 M1 M1 A1 M1 M1 A1 M1 E1 [11]																																																
3(i)	$r = \frac{(6670 - \frac{1}{10} \times 388 \times 164)}{\sqrt{(15358 - \frac{1}{10} \times 388^2)} \times \sqrt{(3428 - \frac{1}{10} \times 164^2)}} = 0.64797\dots$	M1 A1 [2]																																																
3(ii)	$H_0: \rho = 0$ where ρ is the population correlation coefficient $H_1: \rho > 0$	B1 B1 B1 [3]																																																
3(iii)	Critical value is 0.5494 (from tables) $0.6502 > 0.5494$ so test is significant The evidence supports the alternative hypothesis of positive correlation	B1 E1 E1 [3]																																																
3(iv)	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> <th>I</th> <th>J</th> </tr> </thead> <tbody> <tr> <td>Aptitude</td> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> </tr> <tr> <td>Progress</td> <td>y</td> <td>1</td> <td>9</td> <td>6</td> <td>2</td> <td>8</td> <td>5</td> <td>3</td> <td>7</td> <td>4</td> <td>10</td> </tr> <tr> <td>Diff</td> <td>$d = y - x$</td> <td>0</td> <td>7</td> <td>3</td> <td>-2</td> <td>3</td> <td>-1</td> <td>-4</td> <td>-1</td> <td>-5</td> <td>0</td> </tr> </tbody> </table>			A	B	C	D	E	F	G	H	I	J	Aptitude	x	1	2	3	4	5	6	7	8	9	10	Progress	y	1	9	6	2	8	5	3	7	4	10	Diff	$d = y - x$	0	7	3	-2	3	-1	-4	-1	-5	0	M1 A1
		A	B	C	D	E	F	G	H	I	J																																							
Aptitude	x	1	2	3	4	5	6	7	8	9	10																																							
Progress	y	1	9	6	2	8	5	3	7	4	10																																							
Diff	$d = y - x$	0	7	3	-2	3	-1	-4	-1	-5	0																																							
	$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$ $= 1 - \frac{6 \times 114}{10 \times (10^2 - 1)} = 0.3091 \text{ (to 4 s.f.)}$	B1																																																
	Critical value for $n = 10$ at the 5% level is 0.5636 Since $0.3091 < 0.5636$ the null hypothesis is accepted There is insufficient evidence to suggest that aptitude and progress are associated	B1 E1 E1 [6]																																																
3(v)	There are two clear outliers which dominate the calculation. These outliers do not carry such extreme weight in the Spearman formula and therefore do not influence the calculation excessively	E1 E1 [2]																																																

Section A Total: 47

Qu	Answer	Mark
Section B		
4(i)	<p>It might be difficult to get suitable samples from such a small population Some of those involved could be employees of the company and so their responses could be biased in some way [Or other sensible reason, but some reason for possibly being untypical must be given]</p>	<p>B1 B1 [2]</p>
4(ii)	<p>The company does not want to wait 15 years for results It would be hard to keep track on individuals over 15 years; some would move away or die</p>	<p>B1 B1 [2]</p>
4(iii)	<p>Large samples are usually more representative of the population as a whole and this is a good thing Large samples cost more to administer and this is a reason against them</p>	<p>B1 B1 [2]</p>
4(iv)	<p>Choosing random telephone numbers from the town and asking about the ages of people at that address, until they have enough of the right ages (a form of quota sampling) requires a response but some people do not take cold calls nor do people necessarily tell the truth The electoral register is a suitable sampling frame, providing people are chosen from it using a random sampling procedure Employing students to catch people coming out of the town's supermarkets could be a form of quota sampling with the students asked to find so many of particular ages and sexes Some bias may be involved in the location which favours more active people against those who are house-bound There is an assumption that people tell the students the truth about their ages and this may not be justified</p>	<p>B1 B1 B1 B1 B1 [3]</p>
4(v)	<p>H_0: The mean scores of the populations from which the two samples are drawn are equal H_1: The mean score of the population from which the second sample is drawn is less</p>	<p>B1 B1 [2]</p>
4(vi)	<p>Attempt to use 2-sample test based on $N(0,1)$ $\bar{x}_1 = \frac{2730}{88} = 31.0227\dots, \quad \bar{x}_2 = \frac{2252}{80} = 28.15$ $s_1^2 = \frac{88768 - \frac{1}{88} \times 2730^2}{88 - 1} = 46.8500\dots$ $s_2^2 = \frac{64968 - \frac{1}{80} \times 2252^2}{80 - 1} = 19.9265\dots$ <p>Because the samples are large the value of σ_1^2 and σ_2^2 are taken to be s_1^2 and s_2^2 Test statistic = $\frac{31.0227 - 28.15}{\sqrt{\frac{46.8500\dots}{88} + \frac{19.9265\dots}{80}}} = 3.25$ Critical value = 1.645 for 1-tail test Since $3.25 > 1.645$, the null hypothesis is rejected The evidence supports the hypothesis that there is a deterioration of memory</p> </p>	<p>M1 B1 M1 A1 M1 M1 A1 M1 M1 E1 E1 [11]</p>

Qu	Answer	Mark
Section B (continued)		
4(viii)	All the subjects scored higher or the same on the second test so the drug seems to have worked It could be that the subjects have got used to the test There is no control group	B1 B1 B1 [3]
		Section B Total: 25
		Total: 72

AO	Range	Total	Question Number			
			1	2	3	4
1	8-14	10	2	3	3	2
2	8-14	11	3	2	2	4
3	22-28	25	5	7	4	9
4	15-21	21	5	2	5	9
5	4-10	5	1	1	2	1
Totals		72	16	15	16	25