GCE

Statistics (MEI)

Unit **G243**: Statistics 3 (Z3)

Advanced Subsidiary GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

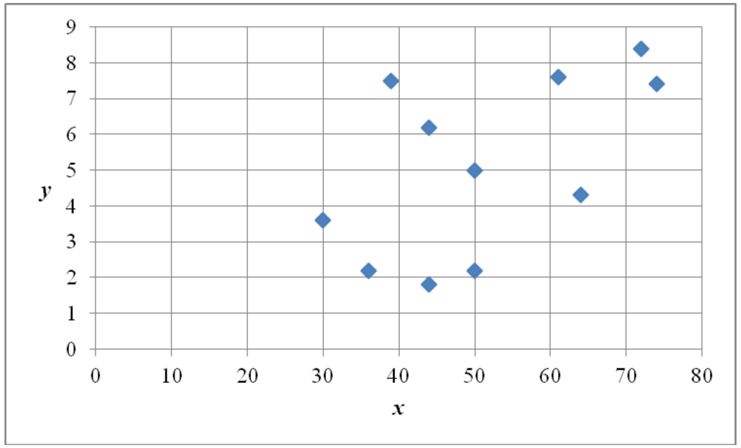
A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		Unpaired t test –allow 2-sample t test	E1	
			Wilcoxon rank sum test (or Mann-Whitney 2-sample test)	E1	
			If the two populations were Normally distributed with equal variances then the unpaired t test would be preferable. Otherwise the Wilcoxon rank sum test would have to be used.	E1	For either Normally distributed or equal variances associated with unpaired t test (or for both associated with t test)
				E1	For full answer. Condone 'symmetrically distributed about the median'
				[4]	
1	(ii)		H_0: the medians of the two populations are the same	B1	For medians
			H_1: the median of the afternoon population is lower	B1	Need population for second mark Note: Explicit statement re shapes of distributions (eg that they are the same shape) is not required.
			Wilcoxon rank sum test (or Mann-Whitney form thereof)	M1	
			Ranks are Mor Rank 3 5 8 9 10 14 15 17 18 19 21 22 Aft Rank 1 2 4 6 7 11 12 13 16 20	M1	Combined ranking
			Rank sum for smaller sample (for Afternoon) is 92	A1	FT (M-W stat = 0 + 0 + 1 + 2 + 2 + 5 + 5 + 5 + 7 + 10 = 37)
			Refer to (10,12) table	B1	
			1–tail 5% critical value is 89 [or 34 for M–W]	B1	
			92 > 89	M1	For sensible comparison leading to a correct conclusion
			Not significant	A1	
			There is insufficient evidence to suggest that, on average, the afternoon waiting times are shorter than the morning waiting times	E1	
				[10]	
1	(iii)		A paired design can only be used if there is some natural data pairing, such as the same people at both a morning and an afternoon appointment.	E1	
			This is very unlikely and so a paired design is not appropriate.	E1	For not appropriate (with justification)
				[2]	

Question			Answer	Marks	Guidance
2	(i)		The pairing will eliminate any differences in individual subjects' abilities and so will compare the times with and without the incentive scheme	E1 E1 **[2]**	
2	(ii)		The Wilcoxon signed rank test	E1 **[1]**	
2	(iii)		The population of differences of times must be Normally distributed	E1 E1 **[2]**	For population of differences For Normally distributed
2	(iv)		H_0: $\mu_D = 0$ H_1: $\mu_D < 0$ Where μ_D denotes the population mean for differences. No further marks unless paired comparison *t* test Differences (after incentive – before incentive) are –0.9 –1.4 –0.5 0.3 –0.2 0.5 –0.6 –1.0 –1.3 0.5 –0.9 $\bar{d} = -0.50$ $s_{n-1} = 0.6899$ Test statistic is $\dfrac{-0.50 - 0}{0.6899/\sqrt{11}} = -2.404$ Refer to t_{10} One tailed 5% critical value is –1.812 So significant There is sufficient evidence to suggest that the mean picking time after the incentive scheme is introduced is less than it was before	B1 B1 B1 M1 A1 M1 A1 M1 A1 E1 E1 **[11]**	 Condone absence of "population" if correct notation "μ" has been used, but do NOT accept $\bar{D}$ or similar unless explicitly stated to be population means. Hypotheses explained in words only must include "population" For differences For both FT their $\bar{d}$ and s_{n-1} CAO but FT from here if M1 awarded For t_{10} Must be minus 1.812 unless absolute values or (before – after) are being compared. No FT if wrong. M1 can be implied by correct cv

Question		Answer	Marks	Guidance
3	(i)		G1 G1 G1 **[3]**	Linear axes, including labels Correct zero or clear broken scale All points correct (allow 2 errors)
3	(ii)	r = 0.5609	M1 A1 **[2]**	
3	(iii)	The population should have a bivariate Normal distribution. In this case the points appear to lie in an elliptical pattern which suggests that the population may have a bivariate Normal distribution.	E1 E1 **[2]**	For bivariate Normal For roughly elliptical
3	(iv)	H_0: $\rho = 0$ H_1: $\rho > 0$ (one-tailed test) where ρ is the correlation coefficient for the underlying bivariate population. For $n = 11$, one tailed 1% critical value = 0.6851 Not significant There is insufficient evidence to suggest that there is positive correlation between total rainfall and weight of crop harvested per square metre.	B1 B1 B1 B1 E1 E1 **[6]**	 FT their cv and test statistic from part (ii) (provided | test statistic | ≤ 1) Condone '…positive correlation between x and y'

Question			Answer	Marks	Guidance
3	(v)		If the locations were not chosen randomly, then making inferences from the sample would not be valid.	E1 E1 [2]	For 'making inferences' For 'not be valid' Allow other reasonable answers
3	(vi)		A test based on Spearman's rank correlation coefficient.	E1 [1]	
4	(i)		Method C Because with method A) a particular fisherman might just take fish from one area and with method B) if all fish are caught on a particular day, the larger or the smaller fish might not be available to catch	E1 E1 [2]	Allow other sensible comments
4	(ii)		Allocate numbers 1 to 120 to the fish. Use random numbers to choose 10 random numbers. If any repeats appear, choose further random numbers to replace them.	E1 E1 E1 [3]	
4	(iii)		Because if she samples on different days, it might be that weather conditions vary and this may result in longer or shorter fish being caught.	E1 E1 [2]	Allow other sensible comments Max 1 mark unless reference to lengths
4	(iv)		$\bar{x} = \frac{7683.5}{50} = 153.67$ $s^2 = \frac{1191300 - \frac{7683.5^2}{50}}{49} = \frac{10576.55}{49} = 215.85$ $s = \sqrt{215.85} = 14.69$	B1 M1 A1 [3]	 For S_{xx}

Question			Answer	Marks	Guidance
4	(v)		Because both samples are large	E1 **[1]**	Condone 'large sample(s)'
4	(vi)		H_0: $\mu_P = \mu_Q$ H_1: $\mu_P \neq \mu_Q$ Where μ_P, μ_Q denote the population mean lengths of fish in Lakes P and Q respectively	B1 B1 B1	Condone absence of "population" if correct notation "μ" has been used, but do NOT accept $\overline{X}$ and $\overline{Y}$ or similar unless explicitly stated to be population means. Accept hypothesis explained in words, provided "population" appears. Condone μ = population mean
			2–sample test based on N(0,1)	E1	
			Test statistic is $\dfrac{153.67-151.7}{\sqrt{\dfrac{215.85}{50}+\dfrac{15.6^2}{50}}}=\dfrac{1.97}{3.03}=0.650$	M1 M1 A1	Numerator Denominator CAO
			2–tailed 10% point of N(0,1) is 1.645	B1	
			0.650 < 1.645	M1	FT their cv and test statistic
			Not significant	A1	
			There is insufficient evidence to suggest that on average the lengths of fish in the two lakes are different.	E1 **[11]**	
4	(vii)		The result of a hypothesis test always has an element of doubt. For example, at the 5% level, 5 times in every 100, the null hypothesis will be rejected when it is in fact true.	E1 E1 **[2]**	Allow E1 for 'sample might not be representative'

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