ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Statistics

SPECIMEN UNITS AND MARK SCHEMES

AQA ADVANCED SUBSIDIARY GCE STATISTICS (5381)
AQA ADVANCED GCE STATISTICS (6381)

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS SS1A
Unit Statistics 1A

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS1A.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Ten per cent of coloured beads used in costume jewellery are orange.

(a) Find the probability that in a string of 40 beads, 4 or fewer beads are orange. *(3 marks)*

(b) Calculate the probability that in a string of 35 beads, exactly 2 beads are orange. *(3 marks)*

(c) State **one** assumption that you have made in answering parts (a) and (b). *(1 mark)*

2 The weights of bags of red gravel may be modelled by a normal distribution with mean 25.8 kg and standard deviation 0.5 kg.

(a) Determine the probability that a randomly selected bag of red gravel will weigh less than 25 kg. *(3 marks)*

(b) Determine, to two decimal places, the weight exceeded by 10% of bags. *(4 marks)*

3 (a) A sample of people, who commute regularly from a town in Surrey into London, was asked for an estimate of the time taken on their most recent journey. The replies are summarised below.

Time (minutes)	**Frequency**
35 -	12
45 -	54
55 -	68
65 -	41
85 - 105	23

Calculate estimates of the mean and the standard deviation of these times. *(5 marks)*

(b) A sample of people who commute regularly from a town in Essex into London was also asked for an estimate of the time taken on their most recent journey. Their replies had a mean of 64 minutes and a standard deviation of 21 minutes. Compare, briefly, the journey times estimated by commuters from the two towns. *(2 marks)*

(c) Give **two** reasons why the data presented in parts (a) and (b) may not adequately represent typical commuting times from the two towns. *(2 marks)*

4 A cricket team meets for fielding practice. One exercise consists of a cricket ball being thrown at different heights, speeds and angles to one side of a fielder who tries to catch it using one hand.

Each member of the team attempts 25 catches with each hand. The number of successful catches are given in the following table.

Fielder	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**
Left hand	11	13	9	17	21	16	14	8	19	19	20
Right hand	18	17	20	22	14	19	21	15	10	24	23

(a) Calculate the value of the product moment correlation between the number of catches with the left hand and the number of catches with the right hand. *(3 marks)*

(b) Comment on the performance of fielders **E** and **I**. *(2 marks)*

(c) When fielders **E** and **I** are omitted from the calculation, the value of the product moment correlation coefficient between the number of left-handed catches and the number of right-handed catches is 0.812, correct to three decimal places. Comment on this value and the value you calculated in part (a) *(2 marks)*

5 Pencils produced on a certain machine have lengths, in millimetres, which are distributed with a mean of μ and a standard deviation of 3. A random sample of 90 pencils was taken and the length of each pencil measured. The mean length was found to be 178.5 millimetres.

(a) Construct a 99% confidence interval for μ. *(5 marks)*

(b) State why, in answering part (a), it is not necessary to assume that the length of pencils are normally distributed. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

6 Last year the employees of a firm either received no pay rise, a small pay rise or a large pay rise. The following table shows the number in each category, classified by whether they were weekly paid or monthly paid.

	No pay rise	**Small pay rise**	**Large pay rise**
Weekly Paid	25	85	5
Monthly paid	4	8	23

A tax inspector decides to investigate the tax affairs of an employee selected at random.

D is the event that a weekly paid employee is selected.
E is the event that an employee who received no pay rise is selected.
E' is the event not E.

(a) Find the value of:

(i) $\mathrm{P}(D)$;

(ii) $\mathrm{P}(D \mid E)$;

(iii) $\mathrm{P}(D \cap E')$. *(5 marks)*

(b) The tax inspector now decides to select three employees. Find the probability that they are all weekly paid if:

(i) one is selected at random from those who had no pay rise, one from those who had a small pay rise and one from those who had a large pay rise; *(3 marks)*

(ii) they are selected at random (without replacement) from all the employees of the firm. *(2 marks)*

7 **[**A sheet of graph paper is provided for use in this question.**]**

Andrew (**A**), Charles (**C**) and Edward (**E**) are employed by the Palace Hotel. Each is responsible for one floor of the building and their duties include cleaning the bedrooms. The number of bedrooms occupied on each floor varies from day to day.

The following table shows 10 observations of the number, x, of bedrooms to be cleaned and the time taken, y minutes, to carry out the cleaning. The employee carrying out the cleaning is also indicated.

Employee	**A**	**C**	**E**	**E**	**C**	**A**	**A**	**E**	**C**	**C**
x	8	22	12	24	19	14	22	16	10	21
y	110	211	132	257	184	165	248	171	97	196

(a) Plot a scatter diagram of the data. Identify the employee by labelling each point. *(3 marks)*

(b) Calculate the equation of the regression line of y on x. Draw the line on your scatter diagram. *(6 marks)*

(c) Calculate the residuals for the three observations when Andrew did the cleaning. *(3 marks)*

(d) Comment on the times taken by Andrew to carry out his cleaning. *(1 mark)*

END OF QUESTIONS

SS1A Specimen

Question	Solution	Marks	Total	Comments
1(a)	Binomial $n = 40$ $p = 0.1$	B1B1		
	P(4 or fewer) = 0.629	B1	3	
(b)	$P(2) = (35 \times 34/2) \times 0.1^2 \times 0.9^{33}$	B1M1		
	= 0.184	A1	3	0.183 – 0.184
(c)	Beads selected randomly/independently	E1	1	
	Total		**7**	
2(a)	$z = (25 - 25.8)/0.5 = -1.6$	M1		
	Probability less than 25kg = 1 – 0.94520	M1		
	= 0.0548	A1	3	
(b)	z = 1.2816	B1		
	Weight exceeded by 10% of bags	M1m1		
	$25.8 + 1.2816 \times 0.5 = 26.44$	A1	4	
	Total		**7**	
3(a)	Class mid-mark Frequency 40 12 50 54 60 68 75 41 95 23	M1		Allow m1A1 for mean and s.d. if method shown. 63.2 (63.1 – 63.3)
	$\bar{x} = 63.2$ $s = 15.2$	A2 A2	5	15.2 (15.0 – 15.3)
(b)	Journeys from Surrey have similar duration, on average, but are less variable than those from Essex.	E1 E1	2	
(c)	People asked may not be representative. Times are estimated not measured.	E1 E1	2	Or any other sensible comments e.g. journey time not defined , weather conditions may be extreme etc
	Total		**9**	

Question	Solution	Marks	Total	Comments
4(a)	0.0477	B3	3	0.047 – 0.048 allow M2 A1 if method shown
(b)	E and I held more catches with left than with right hand - all others held more with right than left.	E1 E1	2	
(c)	Correlation coefficient of 0.812 suggests that those who caught a lot of catches with one hand also caught a lot of catches with the other. When E and I (possibly left handers) are included the correlation coefficient of 0.0477 suggests no association between the number of catches with each hand.	E1 E1	2	
	Total		**7**	
5(a)	99% confidence interval for mean $178.5 \pm 2.5758 \times 3/\sqrt{90}$ 178.5 ± 0.8145 177.69 – 179.31	B1M1 m2 A1	5	
(b)	Sample is large. Sample mean may be assumed to be Normally distributed by Central Limit Theorem.	E1 E1	2	
	Total		**7**	
6(a)(i)	115/150 = 0.767	B1	1	acf
(ii)	25/29 = 0.862	M1A1	2	acf
(iii)	90/150 = 0.6	M1A1	2	acf
(b)(i)	$25/29 \times 85/93 \times 5/28 = 0.141$	M1 M1A1	3	0.14 – 0.141
(ii)	$115/150 \times 114/149 \times 113/148 = 0.448$	M1 A1	2	
	Total		**10**	

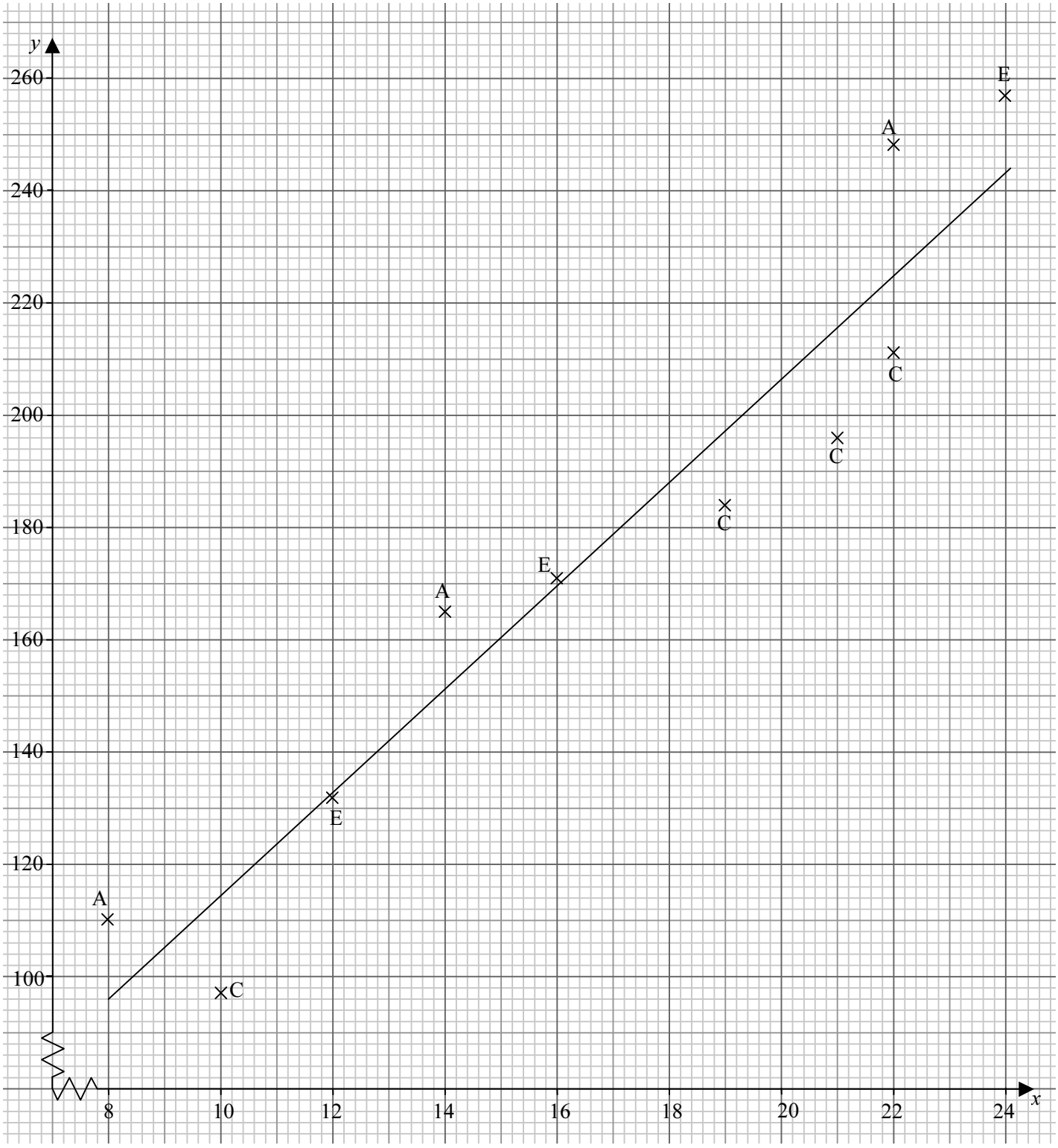
SS1A (cont)

Question	Solution	Marks	Total	Comments
7(a)	See graph on next page	M1 A1 B1	3	
(b)	$y = 22.8 + 9.19x$	B2 B2		22.7 – 22.8 9.18 – 9.2 Allow M1 A1 for a and b if method shown
	$x = 8$ $y = 96.3$ $x = 23$ $y = 234.1$	M1A1	6	+ line on graph
(c)	Residuals $110 - 22.77 - 9.186 \times 8 = 13.7$ $165 - 22.77 - 9.186 \times 14 = 13.6$ $248 - 22.77 - 9.186 \times 22 = 23.1$	M1 A1 A1	3	M1 method - ignore sign, allow read from graph A1 one correct - ignore sign 13.7 (13 – 14) 13.6 (13 – 14) 23.1 (22 – 24) A1 all correct, including sign
(d)	Andrew appears to be slowest (all residuals positive / all times longer than predicted by regression line)	E1	1	
	Total		**13**	
	TOTAL		**60**	

[9]

SS1A (cont)

Graph for Question 7



[10]

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

STATISTICS SS1B
Unit Statistics 1B

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables;
- a sheet of graph paper for use in Question 6;
- a ruler.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS1B.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

1 Jeremy sells a magazine which is produced in order to raise money for homeless people. The probability of making a sale is 0.09 for each person he approaches.

(a) Given that he approaches 40 people, find the probability that he will make:

(i) 2 or fewer sales; *(3 marks)*

(ii) more than 5 sales. *(2 marks)*

(b) Find the probability that he will make two sales given that he approaches 16 people. *(3 marks)*

(c) State **one** assumption you have made in answering parts (a) and (b). *(1 mark)*

2 (a) A sample of people, who commute regularly from a town in Surrey into London, was asked for an estimate of the time taken on their most recent journey. The replies are summarised below.

Time (minutes)	**Frequency**
35-	12
45-	54
55-	68
65-	41
85-105	23

Calculate estimates of the mean and the standard deviation of these times. *(5 marks)*

(b) A sample of people who commute regularly from a town in Essex into London was also asked for an estimate of the time taken on their most recent journey. Their answers had a mean of 64 minutes and a standard deviation of 21 minutes. Compare, briefly, the journey times estimated by commuters from the two towns. *(2 marks)*

(c) Give **two** reasons why the data presented in parts (a) and (b) may not adequately represent typical commuting times from the two towns. *(2 marks)*

3 A cricket team meets for fielding practice. One exercise consists of a cricket ball being thrown at different heights, speeds and angles to one side of a fielder who tries to catch it one handed.

Each member of the team attempts 25 catches with each hand. The number of successful catches are given in the following table.

Fielder	**A**	**B**	**C**	**D**	**E**	**G**	**H**	**I**	**J**	**K**	**L**
Left hand	11	13	9	17	21	16	14	8	19	19	20
Right hand	18	17	20	22	14	19	21	15	10	24	23

(a) Calculate the value of the product moment correlation between the number of catches with the left hand and the number of catches with the right hand. *(3 marks)*

(b) Comment on the performance of fielders **E** and **J**. *(2 marks)*

(c) When fielders **E** and **J** are omitted from the calculation, the value of the product moment correlation coefficient between the number of left-handed and the number of right-handed catches is 0.812, correct to three decimal places. Comment on this value and the value you calculated in part (a) *(2 marks)*

4 The weights of the contents of jars of honey may be assumed to be normally distributed with the standard deviation 3.1 grams. The weights of the contents, in grams, of a random sample of eight jars were as follows:

458 450 457 456 460 459 458 456

(a) Calculate a 95% confidence interval for the mean weight of the contents of all jars. *(6 marks)*

(b) On each jar it states "Contents 454 grams". Comment on this statement using the given sample and your results in part (a). *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

5 Last year the employees of a firm either received no pay rise, a small pay rise or a large pay rise. The following table shows the number in each category, classified by whether they were weekly paid or monthly paid.

	No pay rise	**Small pay rise**	**Large pay rise**
Weekly Paid	25	85	5
Monthly paid	4	8	23

A tax inspector decides to investigate the tax affairs of an employee selected at random.

D is the event that a weekly paid employee is selected.
E is the event that an employee who received no pay rise is selected.
E' is the event "not E".

(a) Find the value of:

(i) P (D);

(ii) P ($D \mid E$) ;

(iii) P ($D \cap E'$). *(5 marks)*

(b) The tax inspector now decides to select three employees. Find the probability that they are all weekly paid if:

(i) one is selected at random from those who had no pay rise, one from those who had a small pay rise and one from those who had a large pay rise; *(3 marks)*

(ii) they are selected at random (without replacement) from all the employees of the firm. *(2 marks)*

6 **[**A sheet of graph paper is provided for use in this question.**]**

Andrew (**A**), Charles (**C**) and Edward (**E**) are employed by the Palace Hotel. Each is responsible for one floor of the building and their duties include cleaning the bedrooms. The number of bedrooms occupied on each floor varies from day to day.

The following table shows 10 observations of the number, x, of bedrooms to be cleaned and the time taken, y minutes, to carry out the cleaning. The employee carrying out the cleaning is also indicated.

Employee	**A**	**C**	**E**	**E**	**C**	**A**	**A**	**E**	**C**	**C**
x	8	22	12	24	19	14	22	16	10	21
y	110	211	132	257	184	165	248	171	97	196

(a) Plot a scatter diagram of the data. Identify the employee by labelling each point. *(3 marks)*

(b) Calculate the equation of the regression line of y on x. Draw the line on your scatter diagram. *(6 marks)*

(c) Use your regression equation to estimate the time which would be taken to clean 18 bedrooms. *(1 mark)*

(d) Calculate the residuals for the three observations when Andrew did the cleaning. *(3 marks)*

(e) Modify your estimate in part (c), given that the 18 bedrooms are to be cleaned by Andrew. *(2 marks)*

7 A gas supplier maintains a team of engineers who are available to deal with leaks reported by customers. Most reported leaks can be dealt with fairly quickly but some require a long time. The time (excluding travelling time), X, taken to deal with reported leaks is found to have a mean of 65 minutes and a standard deviation of 60 minutes.

(a) Assuming that the times may be modelled by a normal distribution, find the probability that it will take:

(i) more than 185 minutes to deal with a reported leak; *(3 marks)*

(ii) between 50 minutes and 125 minutes to deal with a reported leak. *(4 marks)*

(b) The mean of the times taken to deal with each of a random sample of 90 leaks is denoted by $\overline{X}$.

(i) State the distribution of $\overline{X}$. *(3 marks)*

(ii) Find the probability that $\overline{X}$ is less than 70 minutes. *(2 marks)*

(c) A statistician consulted by the gas supplier stated that, as the times had a mean of 65 minutes and a standard deviation of 60 minutes, the normal distribution would not provide an adequate model.

(i) Explain the reason for the statistician's statement. *(2 marks)*

(ii) Give a reason why, despite the statistician's statement, your answer to part (b)(ii) is still valid. *(2 marks)*

END OF QUESTIONS

SS1B Specimen

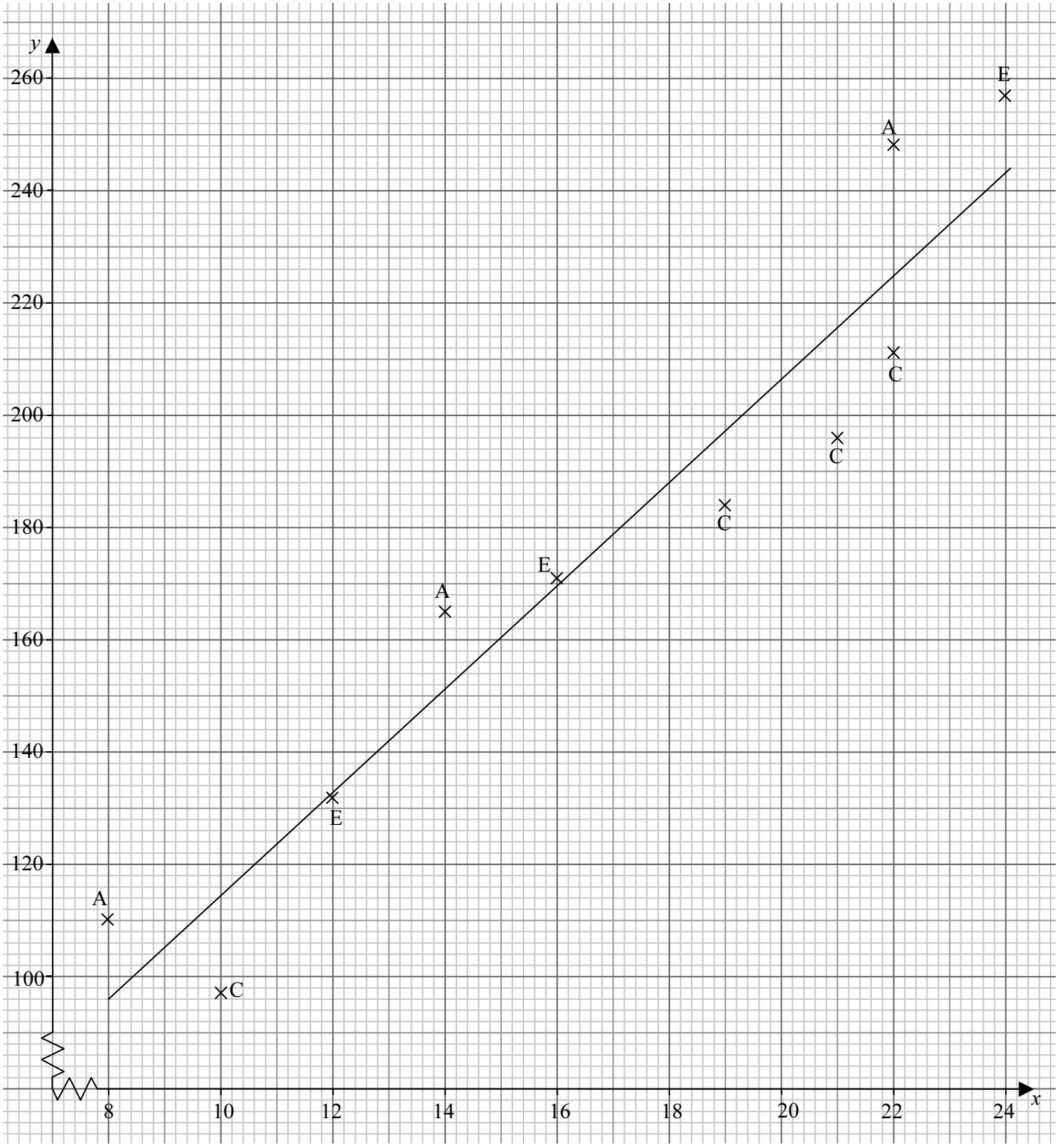
Question	Solution	Marks	Total	Comments
1(a)(i)	Binomial $n = 40$ $p = 0.09$	B1B1		
	P(2 or fewer) = 0.2894	B1	3	0.289 - 0.29
(ii)	P(>5) = 1– P(5 or fewer)	M1		
	= 1– 0.8535 = 0.1465	A1	2	0.146 - 0.147
(b)	$P(2) = (16\times15/2) \times 0.09^2\times 0.91^{14}$	B1M1		
	= 0.260	A1	3	0.259 – 0.26
(c)	probabilities independent/people selected at random/equivalent	E1	1	
	Total		**9**	
2(a)	Class mid-mark Frequency 40 12 50 54 60 68 75 41 95 23	M1		Allow m1A1 for mean and s.d. if method shown.
	$x = 63.2$ $s = 15.2$	A2A2	5	63.2 (63.1 – 63.3) 15.2 (15.0 - 15.3)
(b)	Journeys from Surrey have similar duration, on average, but are less variable than those from Essex.	E1 E1	2	
(c)	People asked may not be representative. Times are estimated not measured.	E1 E1	2	Or any other sensible comments e.g. journey time not defined , weather conditions may be extreme etc
	Total		**9**	
3(a)	0.0477	B3	3	0.047 – 0.048 allow M2A1 if method shown
(b)	**E** and **J** held more catches with left than with right hand - all others held more with right than left.	E1 E1	2	
(c)	Correlation coefficient of 0.812 suggests that those who caught a lot of catches with one hand also caught a lot of catches with the other. When **E** and **J** (possibly left handers) are included the correlation coefficient of 0.0477 suggests no association between the number of catches with each hand.	E1 E1	2	
	Total		**7**	

SS1B (cont)

Question	Solution	Marks	Total	Comments
4(a)	$\bar{x}$ = 456.75	B1		
	95% confidence interval for mean	B1M1		
	456.75 ± 1.96 ×3.1/√8	M2		
	456.75 ± 2.15			
	454.60 – 458.90	A1	6	
(b)	The confidence interval provides evidence that the mean contents are greater than 454 grams. However the sample shows that some jars will contain less than 454 grams.	E1 E1 E1	3	E1 confidence interval refers to **mean** contents E1 evidence mean >454 E1 some individual contents <454
	Total		**9**	
5(a)(i)	115/150 = 0.767	B1	1	acf
(ii)	25/29 = 0.862	M1A1	2	acf
(iii)	90/150 = 0.6	M1A1	2	acf
(b)(i)	25/29 × 85/93 × 5/28 = 0.141	M1 M1A1	3	0.14 - 0.141
(ii)	115/150 × 114/149 × 113/148 = 0.448	M1 A1	2	
	Total		**10**	
6(a)	See graph on next page	M1 A1 B1	3	
(b)	$y = 22.8 + 9.19x$	B2B2		22.7 – 22.8 9.18 – 9.2 Allow M1A1 for a and b if method shown
	$x = 8$ $y = 96.3$ $x = 23$ $y = 234.1$	M1A1	6	+ line on graph
(c)	188	B1	1	188 – 188.3, allow 190
(d)	Residuals	M1		M1 method - ignore sign, allow read from graph
	110 – 22.77 – 9.186 × 8 =13.7 165 – 22.77 –-9.186 × 14 = 13.6 248 – 22.77 – 9.186 × 22 = 23.1	A1		A1 one correct - ignore sign 13.7 (13 – 14) 13.6 (13 – 14) 23.1 (22 – 24)
		A1	3	A1 all correct, including sign
(e)	188 + 17 = 205	M1 A1	2	Any sensible method 201 – 211
	Total		**15**	

SS1B (cont)

Graph for Question 6



SS1B (cont)

Question	Solution	Marks	Total	Comments
7(a)(i)	$z = \frac{(185-65)}{60} = 2.0$	M1		
	P($X > 185$) = 1 – 0.97725	M1		
	= 0.02275	A1	3	0.0227 – 0.023
(ii)	$z_1 = \frac{(50-65)}{60} = -0.25$	M1		
	$z_2 = \frac{(125-65)}{60} = 1.0$	m1		
	P($50 < X < 125$)=	M1		
	0.84134 – (1 – 0.59871) = 0.440	A1	4	
(b)(i)	Normal, mean 65, s.d. 60/√90 = 6.32	B1 B1 B1	3	normal may be implied in (b)(ii)
(ii)	$z = \frac{(70-65)}{\frac{60}{\sqrt{90}}} = 0.7906$	M1		
	Probability mean of 90 less than 70 is 0.785	A1	2	0.785 – 0.786
(c)(i)	Mean is only a little more than one standard deviation above zero. For normal this implies substantial proportion of times would be negative. This is impossible so model must be inadequate.	E1 E1	2	
(ii)	Mean of large sample will be approximately normally distributed even if parent distribution is not.	E1 E1	2	
	Total		**16**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS SS02
Unit 2

In addition to this paper you will require:
- an 8-page answer book;
- an insert for use in Question 3 (enclosed);
- one sheet of graph paper for use in Question 5;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS02.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
Unless stated otherwise, formulae may be quoted, without proof, from the booklet

Answer **all** questions.

1 John routinely uses an electric saw in his cabinet-making business. The number of problems with the electric saw follows a Poisson distribution with mean 1.2 per week.

(a) Find the probability that in a particular week:

(i) one or fewer problems occur;

(ii) exactly one problem occurs. *(3 marks)*

(b) John also uses a sanding tool. The number of problems with the sanding tool follows a Poisson distribution with mean 1.8 per week. Find the probability that a total of 4 or more problems will occur with these two tools next week. (Assume that the problems with each tool occur independently.) *(4 marks)*

2 A machine dispenses salad dressing into bottles. When it is working satisfactorily, the mean volume dispensed is 235 ml. The amount dispensed may be assumed to follow a normal distribution with a standard deviation of 5 ml.

A sample of bottles is selected periodically and the machine is stopped if there is evidence that the mean volume dispensed is less than 235 ml.

A random sample of 9 bottles was selected with the following results.

230 232 235 224 238 233 236 225 235

(a) Stating your null and alternative hypotheses, investigate, at the 5% significance level, whether the machine should be stopped. *(7 marks)*

(b) A new policy is being considered for stopping the machine, since it is undesirable for the bottles to be overfilled. If, in part (a), you had been asked to consider whether the amount of salad dressing dispensed is equal to 235 ml and to use the 5% significance level, state the changes, if any, that you would have made to:

(i) the null hypothesis;

(ii) the alternative hypothesis;

(iii) the critical value;

(iv) the conclusion. *(4 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

A college has a total of 600 networked computers located in learning resource centres across the campus. The Systems Administrator recorded the number of computers used during the time period 2-3pm each day for the first three weeks of a 10-week term. The data are plotted on **Figure 1**.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Week 1	133	160	196	460	316
Week 2	180	200	226	508	360
Week 3	226	230	142	554	422

(a) On one day during the 3-week period, a fault occurred during the time period 2-3pm resulting in all 600 computers being shut down. When does the graph suggest that this shut down occurred? *(1 mark)*

(b) The 5-point moving averages for the above data are tabulated below.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Week 1			**P**	262.4	270.4
Week 2	276.4	286.0	294.8	304.0	**Q**
Week 3	293.2	302.4	314.8		

(i) Find the values of **P** and **Q**. *(3 marks)*

(ii) Plot **all** the 5-point moving averages on **Figure 1** and hence draw a trend line. *(3 marks)*

(iii) Explain why the trend shown by the moving averages cannot continue. *(2 marks)*

(c) (i) Calculate an estimate of the seasonal effect for Tuesday. *(3 marks)*

(ii) Hence estimate the number of computers that will be used on Tuesday of Week 4. *(3 marks)*

Turn over ►

4 The amount charged, £X, for entry to an exhibition depends on the status of the visitor. The following table shows the charges together with the probability that a visitor will have a particular status.

Status	**Charge** (£x)	**P($X = x$)**
Child under 16	2.00	0.36
Student	2.50	0.20
Senior citizen	3.00	0.16
Adult	4.00	0.28

(a) For entrance charges paid by visitors to the exhibition, calculate:

(i) the mean;

(ii) $E(X^2)$;

(iii) the standard deviation. *(6 marks)*

(b) Find the probability that the charge for a randomly selected visitor will be greater than or equal to:

(i) the mean;

(ii) the mode. *(4 marks)*

(c) Children under 5 are admitted free and have been omitted from the probability distribution shown above. If they were included in the probability distribution, explain whether:

(i) the mean would increase, stay the same or decrease;

(ii) the standard deviation would increase, stay the same or decrease. *(4 marks)*

5 [A sheet of graph paper is provided for use in this question]

The following table is copied from the *Annual Abstract of Statistics 2000*, ONS.

Duration of unemployment in the United Kingdom

Thousands, Spring each year, not seasonally adjusted

	Duration of unemployment										
										All 1 year or more	
	All unemployed	Less than 3 months	3 months & less than 6 months	6 months & less than 1 year	1 year & less than 2 years	2 years & less than 3 years	3 years & less than 4 years	4 years & less than 5 years	5 years or more	Number	As % of total
Year											
1989	2 075	647	306	333	252	133	103	71	229	788	38.0
1990	1 974	686	324	310	211	107	73	51	210	653	33.1
1991	2 414	834	466	434	276	113	67	41	179	676	28.0
1992	2 769	668	500	607	529	174	75	37	179	993	35.9
1993	2 936	600	474	599	612	287	109	57	196	1 262	43.0
1994	2 736	609	388	488	514	310	166	81	179	1 249	45.7
1995	2 454	568	386	422	404	243	143	102	182	1 074	43.8
1996	2 334	600	381	419	344	189	128	85	185	931	39.9
1997	2 034	599	317	326	288	148	83	72	197	789	38.8
1998	1 766	592	325	263	217	109	68	42	148	584	33.1
1999	1 741	620	326	276	209	87	45	39	138	518	29.7

Source: Labour Force Survey, ONS

(a) How many people in the United Kingdom had been unemployed for 5 years or more in 1990? *(2 marks)*

(b) How many people in the United Kingdom had been unemployed for 3 years or more in 1998? *(3 marks)*

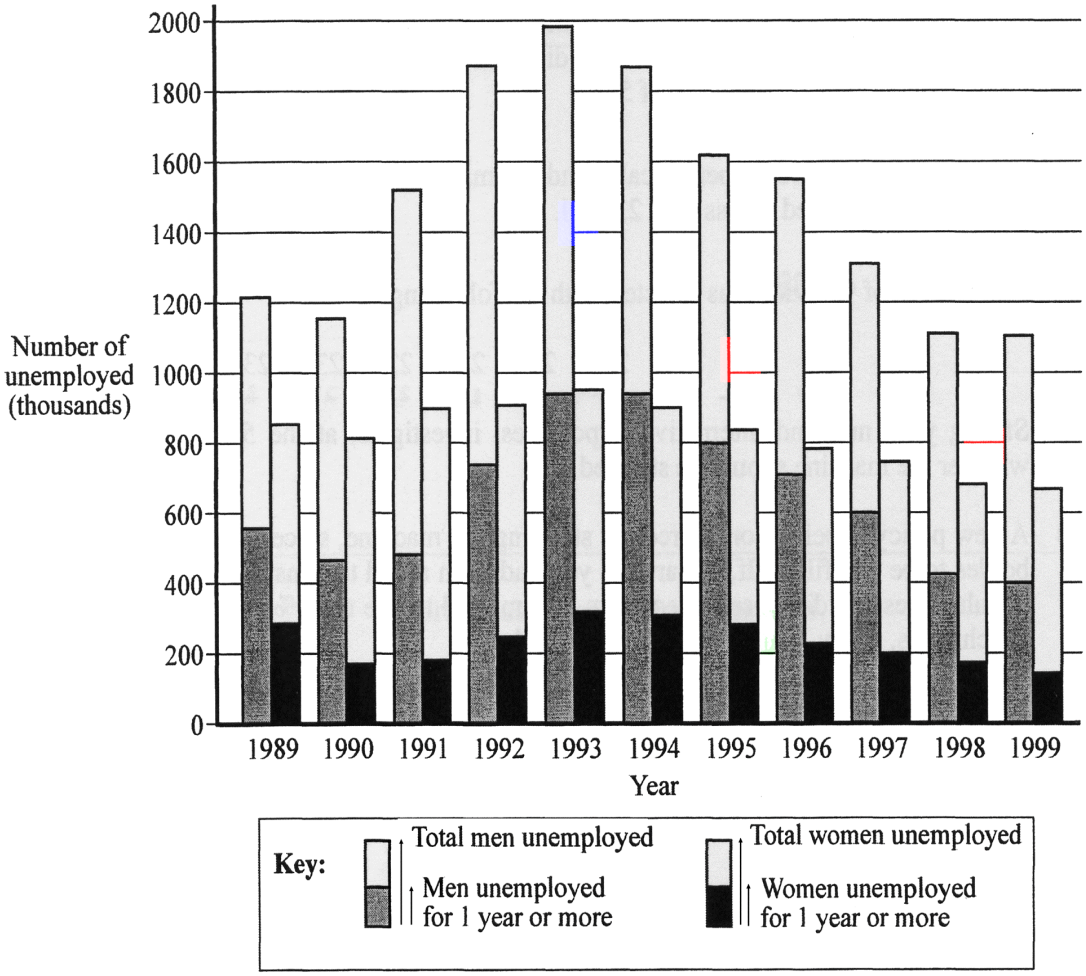
(c) Draw a histogram showing the duration of unemployment in the United Kingdom in 1999. Omit the category "5 years or more". *(6 marks)*

Turn over ►

(d) The bar chart below shows for each of the years 1989 to 1999:

- The total unemployed men;
- Men unemployed for 1 year or more;
- The total unemployed women;
- Women unemployed for 1 year or more.

Number of unemployed men and women in the United Kingdom



Comment on the unemployment of men and women over the period 1989 to 1999, making **four** distinct points. *(4 marks)*

6 A business, concerned with supporting the local sourcing of farm produce, holds a database with the number of producers in each region as shown in the table below.

Region	Number of producers on database
East Anglia	92
East Midlands	67
North	42
North West	54
South East	240
South West	270
West Midlands	38
Yorkshire & Humberside	97
Total	**900**

From the database, a sample of 60 producers is to be selected. The following suggestions are made as to how the sample could be selected.

Suggestion A
The producers are selected using a simple random sample.

Suggestion B
The producers are divided into three groups: South East, South West and all other regions. The producers are selected at random from each group. The number selected from each group is proportional to the number of producers in that group.

Suggestion C
A cluster sample is obtained by selecting four of the nine regions at random. Fifteen producers are selected at random from each of these four selected regions.

(a) Describe how random numbers could be used to select a sample for Suggestion A. *(5 marks)*

(b) Name the type of sampling described in Suggestion B. *(1 mark)*

(c) Give one reason for using Suggestion B in preference to Suggestion A. (*1 mark*)

(d) In Suggestion B, how many producers from the South West will be included in the sample? *(2 marks)*

(e) Discuss the suitability of Suggestion C in context. *(4 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS **SS02**
Unit Statistics 2

Insert for use in answering Question 3.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

TURN OVER FOR FIGURE 1

Turn over ▶

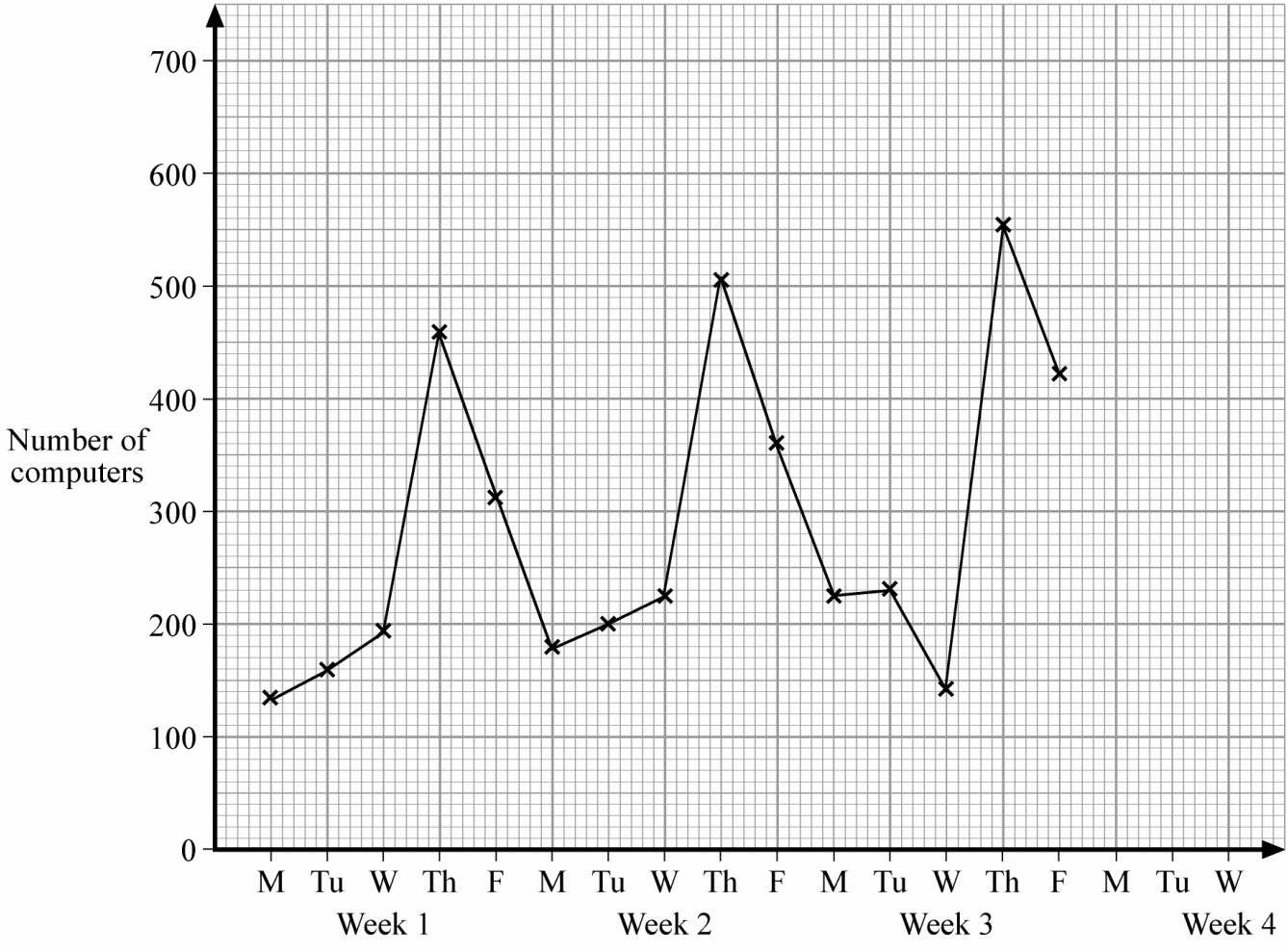


Figure 1 (for Question 3)

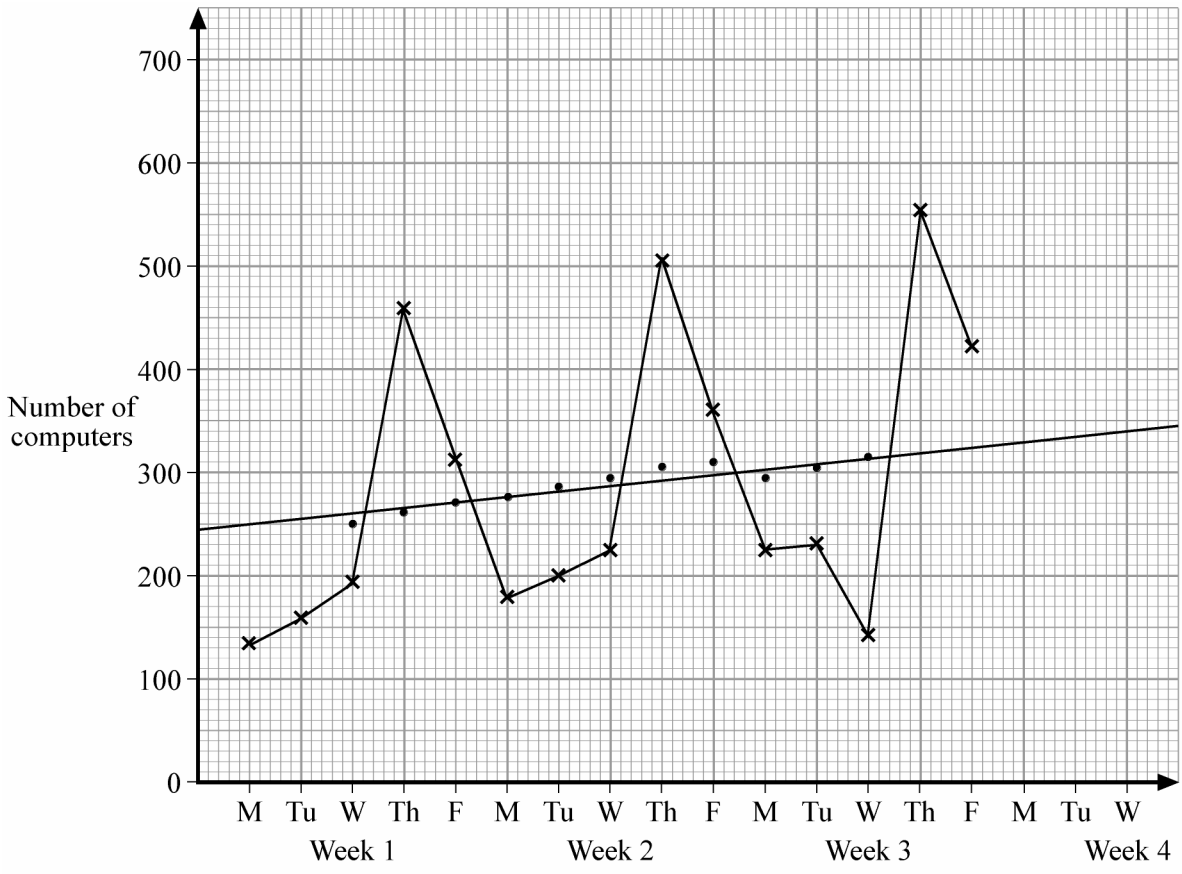
AQA
ASSESSMENT and QUALIFICATIONS ALLIANCE

SS02 Specimen

Question	Solution	Marks	Total	Comments
1	$\lambda = 1.2$			
(a)(i)	$P(X \leq 1) = 0.6626 = 0.663$ (3s.f.)	B1		
(ii)	$P(X = 1) = 0.6626 - 0.3012$	M1		
	$= 0.3614 = 0.361$ (3s.f.)	A1	3	
(b)	$\lambda = 1.2 + 1.8$	M1		Sum of Poissons
	$= 3.0$	A1		
	$P(X \geq 4) = 1 - P(X \leq 3)$	M1		
	$= 1 - 0.6472$			
	$= 0.3528 = 0.353$ (3s.f.)	A1	4	
	Total		**7**	
2(a)	$\bar{x} = \frac{2088}{9}$	B1		
	$= 232$			
	$H_0 : \mu = 235$	B1		
	$H_1 : \mu < 235$	B1		
	$z = \frac{232 - 235}{\frac{5}{\sqrt{9}}}$	M1		
	$= -1.8$	A1		
	c.v. $= -1.645$	B1		
	Reject H_0 and conclude machine should stop.	A1√	7	
(b)(i)	$H_0 : \mu = 235$ no change	B1		Requires correct null hypothesis in (a) or correctly stated null hypothesis here.
(ii)	$H_1 : \mu \neq 235$	B1		
(iii)	$\pm$ 1.96	B1		
(iv)	Accept H_0	A1√	4	Requires previous
	Total		**11**	

SS02 (cont)

Question	Solution	Marks	Total	Comments
3(a)	Wednesday Week 3	B1	1	
(b)(i)	$P = \frac{133 + 160 + 196 + 460 + 316}{5}$	M1		
	$= 253$	A1		
	$Q = \frac{226 + 508 + 360 + 226 + 230}{5}$	A1	3	
	$= 310$			
(ii)	See graph	M1		Plot AND trend in correct position
		A1		Allow one small error
		A1	3	At least one point either side



[30]

SS02 (cont)

Question	Solution	Marks	Total	Comments
(iii)	Trend is upward approximately linear.	E1		
	Cannot continue because there is a max. of 600 computers. Number of computers in use on Thurs is already approaching 600.	E1	2	
(c)(i)	$160 - 254 = -94$	M1		
	$200 - 282 = -82$			
	$230 - 310 = -80$	M1		Mean of 3 Tues
	Seasonal effect $= \dfrac{-256}{3}$			
	$= -85$ (below line)	A1	3	$-92 \sim -78$
(ii)	Tues week 4 = 339 – 85	M1		Read from line or calc.
	= 254	M1		– seasonal effect
		A1	3	246 ~ 262
	Total		**15**	
4(a)(i)	E(X) = 2 (0.36) + 2.5 (0.2) + 3 (0.16) + 4 (0.28)	M1		
	= £2.82			
(ii)	$E(X^2) = 2^2(0.36) + 2.5^2(0.2) + 3^2(0.16) + 4^2(0.28)$	M1		
		A1		
	= 8.61			
(iii)	s.d. $= \sqrt{8.61 - 2.82^2}$	M1		Method for variance (but not if called s.d.)
	= 0.8109	M1		Method for s.d.
	≈ 0.811	A1	6	0.811 (0.810 ~ 0.811) Allow 0.658 (0.657 ~ 0.658) if called variance
(b)(i)	P (X ≥ 2.82) = 0.16 + 0.28	M1		
	= 0.44	A1		
(ii)	Mode = 2	B1		
	P (X ≥ 2) = 1	B1	4	
(c)(i)	Decrease	B1		
	Children under 5 are charged the least (zero).	E1		
(ii)	Increase	B1		
	Spread of distribution is increased.	E1	4	
	Total		**14**	

SS02 (cont)

Question	Solution	Marks	Total	Comments
5(a)	210 000	B2	2	B1 210
(b)	(68 + 42 + 148) thousand	M1		
		A1		
	= 258 thousand	A1	3	
(c)				
	<3m 620 2480 3 – 6m 326 1304 6m – 1y 276 552 1 – 2y 209 209 2 – 3y 87 87 3 – 4y 45 45 4 – 5y 39 39	M1		Class widths
		M1		Frequency density
		A1		
	Graph	M1		Frequency density
		B1		Scale and label
		A1	6	Allow one small error
(d)	Total male unemployment is higher than total female unemployment.	B1		
	More men than women unemployed 1 year or more.	B1		
	Numbers rose from 1990 to 1993 then fell to 1999 for both men and women.	B1		
	Ratio $\frac{\text{unemployed 1 year or more}}{\text{total unemployed}}$ is higher for men than women.	B1	4	
	Etc.			
	Total		**15**	

[32]

SS02 (cont)

Question	Solution	Marks	Total	Comments
6(a)	List producers in alphabetical order. Number producers 000 – 899 Select 3-digit random numbers Ignore repeats and numbers > 899 Select 60 producers corresponding to random numbers.	E1 E1 E1 E1 E1	5	
(b)	Stratified random sampling	B1	1	
(c)	Ensures regions are fairly represented.	E1	1	
(d)	$\frac{270}{900} \times 60$ = 18	M1 A1	2	
(e)	Easier / cheaper Assuming each cluster (region) as being typical of the population – highly unlikely. Not a random sample of all producers on the database because (1) All producers on the database not equally likely e.g. S.W. and W. Midlands are selected, producers in W. Midlands have more chance (15/38) of being selected than producers in S.W. (15/270). (2) Not all combinations are possible e.g. cannot have 60 from S.W. Conclude not a suitable method in this context.	E1 E1 E1 E1	4	Allow any other 4 valid points
	Total		**13**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS SS03
Unit 3

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS03.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 As part of a transport survey, a random sample of 200 electors from three different areas, A, B and C, of a large city is asked whether they would prefer money to be spent on general road improvements or on improving public transport. The replies are summarised in the following table where the entries indicate the percentages in each category.

Preference \ Area	**A**	**B**	**C**
Road improvement	33	19	10
Improving public transport	9	14	15

(a) Explain why a χ^2 test for association between area and preference cannot be carried out on the data given in the table above. *(2 marks)*

(b) Construct a new table, using the given information, that can be analysed using a χ^2 test for association between area and preference. You are **not** required to carry out any analysis. *(3 marks)*

2 The coursework grades, on a scale of A* to G, where A* is the highest grade, and the examination marks of a sample of nine randomly selected students at a college are given in the following table.

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Coursework grade	A	A*	E	C	B	F	G	D	E
Examination mark	76	85	69	39	72	75	45	82	58

(a) Calculate the value of Spearman's rank correlation coefficient between coursework grades and examination marks. *(6 marks)*

(b) Investigate, at the 1% level of significance, whether your value calculated in part (a) indicates a positive association between coursework grades and examination marks. *(4 marks)*

(c) Give **one** reason why Spearman's rank, rather than the product moment, correlation coefficient is appropriate for the analysis in part (a). *(1 mark)*

3 Each member of a random sample of 15 adults was asked to assess a new alcoholic drink. The drink was to be awarded marks in each of five categories. The maximum mark in each category was 20. The total marks awarded were as follows:

88 65 75 48 75 75 68 50 67 85 89 92 61 45 95

A similar alcoholic drink, produced by a different manufacturer and assessed in the same way, had a median total mark of 65.

(a) Use the sign test at the 5% level of significance to investigate the claim that the median total mark given to the new alcoholic drink is greater than 65. *(6 marks)*

(b) Explain, in the context of the test carried out in part (a), the meaning of a Type II error. *(2 marks)*

(c) Give **one** reason why the Wilcoxon signed-rank test might be preferred to the sign test for investigating the claim in part (a). *(1 marks)*

TURN OVER FOR THE NEXT QUESTION

4 A university decided to introduce an enrichment programme for final year science undergraduates. The programme required them to study scientific terms and translate articles in a different language. The undergraduates were offered the choice of studying in either German or Japanese.

In the first year of the programme, a random sample of 90 final year science undergraduates was taken and the following table gives the gender of each undergraduate involved together with their choices.

	German	**Japanese**
Male	39	16
Female	21	14

(a) Investigate whether choice of language is independent of gender using a χ^2 test at the 5% significance level.

Interpret your conclusion in context. *(9 marks)*

(b) After the first year of running the enrichment programme, it was decided that two subjects, Ancient History and Archaeology, would be offered to final year science undergraduates, in addition to German and Japanese.

Following this decision, a random sample of final year science undergraduates was taken from the second year of the enrichment programme, and the following table gives the gender of each undergraduate involved together with their option choices.

	German	**Japanese**	**Ancient History**	**Archaeology**	**Total**
Male	8	2	5	20	**35**
Female	17	10	39	14	**80**
Total	**25**	**12**	**44**	**34**	**115**

(i) Calculate expected values for use in the analysis of this contingency table. *(2 marks)*

(ii) Give a reason why it may be necessary to combine columns in a contingency table. *(1 mark)*

(iii) Explain why German and Japanese are the appropriate columns to combine in the given table. *(2 marks)*

(iv) Carry out a test, at the 1% level of significance, to investigate whether option choice is independent of gender.

Interpret your conclusion in context. *(8 marks)*

[37]

5 One of the side effects of a drug treatment for a serious disease is that nerves can be damaged. A group of 12 patients, all suffering from this disease, is selected at random from those diagnosed at a large hospital. A measurement of the nerve amplitude, in micro amps, is taken from each patient before the drug treatment begins and again after six months of treatment.

Any **decrease** in amplitude would indicate that nerve damage has occurred. The results are given in the following table.

Patient	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Before	15.1	9.2	10.8	12.7	11.8	15.7	14.5	11.5	16.9	7.1	10.1	15.0
After	16.0	9.4	6.9	8.7	11.0	14.3	14.6	11.2	15.7	3.4	7.2	12.7

(a) Explain why it is preferable to obtain measurements from the **same** group of 12 patients before and after treatment, rather than obtaining measurements from one group of patients before treatment is started and another group of patients after six months of treatment.

(2 marks)

(b) Carry out a Wilcoxon signed-rank test, at the 1% significance level, to investigate whether the drug treatment reduces nerve amplitude. Interpret your conclusion in context.*(9 marks)*

6 An investigation is carried out into the effect of depressant and stimulant drugs on arithmetic ability. A sample of 16 students, of similar ability, who had agreed to take part in the investigation, are randomly assigned to be given either a depressant drug, a stimulant drug or a placebo which contains no active ingredient.

Each student was given the same 80 arithmetic problems. The number of these problems solved correctly by each student in one hour is given in the table below.

Depressant drug	Stimulant drug	Placebo
0	46	42
1	58	49
39	64	52
45	68	56
50	79	78
	80	

(a) Give a reason, based on the data, why a normal distribution is unlikely to provide an adaquate model for the number of correctly solved problems in each group. *(2 marks)*

(b) Carry out a Kruskal-Wallis test, using a 5% significance level, to investigate whether there is any difference between average numbers of correctly solved problems for students taking a depressant or stimulant drug, or a placebo.

Interpret your conclusion in context. *(15 marks)*

You may assume that the test statistic, H, for a Kruskal-Wallis test is given by

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{T_i^{\,2}}{n_i} - 3(N+1)$$

END OF QUESTIONS

Question	Solution	Marks	Total	Comments
1(a)	Raw frequencies are not supplied – only percentage given.	E1		Percentage mentioned
	χ^2 test requires actual frequencies, not percentages	E1	2	Requirements of χ^2 test
(b)	**Area Pref** / **A** / **B** / **C**	M1		Conversion to raw data
	Road Imp 66 38 20	A1		One row correct
	Imp public transport 18 28 30	A1	3	All correct
	Total		**5**	
2(a)	**Student** / **1** / **2** / **3** / **4** / **5** / **6** / **7** / **8** / **9**			
	Coursework rank 2 1 6.5 4 3 8 9 5 6.5			
	Exam rank 3 1 6 9 5 4 8 2 7			
		M1		Attempt at ranks
		M1		tied ranks
	$r_s = 0.527$ (3 sf from calc)	B3		
		A1	6	
				alternative
				$d = 1, 0, ½, 5, 2, 4, 1, 3, ½$
				$\sum d^2 = 56½$ B1
				$r_s = 1 - \frac{6\times56.5}{9\times80} = 0.529$ M1, A1
(b)	H_o Coursework grades and Examination marks are independent	B1		
	H_1 Coursework grades and Examination marks have a positive association			
	1 tail 1%			
	cv = 0.7667			
	r_s < cv	B1		For cv
	Accept H_o No significant evidence at 1% level to suggest a positive association between coursework grades and exam marks.	M1		For comparison ts/cv
				$r_s = 0.527$ or 0.529
		A1	4	
(c)	PMCC requires measured data and grades only are given for coursework.	E1	1	
	Total		**11**	

[40]

SSO3 (cont)

Question	Solution	Marks	Total	Comments
3(a)	H_o pop median = 65 H_1 pop median > 65 1 tail 5%	B1		
	Signs + . + – + + + –+ + + + – – +	M1		Signs
	n = 14 ts = 10+/ 4 –	A1		test stat correct and identified
	Binomial model B (14, 0.5)	M1		Binomial model used and Probability attempted
	P (≥ 10+) = P(≤4 -) = 0.0898 > 0.05 for one tail test	M1		Comparison of Binomial probability with 0.05
	Accept H_o. There is insufficient evidence, at the 5% level, to suggest that the median is greater than 65	A1	6	Conclusion in context
(b)	To conclude that the median is unchanged at 65 when, in fact, the median grade for the new drink is actually greater than 65.	E1 E1	2	General explanation of Type II error Explanation in context
(c)	Wilcoxon uses the rank order of the differences, not just the sign.	E1	1	Or Type II error is less likely to occur.
	Total		**9**	
4(a)	H_o Gender not associated with choice of language H_1 Gender associated with choice of language 1 tail 5%	B1		
	German **Japanese** **Male** 39 36.67 16 18.33 **Female** 21 23.33 14 11.67	M1		Totals used for E E method correct
	$ts = \sum \frac{(\lvert O-E\rvert - 0.5)^2}{E} =$	M1		Use of Yates
	$\frac{1.83^2}{36.67}+\frac{1.83^2}{18.33}+\frac{1.83^2}{23.33}+\frac{1.83^2}{11.67} = 0.704$	m1 A1		Ts sum with correct denominators For ts in range 0.70 – 0.71
	cv df = 1 5% cv = 3.84	B1		For cv
	ts < 3.84	m1		For comparison ts/cv
	Accept H_o No sig evidence to suggest an association	A1		
	Females are no more or less likely than males to select either of the 2 languages offered.	E1	9	

SSO3 (cont)

Question	Solution	Marks	Total	Comments
4(b)(i)	<table><tr><th></th><th>Germ</th><th>Japan</th><th>Ancient History</th><th>Arch</th></tr><tr><td>**M**</td><td>8 7.61</td><td>2 3.65</td><td>5 13.39</td><td>20 10.35</td></tr><tr><td>**F**</td><td>17 17.39</td><td>10 8.35</td><td>39 30.61</td><td>14 23.65</td></tr></table>	M1 A1	 2	E method correct Row total × column total / grand total All correct
(ii)	When calculations from low observed frequencies result in an E < 5	B1	1	
(iii)	The expected value for the cell Male/Jap is 3.65 which is less than 5. German is the appropriate choice to combine with Japanese as it is also a foreign language.	E1 E1	 2	Identifying E < 5 Both foreign languages
(iv)	H_o Gender not associated with choice of subject H_1 Gender associated with choice of subject 1 tail 1%	B1		
	ts = $\sum \frac{(O-E)^2}{E}$ = $\frac{1.26^2}{11.26}+\frac{8.39^2}{13.39}+\frac{9.65^2}{10.35}+\frac{1.26^2}{25.74}+\frac{8.39^2}{30.61}+\frac{9.65^2}{23.65}=20.7$	m1 A1		Use of $(O-E)^2$ Sum with correct denominator For ts in range 20.4 – 30.0
	cv df = 2 1% cv = 9.21 ts > 9.21	B1 m1		For cv For comparison ts/cv
	Reject H_o Sig evidence to suggest an association	A1		
	Proportions of males/females studying language are similar to those expected but far more males study Archaeology and far fewer males study Ancient History than expected (and the reverse is true for females)	E1 E1	 8	Mention strong association between males/Archaeology & females/Ancient History
	Total		**22**	

SSO3 (cont)

Question	Solution	Marks	Total	Comments
5(a)	A paired trial will reduce experimental error since the same patient is measured both before and after treatment.	E1		Idea of elimination of differences.
	If two separate groups of patients are used, then any differences identified in nerve amplitudes might be due to individual differences rather than the effect of the treatment.	E1	2	Full explanation showing understanding of reduction of experimental error in context
(b)	H_0 $\eta_{\text{difference}} = 0$ H_1 $\eta_{\text{difference}} < 0$ 1 tail 1%	B1		Or H_0 $\mu_{\text{difference}} = 0$ H_1 $\mu_{\text{difference}} < 0$
	Patient / **Difference B – A** / **Rank** – / +			
	1 / – 0.9 / 5			
	2 / – 0.2 / 2	M1		For differences
	3 / + 3.9 / 11	A1		(B – A or A – B)
	4 / + 4.0 / 12			
	5 / + 0.8 / 4			
	6 / + 1.4 / 7			
	7 / – 0.1 / 1			
	8 / + 0.3 / 3			
	9 / + 1.2 / 6	m1		For ranks (1 = smallest $\lvert$diff$\rvert$)
	10 / + 3.7 / 10	A1		
	11 / + 2.9 / 9			
	12 / + 2.3 / 8			
	Rank totals $T_- = 8$ $T_+ = 70$	m1		For total of + / - ranks
	Test stat T = 8 critical value = 10	B1		For cv
	T < 10 Reject H_0	M1		For comparison ts/cv
	There is significant evidence to suggest that the drug treatment does cause nerve damage (since there is evidence that the median/mean nerve amplitude decreases after treatment)	A1	9	
	Total		**11**	

SSO3 (cont)

Question	Solution	Marks	Total	Comments
6(a)	Normal distribution is unlikely to be an adequate model because there is evidence that the numbers of correctly solved problems in each group are not symmetrically distributed.	E1		Mention of distribution not symmetrical
	Each distribution contains scores near 0 or 80, suggesting that each distribution is not tailing off as it would for a normal distribution (given that the distributions are discontinuous at 0 and 80).	E1	2	Clear identification of unusual distribution at low/high scores suggesting it is very unlikely to be normal.
(b)	H_0 Samples from identical populations H_1 Samples not from identical populations 5% sig level			or H_0 $\eta_D = \eta_S = \eta_P$ H_1 at least two of η_D, η_S, η_P do differ
	Ranks Depressant: 1, 2, 3, 5, 8 Stimulant: 6, 11, 12, 13, 15, 16 Placebo: 4, 7, 9, 10, 14	M1 A2		Ranks as one group
	$T_D = 19$ $T_S = 73$ $T_P = 44$ $n_D = 5$ $n_S = 6$ $n_P = 5$	m1 A1		Totals Any one corect
	$\sum_{i=1}^{k} \frac{T_i^2}{n_i} = \frac{19^2}{5} + \frac{73^2}{6} + \frac{44^2}{5} = 1347.57$	m1		
	$H = \frac{12}{16 \times 17} \times 1347.57 - (3 \times 17) = 8.45$	m1 A1		test stat H = $\frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{T_i^2}{n_i} - 3(N+1)$
	Critical value from $\chi^2_2 = 5.99$ $H > 5.99$	B1 M1		
	Sig evidence to reject H_0	A1		
	There is significant evidence that at least two of the median number of correctly solved scores (from those on a depressant or stimulant drug or placebo) do differ.	E1		
	It would appear that those students taking the depressant drug perform significantly worse (median score lower) that those taking the stimulant drug	E1	15	
	Total		**17**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS SS04
Unit 4

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS04.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The random variable, R, may be modelled by a binomial distribution with $n = 104$ and $p = 0.008$:

(a) name the distribution which may by used as a suitable approximation for R;

(b) evaluate the mean and the standard deviation of this approximating distribution. *(3 marks)*

2 In a traffic study, carried out prior to the introduction of a park and ride scheme, it had been established that 31% of cars entering a city centre carried passengers (in addition to the driver). Following the introduction of the scheme, among a random sample of 200 cars entering the city centre, 84 carried passengers.

(a) Calculate an approximate 90% confidence interval for the proportion of cars entering the city centre which carried passengers after the introduction of the scheme. *(7 marks)*

(b) Comment on the claim that, following the introduction of the scheme, the proportion of cars with passengers entering the city centre has not changed. *(2 marks)*

(c) Give two reasons why the confidence interval you have calculated in part (a) is approximate rather than exact. *(2 marks)*

3 A supermarket stocks two types of bottled water; still and sparkling. During autumn, the daily volume, X litres, of still water sold is a normal random variable with mean 86 and standard deviation 15. Independently and during the same period, the daily volume, Y litres, of sparkling water sold is also a normal random variable but with mean 72 and standard deviation 10.

(a) Determine the probability that, in total over a five day autumn period, the supermarket sells less than 400 litres of still water. *(5 marks)*

(b) State the distribution of $X + Y$, the total amount of bottled water sold during an autumn day. *(3 marks)*

(c) Hence determine the probability that, during a randomly chosen autumn day, the supermarket sells more than 200 litres of bottled water. *(2 marks)*

(d) Calculate the probability that, on a randomly chosen autumn day, the supermarket sells at least 25 per cent more still water than sparkling water. *(5 marks)*

4 A 'safer routes to school' campaign is to be undertaken by a city council which wishes to encourage parents and children to walk or cycle to school rather than to use private cars. As a first step it is decided to estimate the mean distance travelled to school by junior school children. In a pilot study the following distances in miles travelled by a sample of children in the city were obtained.

1.2 0.1 0.7 0.8 0.2 0.1 3.9 0.3 0.1 1.1

(a) Calculate a 95% confidence interval for the mean distance travelled to school by all junior school children in the city. *(8 marks)*

(b) State two assumptions you needed to make in order to answer part (a). *(2 marks)*

(c) Do the data provide any reason to suspect that one necessary assumption may not be true? Explain your answer. *(2 marks)*

In a larger survey, the distances travelled to school by 140 randomly selected junior school children in the city were found to have a mean of 1.01 miles and a standard deviation of 0.980 miles.

(d) Calculate a 99% confidence interval for the mean distance travelled to school by junior school children in the city. *(4 marks)*

(e) Explain why you did not need to make any assumptions to calculate the confidence interval in part (d). *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

5 A hotel group buys large quantities of towels for use by guests. A large batch is ordered from a new supplier and it is specified that:

(i) the mean length of the towel should be 1100 mm;
(ii) the mean number of visual defects per towel should not exceed 1.8;
(iii) not more than 7% of the towels should fail a test of water absorption.

(a) Towels which are too short may lead to customer complaint. Towels which are too long will lead to increased laundry costs. The lengths, in mm, of a random sample of nine towels from the batch were as follows:

1125 1102 1086 1134 1114 1127 1083 1131 1116

Test the hypothesis that the mean length of towels is 1100 mm. Use the 10% significance level and assume that the lengths are normally distributed. *(10 marks)*

(b) A random sample of 20 towels from the batch was inspected and 44 visual defects were observed in total. Using the 5% significance level, examine whether the mean number of visual defects per towel exceeds 1.8. You may assume that visual defects are distributed independently at random at a constant average rate. *(9 marks)*

(c) A random sample of 30 towels from the batch was subjected to the test of water absorption. Four failed and the rest passed. Using the 5% significance level, examine whether more than 7% of towels in the batch would fail the water absorption test. *(6 marks)*

(d) Comment, briefly, on the suitability of this batch of towels. *(3 marks)*

END OF QUESTIONS

SS04 Specimen

Question	Solution	Marks	Total	Comments
1(a)	Poisson	B1		
(b)	mean $104 \times 0.008 = 0.832$	M1		
	s.d. $\sqrt{0.832} = 0.912$	A1	3	
	Total		**3**	
2(a)	Binomial $n = 200$ $p = \frac{84}{200} = 0.42$	B1 B1		
	Approximate by normal, mean 0.42, s.d. $\sqrt{\frac{0.42 \times 0.58}{200}} = 0.03490$	M1 m1		
	Approximate 90% confidence interval $0.42 \pm 1.6449 \times \sqrt{\frac{0.42 \times 0.58}{200}}$	B1 m1		
	0.42 ± 0.0574 $0.363 - 0.477$	A1	7	
(b)	Lower limit of confidence interval is above 0.31, indicating substantial evidence that the proportion has increased.	E1 E1	2	
(c)	Normal distribution used as an approximation to binomial.	E1		
	Value of p used to calculate standard deviation was estimated from the sample and will not be exact.	E1	2	
	Total		**11**	

Question	Solution	Marks	Total	Comments
3(a)	Distribution of 5-day total of still water is Normal mean $5 \times 86 = 430$	B1		
	variance $5 \times 15^2 = 1125$ (s.d. 33.54)	B1		
	$z = \dfrac{(400 - 430)}{\sqrt{1125}} = -0.894$	M1		
	probability less than 400 litres still water	m1		
	is $1 - 0.814 = 0.186$	A1	5	0.184 - 0.187
(b)	Normal,mean 86 + 72 = 158	B1		
	variance $15^2 + 10^2 = 325$ (s.d. 18.03)	B1		
		B1	3	
(c)	z = (200 - 158)/√325 = 2.330			
	Probability sells more than 200 litres	M1		
	= 1 - 0.9901 = 0.0099	A1	2	0.0099 - 0.01
(d)	Supermarkety sells at least 25% more still than sparkling water if X>1.25Y. i.e. X - 1.25Y>0	B1		
	Distribution of $X - 1.25Y$ is Normal mean $86 - 1.25 \times 72 = -4$	M1		
	variance $15^2 + 1.25^2 \times 10^2 = 381.25$ (s.d. 19.53)	m1		
	$z = \dfrac{(0 - (-4))}{\sqrt{381.25}} = 0.205$	m1		
	probability $1 - 0.581 = 0.419$	A1	5	$0.417 - 0.421$
	Total		**15**	
4(a)	$\bar{x} = 0.85 \quad s = 1.151$	B1		
	95% confidence interval	B1		
	$0.85 \pm 2.262 \times \dfrac{1.151}{\sqrt{10}}$	M1		
	0.85 ± 0.823	m1		
	$0.027 - 1.673$	m1		
		B1		
		B1√		
		A1	8	
(b)	Distribution normal	E1		
	Sample random	E1	2	
(c)	Yes - 3.9 is much larger than other observations - suggesting a (positive)	E1		
	skew distribution.	E1	2	any reasonable explanation
(d)	$1.01 \pm 2.5758 \times \dfrac{0.980}{\sqrt{140}}$	M1		
		B1		
	1.01 ± 0.213	m1		
	$0.797 - 1.223$	A1	4	
(e)	Sample large → mean normally	E1		
	distributed.	E1	2	
	Total		**18**	

SS04 (cont)

Question	Solution	Marks	Total	Comments
5(a)	$H_0 : \mu = 1100$	B1		
	$H_1 : \mu \neq 1100$	B1		
	$\bar{x} = 1113.1$ $\quad s = 18.92$	B1 B1		
	$t = \dfrac{(1113.1 - 1100)}{\left(\dfrac{18.92}{\sqrt{9}}\right)} = 2.08$	M1 m1 A1		
	critical value t_8 is 1.86	B1		
	Reject H_0, conclude significant evidence that mean not equal to (greater than) 1100.	B1√ A1√	10	
(b)	H_0: Poisson, $\mu = 1.8$ per towel	B1		
	H_1: $\mu > 1.8$ per towel	B1		
	If H_0 true the number of visual defects on 20 towels will be Poisson mean $20 \times 1.8 = 36$	B1		
	Approximate by normal mean 36 standard deviation $\sqrt{36} = 6$	M1 A1		
	$z = \dfrac{(43.5 - 36)}{6} = 1.25$	m1		Allow without continuity correction (z=1.33)
	Critical value 1.6449	B1A1		
	Accept H_0 no significant evidence that mean number of visual defects per towel exceeds 1.8.	A1√	9	
(c)	H_0: Binomial $n = 30$ $\quad p = 0.07$	B1		
	H_1: $p > 0.07$	B1		
	P(4 or more failing test) $= 1 - 0.8450$	M1		
	$= 0.155$	m1 A1		
	> 0.05			
	Accept H_0 no significant evidence that proportion failing would exceed 7%.	A1√	6	
(d)	Evidence mean length not equal to 1100. Sample mean exceeds 1100 but only by a relatively small amount. Laundry costs increased but probably not by much.	E1		
	Insufficient evidence of too many visual defects (although observed number exceeded 1.8 per towel) or too many failing water absorption test (although observed proportion exceeded 0.07).	E1		Any three appropriate comments
	Fairly satisfactory.	E1	3	
	Total		**28**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS SS05
Unit 5

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SSO5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The volume, in millilitres, of each of a random sample of 8 pints of draught beer, served by an inexperienced bar-person, *I*, was measured. From the measurements, an unbiased estimate of the population variance was calculated as 73.5 ml ^{2}.

The volume, in millilitres, of each of a random sample of 12 pints of draught beer, served by an experienced bar-person, *E*, was measured. From the measurements, an unbiased estimate of the population variance was calculated as 29.4 ml ^{2}.

Assuming measurements to be normally distributed, investigate, at the 5% level of significance, the claim that the variability in the volume of beer served is greater for *I* than for *E*. *(6 marks)*

2 New unsharpened pencils have a nominal length of 180 mm. The cutting machine is such that the length, in millimetres, of pencils may be described by a random variable X with a rectangular distribution on the interval $178 \leqslant x \leqslant 180 + c$, where $c > 0$.

An analysis of the lengths of a random sample of 840 pencils reveals that 400 are less than 180 mm.

(a) Show that the estimate of c is 2.2 mm. *(3 marks)*

(b) The specification requires that the lengths of at least 95% of the pencils should be within 1 per cent of the nominal length.

Determine whether the pencils are likely to conform to this specification. *(4 marks)*

3 A school bus travels the same route each morning. The time taken, T minutes, from its first stop to the school is recorded on each of a random sample of 30 mornings.

The recorded times then gave:

$$\sum (t - \bar{t})^2 = 478.5$$

where $\bar{t}$ denotes the sample mean.

(a) Stating the necessary distributional assumption, construct a 95% confidence interval for the standard deviation of the morning journey time of the bus. *(7 marks)*

(b) Hence comment on the claim that the standard deviation of the morning journey time of the bus is 5 minutes. *(2 marks)*

4 A multiple-choice test, consisting of five questions, is taken by each of a random sample of 250 first-year students. The number of correct answers achieved by these students is summarised in the following table.

Number of correct answers	0	1	2	3	4	5
Number of students	5	13	61	84	72	15

(a) Show that an estimate of p, the probability of a correct answer, is 0.6. *(2 marks)*

(b) Using a χ^2 goodness of fit test and the 10% level of significance, test whether a binomial distribution provides an adequate model for these data. *(11 marks)*

(c) Comment on the suggestion that all of the students had a similar chance of answering each question correctly. *(2 marks)*

Turn over ▸

5 It is claimed that men are faster than women at solving simple number puzzles.

To investigate this claim, the same puzzle was given to random samples of 120 men and 150 women under identical experimental conditions. The time taken, X seconds, by each person to solve the puzzle was recorded.

The results are summarised in the table below, together with known values for the population standard deviations.

	Sample		**Population**
	Size (n)	**Mean** ($\bar{x}$)	**Standard deviation** (σ)
Men	120	232	18
Women	150	237	15

(a) Investigate, at the 1% level of significance, whether the claim can be supported. *(7 marks)*

(b) Explain why **no** assumption regarding the distributions of times was necessary when carrying out your test in part (a). *(2 marks)*

(c) (i) State the additional information you would have needed to carry out the test in part (a), if the values for the population standard deviations were unknown. *(1 mark)*

(ii) Indicate with justification, what, if any, subsequent changes would be required to the test procedure. *(2 marks)*

6 The time, D days, between successive accidents at a factory can be modelled by an exponential distribution with mean 20.

(a) Write down the numerical value for the standard deviation of D. *(1 mark)*

(b) Calculate the probability that the time between successive accidents at the factory is:

(i) more than 25 days;

(ii) between 15 and 30 days. *(5 marks)*

(c) Given that there are no accidents during a 25-day period, determine the probability that there are no accidents during the next 25 days. Justify your answer. *(3 marks)*

(d) Given that the factory is open five days each week, specify the distribution of the **weekly number** of accidents at the factory. *(2 marks)*

7 A commuter drives to work each day by one of two routes, *L* and *S*. Although, in terms of distance, route *S* is the shorter, the commuter suspects that, on average, the journey times for the two routes are the same.

To investigate this suspicion, the commuter records the time, in minutes, taken on each of a random sample of journeys using route *L* and on each of a random sample of journeys using route *S*. The results are shown below.

Route	**Journey times**									
L	25	26	22	23	29	26	27	31	26	20
S	28	30	28	32	21	26				

Journey times by each route may be assumed to be normally distributed with a common variance of σ^2.

(a) Calculate the value for the pooled estimate of σ^2. *(4 marks)*

(b) Hence test, at the 10% level of significance, the commuter's suspicion. *(9 marks)*

(c) Indicate, with a reason, the preliminary test that you would have carried out if a common variance could **not** be assumed. *(2 marks)*

END OF QUESTIONS

SS05 Specimen

Question	Solution	Marks	Total	Comments
1	$H_0 : \sigma_I^2 = \sigma_E^2$ $H_1 : \sigma_I^2 > \sigma_E^2$	B1		Both
	SL $\alpha = 0.05$			
	DF $\nu_1 = 7$ $\nu_2 = 11$	B1		cao both
	CV $F = 3.012$	B1		awrt 3.01
	$F = \frac{s_I^2}{s_E^2}$ if H_0 true	M1		Use of variance ratio
	$= \frac{73.5}{29.4} = 2.5$	A1		cao
	Thus insufficient evidence, at 5% level, to support claim that variability is greater for *I* than for *E*	A1√	6	ft on *F* and CV
	Total		**6**	
2(a)	$P(X < 180) = \frac{180-178}{(180+c)-178}$	M1		Use of ratio or equivalent
	$= \frac{2}{2+c}$	A1		cao or equivalent
	$\Rightarrow \frac{2}{2+c} = \frac{400}{840}$	m1		Equating to $\frac{400}{840}$
	$\Rightarrow c = \frac{880}{400} = 2.2$		3	**ag**
(b)	1% of 180 = 1.8	B1		cao
	$P(178.2 < X < 181.8) =$	M1		Both sides of 180
	$\frac{181.8-178.2}{182.2-178} = \frac{3.6}{4.2} = 0.86$	A1		awrt
	Since 0.86 < 0.95 (95%) pencils are not likely to conform to specification	E1√	4	ft on calculated percentage
	Total		**7**	

Question	Solution	Marks	Total	Comments
3(a)	Assumption: Time, $T \sim$ Normal	B1		
	CI for σ^2 is: $\frac{\sum(t-\bar{t})^2}{\chi^2(\mathrm{U})}$ to $\frac{\sum(t-\bar{t})^2}{\chi^2(\mathrm{L})}$	M1		Use of CI formulae or equivalent ($s^2 = 16.5$)
	DF $\nu = 29$	B1		cao
	95% $\Rightarrow$ 0.025 & 0.975 so χ^2 values are: 16.047 and 45.722	B1		Both, awfw 16.0 to 16.1 awfw 45.7 to 45.8
	CI for σ^2 is: $\frac{478.5}{45.722}$ to $\frac{478.5}{16.047}$	A1√		ft on χ^2 values
	ie (10.465, 29.819)			
	CI for σ is: $\sqrt{(10.465,\ 29.819)}$	M1		Use of $\sqrt{\text{CI for } \sigma^2}$
	ie (3.24, 5.46)	A1	7	awfw 3.23 to 3.24; awrt 5.46
(b)	Accept claim that $\sigma = 5$	B1√		ft on (a)
	Value of 5 $\in$ CI	E1√	2	ft on (a)
	Total		**9**	
4(a)	$p = \frac{\sum fx}{5 \times 250}$	M1		Use of ratio
	$= \frac{750}{1250}$ (= 0.6)	A1	2	cao; **ag**
(b)	H_0: $X \sim$ B(5, 0.6)	B1		Accept equivalent in words
	H_1: not H_0			Not necessary
	0^x O P($X = x$) E $\frac{(O-E)^2}{E}$			
	0 5 } 0.01024 2.56 }	M1		Binomial probabilities
	1 13 } 0.07680 19.2 } 0.650			
	2 61 0.23040 57.6 0.201	M1		E = Probability × 250
	3 84 0.34560 86.4 0.067			
	4 72 0.25920 64.8 0.800	A2,1		6 Es awrt 1dp (A1 for 4 or 5)
	5 15 0.07776 19.44 1.014			
	T 250 1 250 2.732	M1		Combining Es and Os
	$\chi^2 = \sum \frac{(O-E)^2}{E}$ = 2.730 to 2.735	M1 A1		Use of formula awfw
	SL $\alpha = 0.10$			
	DF $\nu = (6 - 1) - 1 - 1 = 3$	B1		cao
	CV $\chi^2 = 6.251$	B1√		awrt 6.25; ft on $\nu \in$ (3, 4, 5)
	Thus insufficient evidence, at 10% level, to conclude that binomial distribution is not adequate model	A1√	11	ft on χ^2 and upper CV
(c)	As binomial model not rejected, it suggests that students had an equal chance of answering each question correctly (p constant)	B1√ B1√	2	ft on conclusion in (b) ft on conclusion in (b)
	Total		**15**	

SS05 (cont)

Question	Solution	Marks	Total	Comments
5(a)	H_0: $\mu_M = \mu_W$	B1		
	H_1: $\mu_M < \mu_W$	B1		
	SL $\alpha = 0.01$			
	CV $z = -2.3263$	B1		awfw –2.33 to – 2.32 Accept + if consistent with H_1
	$z = \dfrac{(\bar{x}_M - \bar{x}_W) - (\mu_M - \mu_W)}{\sqrt{\dfrac{\sigma_M^2}{n_M} + \dfrac{\sigma_W^2}{n_W}}}$	M1		Use of formula
	$= \dfrac{232 - 237}{\sqrt{\dfrac{18^2}{120} + \dfrac{15^2}{150}}} = -2.44$	A1		Substitution of given values
		A1		awfw –2.45 to –2.43; Accept + if consistent with H_1
	Thus evidence, at 1% level, to support claim that men are faster than women	A1√	7	ft on z and CV
(b)	Application of Central Limit Theorem	B1		
	because of large sample sizes	E1	2	
(c)(i)	Sample variances OR Σx^2 values	B1	1	
(ii)	None	B1		
	because of large sample sizes	E1	2	
	Total		**12**	
6(a)	Standard deviation, $\sigma = 20$	B1	1	cao
(b)(i)	$P(D > 25) = 1 - \left(1 - e^{-\frac{25}{20}}\right)$	M1		Use of associated df or pdf
	or $= \left[-e^{-\frac{d}{20}}\right]_{25}^{\infty}$	A1		Correct expression
	$= 0.287$	A1		awfw 0.286 to 0.287
(ii)	$P(15 < D < 30)$			
	$= P(D < 30) - P(D < 15)$	M1		Difference or equivalent
	$= \left(1 - e^{-1.5}\right) - \left(1 - e^{-0.75}\right) = 0.249$	A1	5	awrt
(c)	Exponential distribution has 'no memory' so required probability is	M1		Use of this property
	equal to (b)(i)	A1		Or equivalent
	$= 0.287$	A1√	3	ft on (b)(i)
(d)	Poisson	B1		
	Mean or parameter $= \dfrac{5}{20} = 0.25$	B1	2	cao
	Total		**11**	

SS05 (cont)

Question	Solution	Marks	Total	Comments
7(a)	$s_L^2 = \frac{94.5}{9} = 10.5$	B1		cao
	$s_S^2 = \frac{71.5}{5} = 14.3$	B1		cao
	$s_P^2 = \frac{9\times10.5 \ + \ 5\times14.3}{14} = \frac{166}{14}$	M1		Use of pooling formula
	$= 11.857$	A1	4	awfw 11.8 to 11.9; s_P = 3.43 to 3.45
(b)	H_0: $\mu_L = \mu_S$	B1		
	H_1: $\mu_L \neq \mu_S$	B1		
	SL $\alpha = 0.10$			
	DF $\nu = 14$	B1		cao
	CV $t = \pm1.761$	B1		awrt ±1.76
	$\bar{x}_L = 25.5 \quad \bar{x}_S = 27.5$	B1		cao both
	$z = \frac{(\bar{x}_L - \bar{x}_S) - (\mu_L - \mu_S)}{\sqrt{s_P^2\left(\frac{1}{n_L} + \frac{1}{n_S}\right)}}$	M1		Use of formula
	$= \frac{\pm(25.5 - 27.5)}{\sqrt{11.857\times\left(\frac{1}{10} + \frac{1}{6}\right)}} = \pm1.12$	A1√		ft on $\bar{x}_L$, $\bar{x}_S$ and s_P^2
		A1		awfw ±1.10 to ±1.15
	Thus no evidence, at 10% level, to dispute commuter's suspicion that the journey times for the two routes are the same	A1√	9	ft on *t* and CV
(c)	*F*-test	B1		
	To check that pooling of sample variances is valid	E1	2	Or equivalent
	Total		**15**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS SS06
Unit 6

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS06.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 As part of an investigation into the dental health of children, a dental school identifies sets of triplets. For each set of triplets, one child is given a placebo, one an established fluoride tablet and one a new enhanced fluoride tablet.

(a) Identify those children who constitute:

(i) the control group; *(1 mark)*

(ii) the experimental group. *(1 mark)*

(b) Name the blocking factor. *(1 mark)*

(c) Double blind trials are to be employed in the investigation. Explain, in context, both the meaning and the purpose of such trials. *(3 marks)*

2 In a comparison of the drying times, in hours, of five different paints, each paint was applied to six different surfaces and then allowed to dry under identical conditions.

An analysis of the results provided the following information.

Source of variation	**Sum of squares**
Between paints	2.76
Between surfaces	1.40
Error (Residual)	
Total	6.04

(a) Assuming all drying times to be normally distributed with a common variance, investigate, at the 1% level of significance, for a difference between the mean drying times of the five paints. *(6 marks)*

(b) Suggest a possible next step in the analysis of the original data. *(1 mark)*

3 Large packets of ground almonds have nominal contents of 150 grams.

The weight of ground almonds delivered into a packet by a filling machine is normally distributed with a mean of μ grams and a standard deviation of 2.5 grams.

The production manager decides to set the machine so that $\mu = 153$ and to monitor the weight of ground almonds in packets by selecting random samples of 4 packets at regular intervals.

(a) Calculate to two decimal places, **but do not graph**, upper and lower warning (95%) and action (99.8%) control limits for:

(i) sample means; *(4 marks)*

(ii) sample ranges. *(3 marks)*

(b) State the action, if any, you would recommend if a subsequent sample of packets gave weights of:

(i) 158.5, 152.4, 159.6 and 157.5 grams;

(ii) 151.5, 152.2, 151.8 and 152.5 grams. *(5 marks)*

4 Each of nine items was valued independently by an antique dealer and an antique expert. The nine items consisted of a random sample of eight genuine antiques plus, by mistake, one forgery. The valuations, in pounds, are shown below.

Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Dealer	255	1050	250	475	1250	865	970	460	425
Expert	195	825	325	410	450	675	775	540	355

(a) State why paired, rather than independent samples, were used. *(1 mark)*

(b) Using a Wilcoxon signed-rank test and the 5% level of significance, investigate the claim that the dealer's valuations are, on average, greater than those of the expert. *(9 marks)*

(c) Indicate why **Item 5** is likely to be the forgery. *(1 mark)*

(d) Discarding the results for **Item 5**, the remaining differences (dealer minus expert) have a mean of 81.25, and a value of 13 812.50 for the unbiased estimate of the corresponding population variance.

Stating the necessary distributional assumption, use a paired *t*-test and the 5% level of significance to investigate the claim that the dealer's valuations are, on average, greater than those of the expert. *(6 marks)*

(e) Compare your conclusions to parts (b) and (d). *(2 marks)*

Turn over ▸

5 In a comparison of four different types, *A*, *B*, *C* and *D*, of washing machine tablets, four different makes of washing machine, *W1*, *W2*, *W3* and *W4,* are available.

Two experimental designs are suggested.

Design 1

A	*B*	*C*	*D*
W1	*W2*	*W4*	*W2*
W2	*W3*	*W3*	*W2*
W3	*W4*	*W1*	*W3*
W1	*W1*	*W4*	*W4*

Design 2

A	*B*	*C*	*D*
W1	*W2*	*W2*	*W1*
W4	*W4*	*W1*	*W2*
W3	*W1*	*W4*	*W3*
W2	*W3*	*W3*	*W4*

(Thus, for example, the first column of **Design 2** indicates that tablet *A* is used in each of washing machines *W1*, *W2*, *W3* and *W4*; the order within the column is unimportant.)

(a) State **one** disadvantage of **Design 1**. *(1 mark)*

(b) Write down the name of **Design 2**. *(1 mark)*

(c) Name the technique that you would use to analyse the results from **Design 2**. *(1 mark)*

(d) Subsequently, it was decided to introduce four sources of water of differing hardness, *H1*, *H2*, *H3* and *H4*.

Name and construct an experimental design of 16 observations that could be used to compare the tablets, allowing for effects due to washing machines and hardness but assuming no interactions. *(3 marks)*

6 A cucumber grower has three similar greenhouses: one contains plants of variety A, another contains plants of variety B, and the third contains plants of variety C. The environmental conditions in all three greenhouses are controlled automatically so as to be indistinguishable.

From each greenhouse, the grower chooses five plants at random and keeps a record of the yield, in kilograms, of cucumbers from each plant.

The results are shown below.

Variety		
A	**B**	**C**
4.6	5.9	7.8
4.3	6.4	5.7
6.7	5.0	6.5
5.0	4.4	6.0
3.9	7.3	7.5

(You may assume that $\sum_i \sum_j x_{ij}^2 = 526.00$)

(a) Assuming that all yields are normally distributed with the same variance, investigate, at the 5% level of significance, for a difference in mean yield between the three varieties of cucumber plant. *(11 marks)*

(b) Subsequently, the grower discovered that the night-time temperature controller in the greenhouse containing variety A had been faulty. This would have had the effect of reducing the yields from all cucumber plants in that greenhouse by approximately 1 kilogram.

Without further calculations, indicate, with a reason, the effect, if any, on your conclusion in part (a). *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

7 A car manufacturer claims that the quality of large batches of gaskets from a particular supplier is not meeting the agreed specification.

The manufacturer's quality control inspector designs two alternative sampling plans, **Plan A** and **Plan B**.

(a)

Plan A	Select 50 gaskets at random from a batch. Accept the batch if there are fewer than 4 defective gaskets;otherwise reject the batch.

Using binomial distributions:

(i) show that the probability of accepting a batch containing 15 per cent defective gaskets is approximately 5%; *(2 marks)*

(ii) determine the probability of rejecting a batch containing only 3 per cent defective gaskets. *(2 marks)*

(b)

Plan B	Select 25 gaskets at random from a batch. Accept the batch if the number of defective gaskets, X, in the sample is zero; reject the batch if X is more than 2. If there are 1 or 2 defective gaskets, select another 25 gaskets at random from the batch and count the number, Y, of defective gaskets in this second sample. Accept the batch if $(X + Y)$ is fewer than 4; otherwise reject the batch.

Using binomial distributions, show that the probability of accepting a batch containing 15 per cent defective gaskets is approximately 5%. *(5 marks)*

(c) For **Plan B**, the probability of rejecting a batch containing only 3 per cent defective gaskets is 0.075, correct to three decimal places.

Using this information, together with your results from parts (a) and (b), comment on the relative merits of the two sampling plans. *(3 marks)*

END OF QUESTIONS

SS06 Specimen

Question	Solution	Marks	Total	Comments
1(a)(i)	Children receiving placebo	E1	1	Placebo
(ii)	Children receiving new enhanced fluoride tablet	E1	1	New tablet
(b)	Sets of triplets or families	E1	1	Triplets or families
(c)	Meaning: Patient does not know treatment Administrator does not know treatment Purpose: To eliminate possible bias due to either patient or administrator using feel good/bad factor	 E1 E1 E1	3	Or equivalent
	Total		**6**	
2(a)	Source SS DF MS Ratio Paints 2.76 4 0.69 7.34 Surfaces 1.40 5 0.28 Error 1.88 20 0.094 Total 6.04 29 CV $F_{20}^{4}(0.01) = 4.431$ Thus evidence, at 1% level, of a difference between the mean drying times of the five paints	B1 B1 M1 A1 B1 A1√	6	$SS_E = 1.88$ cao DF = 4 and 20 cao Use of $R = MS_P/MS_E$ $R = 7.34$ awrt awrt 4.43 √ on R and CV
(b)	Test for a difference between surfaces or Test(s) to see which paint(s) is/are best	E1	1	either
	Total		**7**	

SS06 (cont)

Question	Solution	Marks	Total	Comments
3(a)(i)	CL: $\mu \pm z\frac{\sigma}{\sqrt{n}}$	M1		Use of formula
	z-values are 1.96 and 3.09	B1		awrt both
	W(95%): $153 \pm 1.96\times\frac{2.5}{\sqrt{4}}$			
	ie (150.55, 155.45)	A1		awrt both
	A(99.8%): $153 \pm 3.09\times\frac{2.5}{\sqrt{4}}$			
	ie (149.14, 156.86)	A1	4	awrt both
(ii)	CL: $\sigma \times D$	M1		Use of Table 12 and D
	LAL: $2.5 \times 0.199 = 0.50$			
	LWL: $2.5 \times 0.595 = 1.49$	A2, 1	3	A2 for 4 correct awrt
	UWL: $2.5 \times 3.984 = 9.96$			A1 for ≥1 correct awrt
	UAL: $2.5 \times 5.309 = 13.27$			
3(b)	Attempt at sample mean or range	M1		
(i)	$\bar{x} = 157$ $w = 7.2$	A1		cao both
	Mean > UAL so investigate	A1√		√ on values and limits
	(Range within LWL to UWL so OK)			One correct comment
(ii)	$\bar{x} = 152$ $w = 1.0$	A1		cao both
	Mean within LWL to UWL so OK			√ on values and limits
	Range < LWL but this is OK	A1√	5	One correct comment
	Total		**12**	

Question	Solution	Marks	Total	Comments
4(a)	To reduce or remove effect of variation in values of antiques	E1	1	Or equivalent; accept idea of reduce or remove bias
(b)	H_0: $A_D = A_E$	B1		Accept use of μ, η or m
	H_1: $A_D > A_E$	B1		May be scored using μ in (d)
	SL $\alpha = 0.05$			
	SS $n = 9$			
	CV $T = 8$ (or 37)	B1		cao either
	d: 60 225 −75 65 800 190 195 −80 70	M1		Differences
		M1		Ranking ignoring signs
	$r_{\lvert d\rvert}$: 1 8 4 2 9 6 7 5 3	M1		Applying signs
	$T_- = 9$ (or $T_+ = 36$)	A1		cao either
	Comparison of T with CV	M1		Must be consistent and correct tail
	Thus insufficient evidence, at 5% level, to support claim that dealer's valuations are, on average, greater than those of expert	A1√	9	√ on T and CV
(c)	Due to large difference in valuations	E1	1	Or equivalent
(d)	Differences ~ normal	B1		
	SL $\alpha = 0.05$			
	DF $\nu = 9 - 1 - 1 = 7$	B1		cao
	CV $t = 1.895$	B1		awfw 1.89 to 1.90
	$t = \dfrac{\bar{d} - \mu_d}{\sqrt{\frac{s_d^2}{n}}} = \dfrac{81.25}{\sqrt{\frac{13812.5}{8}}} = 1.96$	M1		Use of formula
		A1		awfw 1.95 to 1.96
	Thus evidence, at 5% level, to support claim that dealer's valuations are, on average, greater than those of expert	A1√	6	√ on t and CV
(e)	Wilcoxon test did not support claim but t-test did support claim because:	E1√		√ on (b) and (d)
	t-test uses values rather than ranks t-test more powerful t-test is on means rather than medians	E1	2	Answers to (b) and (d) must be correct
	Total		**19**	

Question	Solution	Marks	Total	Comments
5(a)	Unbalanced Tablet *A* not used in *W*4	E1	1	Or equivalent
(b)	Randomised block design (RBD)	B1	1	Not 2-way anova
(c)	Two-way/two-factor anova	B1	1	Accept anova for RBD Do not accept RBD
(d)	Latin square design (LSD) *A* *B* *C* *D* *H*1 *W*1 *W*2 *W*3 *W*4 *H*2 *W*4 *W*1 *W*2 *W*3 *H*3 *W*3 *W*4 *W*1 *W*2 *H*4 *W*2 *W*3 *W*4 *W*1	B1 B1	3	Introduction of *H*1 to *H*4 Clear and correct LSD Allow interchange of *A* to *D*, *W*1 to *W*4 and *H*1 to *H*4
	Total		**6**	
6(a)	$SS_T = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{n}$ $= 526 - \frac{87^2}{15} = 21.4$ $SS_B = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{n}$ $= 512.7 - \frac{87^2}{15} = 8.1$ $SS_W = 21.4 - 8.1 = 13.3$ Source SS DF MS Ratio Between 8.1 2 4.05 3.65 Within 13.3 12 1.108 Total 21.4 14 H_0: means all equal H_1: means not all equal CV $F_{12}^2(0.05) = 3.885$ Thus insufficient evidence, at 5% level, of a difference in mean yield between the three varieties	M1 A1 M1 A1 M1 B1 M1 A1 B1 B1 A1√	11	Use of formula cao Use of formula or equivalent Accept all $n_i = n \div k$ cao Providing > 0 DF = 2 and 12 cao Use of $R = MS_B/MS_W$ R = 3.65 to 3.66 awfw Or equivalent Or equivalent (not necessary) awfw 3.88 to 3.89 √ on *R* and CV
(b)	Reduction in SS_B (with no change in SS_W) Thus *F*-ratio is reduced so no change in conclusion	B1 B1	2	Or equivalent
	Total		**13**	

SS06 (cont)

Question	Solution	Marks	Total	Comments
7(a)(i)	B(50, 0.15)			
	$P(A) = P(X < 4) = P(X \leq 3)$	M1		Use of B and ≤3
	= 0.0460	A1	2	awfw 0.045 to 0.050 ag of ≈ 5%
(ii)	B(50, 0.03)			
	$P(R) = P(X \geq 4)$			
	$= 1 - P(X \leq 3)$	M1		Use of B and 1 − ≤3
	$= 1 - 0.9372 = 0.0628$	A1	2	awrt 0.063
(b)	A = (0 in S1) or (1 in S1 and 0 to 2 in S2) or (2 in S1 and 0 to 1 in S2)	M1		Sensible attempt at logic; may be implied
	Thus $P(A) = P(X = 0) +$ $P(X = 1) \times P(Y \leq 2) +$ $P(X = 2) \times P(Y \leq 1)$	A1		Convincing deduction
	$P(A) = 0.0172 +$ (0.017) $0.0759 \times 0.2537 +$ (0.019) 0.1606×0.0931 (0.015)	M1 A1		Use of tables or formula for B(25, 0.25) At least 1 value in ( )
	= 0.0514	A1	5	awfw 0.051 to 0.052 ag of ≈ 5%
(c)	15% (poor) Accept / 3% (good) Reject **Plan A** 0.046 / 0.063 **Plan B** 0.051 / 0.075			
	Plan A has lower risk of accepting poor (15%) batches, or equivalent			Accept similar risks or Plan A better
	Plan A has lower risk of rejecting good (3%) batches, or equivalent			Accept similar risks or Plan A better
	Plan B more complicated than Plan A			Or equivalent
	Plan B has less sampling/is less costly than Plan A			Or equivalent
		E3,2,1	3	Any three valid points
	Total		**12**	
	TOTAL		**75**	