A-LEVEL
Statistics

SS04 Statistics 4
Mark scheme

6380
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Version 1.0: Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$H_0 : p = 0.1$ $H_1 : p \neq 0.1$	B1		For both. Or π or in words. Not sample prop or just proportion.
	Under H_0 , the number unable to smell freesias ~ B(20, 0.1)	B1		Binomial distribution with n=20 stated or used to find probability (even if subsequent approximation used).
	Then $P(X \geq 5) = 1\text{-}0.9568$	M1		For finding $P(X \geq 5)$ or $P(X > 5)$ from B(20,0.1)
	= 0.0432	A1		(0.04(0) ~ 0.045)
	Cannot reject / accept H_0 at the 5% level	m1		Either 0.0432 or 0.0113 compared with 0.025 oe
	No evidence of difference from 10 per cent.	E1dep		Correct conclusion in context. Depends on all previous marks.
Notes	(i) Using $P(X > 5)$ gives P= 1–0.9897=0.0113 (0.011~0.012) then reject H_0 for max 4/6 B1B1M1A0m1E0 (ii) May compare 0.9568 or 0.9897 with 0.975 for 6/6 or 4/6 max. Disallow mixture which gets max B1B1 2/6			
			6	
(b)	*For example:* May be visitors not residents. Under-represents those who don't use the high street or flower shop often. Under-represents those who aren't out in the early morning. Flower-lovers more likely to stop May include blood relatives	E1 E1		Oe on 'over-represents' Relevant contextualised reasons based on any two from residence/location/time/interest/genetics Not given for 'small sample'
			2	

		Total	**8**	

Q	Solution	*Marks*	Total	Comments
2(a)(i)	No. of hits in 1 minute (X)~Po(4) $P(X=3)=0.4335-0.2381$ = 0.1954	M1 A1		Using Poisson here or in (ii) Or $P(X=3)=\frac{4^3}{3!}e^{-4}$ 0.195 ~ 0.196
			2	
(ii)	P(3 hits **and** 3 from UK) = 0.1954 × p where p = 0.8^3 = 0.1954 × 0.512 = 0.1(00028) **Alternative** No. of hits in 1 minute from the UK (U)~Po(3.2) No. of hits in 1 minute from outside UK (V)~Po(0.8) Then P(U = 3 and V = 0) = P(U = 3) × P(V = 0) =0.2226 × 0.4493 = 0.1(00028)	M1 B1 A1 (M1) (B1) (A1)		Their (a)(i) × p where 0<p<1 0.8^3 or 0.512 PI, either alone or in a correct binomial expression or as part of (a)(i) × p 0.099 ~ 0.101 Either Po(3.2) or Po(0.8) stated or used. PI Either 0.222 ~ 0.223 or 0.449 ~ 0.450 0.099 ~ 0.101
			3	

Q	Solution	Marks	Total	Comments
2(b)	No. of hits in 1 hour (Y) ~ Po(240) which can be approximated by N(240,240) P(Y>220) = P(Y > 220.5) = $P\left(Z > \frac{220.5-240}{\sqrt{240}}\right)$ = $P(Z > -1.26)$ = 0.89617	B1 B1 M1 A1 A1		Poisson 240. PI For Normal approximation using their λ and $\sqrt{\lambda}$. Disallow λ=4. For standardisation using their λ and $\sqrt{\lambda}$. Disallow λ=4. Condone missing/wrong CC. Ignore sign. For completely correct expression AWRT 0.896
Notes	(i) Missing CC gives answers $z = -1.29$ with prob 0.900~0.903 (ii) Wrong CC gives answers $z = -1.32$ with prob 0.905~0.908 both get B1B1M1 for max(3/5) (iii) Exact Poisson(240) gives 0.897146 for first B1 only			
			5	
(c)	No. hits from outside UK (W) ~ B(240, 0.2) which is approximately N(48, 38.4) $P(W < 40) = P(Z < \frac{39.5-48}{\sqrt{38.4}})$ $= P(Z < -1.37(2))$ = 1- 0.91466 = 0.08534 (from tables using Z = -1.37)	B1 M1 A1 M1 A1 A1		Or for B(240, 0.8) **if** P(W > 200) used subsequently. PI Normal approximation to binomial stated or clearly used. Mean = 48 cao (or 192 **if** P(W > 200) used), variance = AWRT 38.4 (or SD = 6.197 AWRT 6.2). May be implied. Standardizing with their mean and SD (allow missing or wrong CC); ignore sign AWRT -1.37 (or +1.37 **if** other tail used) 0.085 ~ 0.086 (more exact value 0.085081)
Notes	(i) No CC gives Z= –1.29 & p= 0.0985 (0.098~0.099) for max 4/6 (ii) Wrong CC gives Z= –1.21 & p= 0.113 (0.11~0.12) for max 4/6 (iii) Use of exact B(240, 0.2) gives answer 0. 0826 which is not in range and scores 1/6 if first M1 is not earned (iv) **SC** Use of Po(48) normal approx., (ie variance 48) gives Z= –1.23 and prob 0.109~0.11 for max 2/6 (v) Use of exact Po(48) gives answer 0.107 scores 1/6 from first B1 only.			
			6	

		Total	**16**	

Q	Solution	Marks	Total	Comments
3 (a)	$H_0 : \mu = 0.215$ $H_1 : \mu > 0.215$	B1		Both. Or population mean for μ. Next 4 marks are PI.
	$\bar{x} = 0.2343 \quad s = 0.01512$	B1		For AWRT 0.234 and $s_{n-1} = 0.015 \sim 0.016$ or s_n =AWRT 0.014 (ignore labels)
	$(t =)\ \dfrac{0.2343 - 0.215}{0.01512/\sqrt{7}}$	M1		M1 for use of $\dfrac{s_{n-1}}{\sqrt{n}}$ *or* $\dfrac{s_n}{\sqrt{n-1}}$. Condone z=.
		m1		Correct formula, ignore sign for m1. Or $(t =)\ \dfrac{0.2343 - 0.215}{0.014/\sqrt{6}}$
	= 3.37(7)	A1		AWFW 3.31 to 3.41
	Critical value $t_6 = 3.143$	B1		For 6 df (may be implied by 3.14 or 3.71(3.707))
		B1		For 3.14 cao (or –3.14 **if** test stat < 0) **Alternative** for B1B1 $p = 0.00745$ AWFW 0.007 to 0.008 for B1. Comparison of their p with 0.01 B1
	Reject H_0 at 1% level. Evidence does support Olga's suspicion OR thickness of shells has increased OR thickness of shells > 0.215	E1dep		Requires correct TS and critical t (both positive) OR correct p-value and 0.01 but still requires positive t if seen. Depends on all previous marks. In context
Notes	(i) z test gets B1 B1 M1 m1 A1 B0 B0 A0 for max 5/8 (ii) One sided CI or Decision Interval potentially full marks from 0.216 > 0.215 OR 0.233 < 0.234 so rej H_0 (iii) Two-sided test (or CI) gets B0 B1 M1 m1 A1 B1 B0 A0 for max 5/8			
			8	
(b)	Yes, (the suggestion is sensible) because…	B1		Requires *sensible* reason which may not be entirely correct or complete. Just "Yes" is enough.
	…it provides a baseline/control group **or** thickness may have increased for other reasons.	E1		oe Needs idea of comparison, for example: *Can then compare with and without crabs.* Also award E1 for **convincing** argument for "No"/"not sensible"
			2	
(c) (i)	Cannot assume normal distribution.	E1		Mention of non-normality
(ii)	Thus, sample size should change/increase	E1		Mention of sample size (not decrease)
	so can then use large sample approximation.	E1		Consideration of approx distribution of test statistic. Allow mention of z- or t-test or Central Limit Theorem. Requires consideration of sample size.
			3	

		Total	**13**	

Q	Solution	Marks	Total	Comments
4(a)(i)	Sample proportion = $\frac{8}{70} = \frac{4}{35} = 0.114(3)$	B1		Any of these
	Use of z = 1.6449	B1		1.64 ~ 1.65. Here OR in part (ii). PI.
	Use of $\frac{(0.1143)(0.8857)}{70}$ (= 0.038)	M1		Their sample proportion and .
	90% CI: $0.1143 \pm 1.6449\sqrt{\frac{(0.1143)(0.8857)}{70}}$	m1		Their proportion, z and $\sqrt{variance}$
	$= 0.1143 \pm 0.0626$ or (0.0519, 0.1767)	A1		Either form. $0.114 \pm (0.062 \sim 0.063)$ Or (0.051~0.052, 0.176~0.177)
	Alternative (using numbers)			
	Final CI symmetrical about 0.114	(B1)		
	$z = (\pm)1.64(49)$	(B1)		
	Use of $70 \times 0.114 \times 0.886$ (= 7.070)	(M1)		
	$8 \pm (Their\ z) \times \sqrt{Their\ \sigma^2}$	(m1)		
	Answer as above	(A1)		
			5	
(ii)	Use of z = 1.6449	(B1)		1.64 ~ 1.65. Only if NOT in part (a).
	90% CI is $0.118 \pm 1.6449 \times \frac{0.019}{\sqrt{70}}$	M1		Use of $\frac{0.019}{\sqrt{70\ or\ 69}}$ $(= 0.0227\ or\ 0.0229)$
		m1		Correct interval, allow any Z or t_{69} or t_{70}
	$= 0.118 \pm 0.0037$ = (0.114, 0.122)	A1		**Either** for 0.118 ± AWRT(0.004) **or** AWRT 0.114 and 0.122
Note	Using t = 1.667 (1.66 ~ 1.67) gives answer in range for full marks (B1) + 3/3			
			3	
(b)(i)	CI for RGCB includes 0.09	AF1		ft their CI which must include 0.09. Needs M1m1 in (a)(i) and 0.09 specified.
	So no evidence of a difference (between men and women)	AFdep1		Needs above AF1
(ii)	CI for mean TTF excludes 0.125	AF1		ft their CI which must exclude 0.125 Needs M1m1 in (a)(ii) and 0.125 specified.
	So there is evidence of a difference (between men and women)	AFdep1		Needs above AF1
Notes	(i)In (b)(ii),iIf direction of difference is referred to, it must be correct for AFdep1 (eg “women have lower mean”, “women are quicker” oe) (ii) **SC** If 4/4 and **both** conclusions in (b) are too definite, deduct 1 mark.			
			4	
		Total	**12**	

Q	Solution	Marks	Total	Comments
Allow 3dp accuracy for probabilities in this question				
5(a)	Require $P(X>c)<0.01$ using $\lambda=1.6$ From Poisson tables $P(X>3)=1-P(X\le 3)=1-0.9212=0.0788$ $P(X>4)=1-P(X\le 4)=1-0.9763=0.0237$ $P(X>5)=1-P(X\le 5)=1-0.9940=0.006$	M1		Needs Poisson and at least $P(X>4)$ and $P(X>5)$ identified (may not be evaluated)
	This is < 0.01	A1		Any one correct Poisson probability **and** comparison with 0.01
	Thus require c = 5	A1		5 cwo. Needs 2 correct Poisson probabilities (0.024 and 0.006)
	Alternatively Using the complement , require $P(X\le c)>0.99$ (or ≥ 0.99) Reading directly from Poisson tables $P(X\le 5)=0.9940>0.99$ $P(X\le 4)=0.9763<0.99$	(M1)		Needs Poisson and at least $P(X\le 4)$ and $P(X\le 5)$ identified
		(A1)		Any one correct Poisson probability **and** comparison with 0.99
	c = 5	(A1)		5 cwo. Needs 2 correct Poisson probabilities (0.994 and 0.976)
Notes	(i) c =5 stated with no justification gets 0/3 (ii) No **numerical** probabilities given – eg P(X>4)>0.01 and P(X>5) <0.01 so c=5 – gets M1A0A0			
			3	
(b)	$H_0:\lambda=1.6$ $H_1:\lambda>1.6$	B1		For both. Allow μ or "rate".
	Find $P(X\ge 4)$ from **Poisson** tables	M1		Attempt to calculate $P(X\ge 4)$ or $P(X>4)$ (= 1 – 0.9763 = 0.0237)
	= 1 – 0.9212 = 0.078(8)	A1		0.07 ~ 0.08
	This is > 0.05 so do not reject H_0.	M1		Compare their Poisson prob with 0.05.
	There is **no** evidence that the **mean** or **rate** of occurrence of air **bubbles** has **increased**.	E1dep		Correct P-value and 0.05, including conclusion in context. Must accept H_0. Depends on all previous marks.
			5	

		Total	**8**	

Q	Solution	Marks	Total	Comments
6 (a) (i)	E(U) = 1.8 + 1.8 + 1.8 = 5.4 Var(U) = $0.07^2 + 0.07^2 + 0.07^2 = 0.0147$			cao allow 0.015
(ii)	E(V) = 2.4 + 2.4 = 4.8 Var(V) = $0.15^2 + 0.15^2 = 0.045$			cao cao
(iii)	E(U + V) = 5.4 + 4.8 = 10.2 Var(U + V) = 0.0147 + 0.045 = 0.0597			cao Allow 0.06
(iv)	E(U - V) = 5.4 - 4.8 = 0.6 Var(U - V) = 0.0147 + 0.045 = 0.0597			cao Allow 0.06.
		M1		Method for any one mean (may be implied.)
		M1		Method for any one variance (may be implied.) Don't give if only SDs seen.
		B4		½ mark for each of the above 8 answers. Total rounded down.
Notes	*(i)* Method marks for means are for 3×1.8, 2×2.4, means of (i) + (ii) and (i) – (ii) *(ii)* Method marks for variances are for 3×0.07^2, 2×0.15^2, vars of (i) + (ii) and same as (iii)			
			6	
(b)(i)	Total thickness T = U + V ~ N(10.2, 0.0597)	M1		Use of correct normal dist, their mean and variance from (a)(iii)
	$P(T<10)=P(Z<\frac{10-10.2}{\sqrt{0.0597}})$	m1		Standardising. Award here or in (b)(ii). Ignore sign.
	$=P(Z<-0.82)$	A1		-0.82 ~ -0.81 (-0.81855)
	$=1-0.79389=0.20611$ from tables	A1		0.206 ~ 0.208 (0.20652 from calculator)
(ii)	Use of W = U - V ~ N(0.6, 0.0597)	M1		Use of correct normal dist, their mean and variance from (a)(iv).
	Require P(U > V) = P(W > 0)			
	$P(W>0)=P(Z>\frac{0-0.6}{\sqrt{0.0597}})$	(m1)		Award here if not given in (b)(i)
	$=P(Z>-2.45(6))$	A1		-2.46 ~ -2.44 (-2.455637)
	$=0.99305$ from tables	A1		AWRT 0.993 (0.99297 from calculator)
			7	
(c) (i)	$p_1 = [(b)(i)]^4$ $= (0.20611)^4 = 0.0018$	M1 A1		Stated or used with their (b)(i). 0.0018 ~ 0.0019
			2	
(ii)	Expect $p_2 > p_1$ Each biscuit < 10mm implies total < 40mm But there are other ways of total being less than 40mm	B1 E1dep E1dep		 Requires B1 Requires B1 SC p_2 is actually 0.0508. Not required but allow E1 for 0.05 ~0.052 for max 2/3 (B1 E1dep) if no other argument
			3	

		Total	**18**	