# AQA

# AS
# **Statistics**
SS03 Statistics 3
Mark scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk.

**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

SSO3
Incorrect method followed by correct answer ( from calculator) counts as 2 attempts so award average rounded down

Final A or E mark in hypothesis test can only be awarded if both ts and cv ( or p value and sig level comparison) are correct

| Q1 | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1(a) | $H_o$  $\eta_d$  or $\mu_d = 0$<br>$H_1$  $\eta_d$  or $\mu_d \neq 0$<br>     2 tail test   5 % level | B1 | | $H_0$:Population median/mean/average score difference = 0<br>$H_1$:Population median median/mean/average score difference $\neq$ 0 |
| | Differences ( caffeinated – uncaffeinated)<br>A    B    C    D    E    F    G    H    I    J<br>–4  +9  +4  +3  -1  +5  +2  +8  +1  +6<br><br>Ranks<br>5½  10  5½  4  1½  7  3  9  1½  8 | M1<br><br><br>m1 | | Differences (ignore sign) effort<br><br>Ranks (smallest abs diff = rank 1) |
| | $T_+ = 10 + 5½ + 4 + 7+ 3 + 9 + 1½ + 8 = 48$<br>$T_- = 5½ + 1½ = 7$<br>test stat $T = 7$            or test stat $T = 48$<br>      cv = 8                            cv = 47<br><br> 7 < 8                          or  48 > 47<br><br>Reject  $H_o$<br>Significant evidence to suggest that the median/mean/average score gained in the hand-eye coordination tasks is different ( or higher) after the consumption of a caffeinated drink. | m1<br>A1<br><br>B1<br><br>m1<br><br>A1 PI<br><br><br>E1 | 9 | Effort at total for ranks<br>One total correct<br><br>Cao for cv<br>Correct comparison lower/lower or upper/upper tail ts and cv<br><br><br>Conclusion correct<br><br>Conclusion in context<br><br>Need score/hand-eye coordination and drink/caffeine |
| (b)(i) | Students can be regarded as a random sample<br>Differences (in scores) are distributed symmetrically | E1<br><br>E1 | | Disallow ref to 'normal' |
| (ii) | $H_o$ rejected in error.<br><br>Conclusion that there is a difference in median scores is incorrect.<br>Having a caffeinated drink or not makes no difference to median scores gained for the tasks<br> or<br> Having a caffeinated drink does not effect hand-eye coordination | E1 PI<br><br><br>E1 | 4 | Type I error defined<br>Allow $H_1$ accepted in error<br><br><br>In context |
| | | Total | 13 | |

| Q2 | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **2(a)** | $H_o$ The two populations have identical distributions | B1 | | $H_o\, \eta_{painkiller} = \eta_{placebo}$ $H_1\, \eta_{painkiller} < \eta_{placebo}$ oe |
| | $H_1$ The two populations do not have identical distributions (patients taking the recommended painkiller had, on average, fewer days to wait for their back pain to go away completely) | | | Allow 'population median' Disallow use of X, Y or A, B as labels unless identified (check table in stem) |
| | **Ranks** | | | |
| | (see rank table below) | M1 | | Ranks effort – can be reversed – as one group |
| | $T_{pain} = 34\tfrac{1}{2}\ (43\tfrac{1}{2})\quad T_{placebo} = 43\tfrac{1}{2}\ (34\tfrac{1}{2})$ $n_{pain} = 6 \qquad n_{placebo} = 6$ | m1 | | Totals of ranks effort |
| | | m1 | | Method correct for $U$ |
| | $U_{pain} = 34\tfrac{1}{2} - \dfrac{6\times7}{2} = 13\tfrac{1}{2}$ $U_{placebo} = 43\tfrac{1}{2} - \dfrac{6\times7}{2} = 22\tfrac{1}{2}$ | A1 | | Either $U$ correct ( allow reverse for reversed ranks) |
| | Test statistic $U = 13\tfrac{1}{2}$   cv = 5 or  Test statistic $U = 22\tfrac{1}{2}$   cv = 31 | B1 m1 | | Either cv correct cao comparison consistent |
| | Accept $H_o$ | A1 | | Conclusion correct |
| | No significant evidence to suggest that the patients taking the recommended painkiller had, **on average**, fewer days (oe) to wait for their  back pain to go away completely. | E1 | **9** | Explanation in context correct 'on average, painkiller not better' |
| **2(b)** | So that the adults all felt they were treated the same/had similar expectations and the painkiller group did not assume they would recover faster/slower than the placebo group or vice versa | E1 | **1** | Allow comment that both groups 'treated/felt the same' so a difference in drug outcome is more likely to be detected if one exists. Disallow 'no bias' or 'more accurate' Disallow 'demand characteristics' |
| | | **Total** | **10** | |

Rank table:

| Pain killer | 1 12 | 9 4 | 7 6 | 4½ 8½ | 3 10 | 10 3 |
|---|---|---|---|---|---|---|
| Placebo | 4½ 8½ | 8 5 | 6 7 | 2 11 | 12 1 | 11 2 |

| Q3 | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| | $H_0$: Samples from identical populations <br> $H_1$: Samples not from identical populations <br> (or 'at least 2 pops differ')    5% sig level | B1 | | $H_o$ $\eta_{with} = \eta_{without} = \eta_{none}$ <br> $H_1$ at least 2 of $\eta_{with}, \eta_{without}, \eta_{none}$ differ <br> oe |
| | Ranks | M1 | | Effort at ranks as one group – can be reversed |
| | | A1 | | All correct |
| | Totals <br> $T_{with} = 29\frac{1}{2}\ 72\frac{1}{2}$  $T_{wirhout} = 44\frac{1}{2}\ 40\frac{1}{2}$  $T_{None} = 62\ 23$ | m1 | | Attempt at totals of ranks |
| | $n_{with} = 6$     $n_{without} = 5$     $n_{None} = 5$ | A1 | | One total correct |
| | $\sum \frac{T_i^2}{n_i} = \frac{29\frac{1}{2}^2}{6} + \frac{44\frac{1}{2}^2}{5} + \frac{62^2}{5} = 1309.9$ | m1 PI | | effort ft |
| | $H = \frac{12}{16 \times 17} \times 1309.9 - 3 \times 17 = 6.79$ | m1 PI <br> A1 | | Formula for $H$ <br> awfw  6.6 – 6.9 |
| | Critical value from $v = 2$   cv = 5.991 <br>   $H > 5.991$ | B1 | | cao |
| | Significant evidence to reject $H_o$ <br> There is significant evidence of a difference between average scores/correct answers for at least 2 music types. | A1 PI <br><br> E1dep | | Reject $H_o$ <br><br> At least two differ in context of music/types |
| | | E1 not dep | **12** | Or ref to <br> $\sum \frac{T_i^2}{n_i} = \frac{29\frac{1}{2}^2}{6} + \frac{44\frac{1}{2}^2}{5} + \frac{62^2}{5}$ <br> = 145 + 396 + **768.8** <br><br> In context – must identify 'none' best (better than background music with lyrics) and supply some justification (not PI) <br> Disallow ref to totals |
| | **Total** | | **12** | |

**Ranks**

| With lyrics | Without lyrics | None |
|---|---|---|
| 2    15 | 7½    9½ | 12    5 |
| 1    16 | 13    4 | 9    8 |
| 11    6 | 14    3 | 16    1 |
| 5    12 | 6    11 | 15    2 |
| 3    14 | 4    13 | 10    7 |
| 7½    9½ | | |

| | With | | Without | | None | |
|---|---|---|---|---|---|---|
| | rank | score | rank | score | rank | score |
| **Mean** | 4.9 | | 8.9 | | 12.4 | |
| **Med** | 4 | | 7½ | | 12 | |
| | Median score | | Median score | | Median score | |
| | 23.5 | | 27 | | 31 | |

 Reading with no background music results in higher scores for successfully answering questions on a reading task than listening to background music with lyrics.

| Q4 | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| (a) | $H_0$: Food type preferred is independent of the political party supported <br> $H_1$: Food type preferred is not independent of the political party supported <br><br> 1 tail 5% <br> Expected freqs | B1 | | oe <br> ref to No Association and Assoc <br><br> Disallow nonsense statements |

|  | British | French | Italian |
|---|---|---|---|
| A | 26.5 | 24.25 | 24.25 |
| B | 31.8 | 29.1 | 29.1 |
| C | 17.7 | 16.2 | 16.2 |
| D | 12.4 | 11.3 | 11.3 |
| E | 17.7 | 16.2 | 16.2 |

M1, m1 — One exp freq correct or method seen / At least 6 exp freq correct

A1 — All correct – condone small arithmetic/rounding slip

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(29-26.5)^2}{26.5} + \frac{(21-24.25)^2}{24.25} + \ldots \frac{(17-16.2)^2}{16.2}$$

m1 PI — Numerator correct
m1 PI — Denominator correct

$$= 0.24 + 0.44 + 0.02 + 0.25 + \ldots\ldots$$

ts $\chi^2 = 22.9$  — A1 — awfw 20 – 26

df = 8    5%    cv = 15.507  — B1 PI — df correct
ts > 15.507  — B1 — cv correct / Alt or $p = 0.0035 < 0.05$ B1 B1

Reject $H_0$  — A1 — Conclusion
Sig evidence to suggest that (food) type preferred is not independent of the political party supported  — E1 — Correct and in context  — **Total 11**

| Q4 | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| (b) | ts = 0.24 + 0.44 + 0.02 <br> + 0.25 + **3.37** + 1.73 <br> + 1.23 + 1.65 + **5.98** <br> + **4.71** + 0.97 + 1.65 <br> + 0.40 + 0.21 + 0.04 <br> Biggest sources of association are indicated by 3.37 , 5.98, 4.71 <br><br> It appears that: <br> <u>Political party C</u> supporters are <u>more likely</u> than expected to prefer <u>Italian food</u> (26 obs/ 16.2 exp) <br> <u>Political party D</u> supporters are <u>more likely</u> that expected to prefer <u>British food</u> (20 obs / 12.4 exp) <br> <u>Political party B</u> supporters are <u>more likely</u> than expected to prefer <u>French food</u>. (39 obs / 29.1 exp) | B1 <br><br><br><br><br> E1 <br><br> E1 <br><br> E1 | **4** | Ref to contributions to ts / largest $\frac{(O-E)^2}{E}$ <br><br> Must ref to obs or exp frequencies <br><br> Also allow for E1 <br> Political parties A and E have no particular food preferences as all of their obs and expected frequencies are very similar |
| (c) | Survey applies only to : <br> readers of the newspaper; <br> people aged 40 years– 50 years. | B1 <br> B1 | **2** | |
| | | **Total** | **17** | |

| Q5 | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **(a) (i)** | Ranks | | | |

| | | Score rank | | Takings ranks |
|---|---|---|---|---|
| | **A** | 6 | 7 | 2 |
| | **B** | 3 | 10 | 10½ |
| | **C** | 2 | 11 | 9 |
| | **D** | 12 | 1 | 7 |
| | **E** | 11 | 2 | 10½ |
| | **F** | 4 | 9 | 3 |
| | **G** | 8½ | 4½ | 4 |
| | **H** | 1 | 12 | 12 |
| | **I** | 8½ | 4½ | 1 |
| | **J** | 5 | 8 | 8 |
| | **K** | 10 | 3 | 6 |
| | **L** | 7 | 6 | 5 |

**Note PMCC 0.2749 B0**

$r_S = \pm\ 0.312$ from calculator — B3 — scB2 no method $r_S = \pm\ 0.31$

or

$|d| =$ 4, 7½, 7, 5, ½, 1, 4½, 11, 7½, 3, 4, 2
or ( 5, ½, 2, 6, 8½, 6, ½, 0, 3½, 0, 3, 1 ) — or B1 — Differences and $\sum d^2$ effort

$\sum d^2 = 374$ or $\sum d^2 = 196$ — M1 — formula correct

$r_s = 1 - \frac{6 \times 374}{12 \times 143} = \underline{-\ 0.308}$ or $= 1 - \frac{6 \times 196}{12 \times 143} = \underline{0.315}$ — A1 — - 0.308, 0.315

**Total 5**

**(ii)**

$H_o$ Rank orders of gender bias score and takings are independent.
$H_1$ Rank orders of gender bias score and takings are not independent. — B1 — Allow $H_o$ No association $\quad H_1$ Association( or ref to r/ρ) Correct hypotheses stated in (i) or (ii)

2 tail 10%    |cv| = 0.5035 — B1 — cv correct cao
 $|r_S| =$ 0.312 or 0.308 or 0.315 < 0.5035 — M1 — consistent comparison correct SRCC and correct cv +/+ or -/-  PI

Accept $H_o$
No significant evidence at 10% level to doubt that rank orders of gender bias score and takings are independent — E1 — Conclusion in context

**Total 4**

| Q5 | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **(b)** **(i)** **SRCC** **0.754** **B0** | PMCC = 0.813  from calculator sc 0.81 allow M1 M1 A0 (or B2) | B3 | **3** | or $r_s = \dfrac{96124 - \frac{1333 \times 802}{12}}{\sqrt{2728.9 \times 27429.7}} = \dfrac{87253.2}{52.2 \times 165.6}$  $= \dfrac{7035.2}{8651.8} = 0.813 \ (3 \text{ sf})$  M1,  M1,  A1   awfw ( 0.811, 0.815 ) |
| **(ii)** | $H_0$: $\rho = 0$ $H_1$: $\rho > 0$    1 tail    1%  test stat $|r| = 0.813$ $|cv| = 0.6581$ $0.813 > 0.6581$ Reject $H_0$  Significant evidence to suggest that there is a positive correlation between running time and budget or Arthur's belief is correct/supported by the conclusion. | B1  B1 M1   E1 | **4** | Hypotheses oe  cao cv consistent comparison correct PMCC with cv   correct conclusion in context |
| **(iii)** | (Running) times and budgets have a bivariate normal distribution. | B1 E1 | **2** | Bivariate normal Context |
| **(iv)** | $H_0$: $\eta = 106$ $H_1$: $\eta > 106$  1 tail    10%   + + - + + - + - - - + +   ts = 7+ / 5 –  B ( 12, 0.5 )  P( ≤ 5 - ) = 0.387  > 0.10   Accept  $H_0$  No significant evidence to support Arthur's suspicion or no significant evidence to support that the median/average running times were longer in 2010 | B1   M1    m1 m1 dep   E1 | **5** | Hypotheses oe in words Allow $\eta_{2015}$ as substitute for 106  Signs effort – can be reversed   Correct binomial prob Comparison with 0.10  Alt Critical region obtained {0,1,2,3} with prob 0.073 M1 0.073 m1 dep 0.073 < 0.10 cr {0,1,2,3} or {9,10,11,12} |
| | | **Total** | **23** | |