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General Certificate of Education (A-level)
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Statistics SS05

(Specification 6380)

Statistics 5

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$s_x = 11.0548$ or $\sum (x-\bar{x})^2 = 1710.93$	B1		awfw 11.0 to 11.1 or 1710 to 1711
	[90% confidence interval for σ]			
	$6.571 \le \frac{14 \times 11.1^2}{\sigma^2} \le 23.68$	M1		M1 any correct expression – condone one small slip eg $15s^2$ or $14s$ and incorrect χ^2
		m1		m1 completely correct expression – condone incorrect χ^2
	$6.571 \le \frac{1710.9}{\sigma^2} \le 23.68$	B1 B1		B1 14 df (seen or implied by correct cv.) B1 awfw 6.57~ 6.571, awfw 23.6 ~ 23.7 both
	$72.25 \le \sigma^2 \le 260.38$	M1		M1 correct method for setting up interval for σ or σ^2
	$8.50 \le \sigma \le 16.1$	A1 cao	7	awfw 8.40 ~ 8.60 and 16.0 ~ 16.30
(b)	18 is above the upper limit of the CI 18 does not fall inside the CI	E1		comparison with <u>correct</u> CI
	Alan is unlikely to have met his aim	E1dep	2	Dep E1 – accept "No"; or "Alan has not met his aim"
	Total		**9**	

Q	Solution	Marks	Total	Comments
2	$H_0 : \mu_A = \mu_B + 24$	B1		(s.c. B1 for both $H_0 : \mu_A = \mu_B$ and
	$H_1 : \mu_A > \mu_B + 24$	B1		$H_1 : \mu_A > \mu_B$)
	$\bar{x}_A = 473 \quad \bar{x}_B = 438$	B1		B1 both means; awfw 472 ~ 473 and 438 ~ 439
	test statistic $z = \dfrac{473 - 438 - 24}{\sqrt{\dfrac{7^2}{6} + \dfrac{10^2}{8}}}$	M1		M1 Numerator (allow (473 – 438) or (438 – 473 – 24)
		M1		M1 Denominator
	= 2.29	A1		A1 awfw 2.25 ~ 2.45
	cv 5% level 1-tail test z = 1.6449	B1		B1 awfw 1.64 ~ 1.65 (condone $\pm$)
	2.42 > 1.6449 reject H_0	A1		dep A1 for ts (consistent with hypotheses) and B1 for cv
	Evidence at the 5% level to support Nasreen's belief.	E1	9	Correct comment in context dep. on previous A1 - must mention mean or average and some element of doubt. eg Some evidence that **boxes** of eggs from Alaric are **more than 24gm** heavier **on average** than those from Belinda
	Total		**9**	

Q	Solution	Marks	Total	Comments
3(a)	$H_0 : \sigma_M = \sigma_T$ or $H_0 : {\sigma_M}^2 = {\sigma_T}^2$ $H_1 : \sigma_M \neq \sigma_T$	B1		B1 Both. Other suffices must be clearly assigned
	$s_T = 17.8$ or ${s_T}^2 = 317$	B1		B1 17.7 ~ 17.8 or 316 ~ 317
	$s_M = 18.4$ or ${s_M}^2 = 338$	B1		B1 18.3 ~ 18.4 or 338 ~ 339
	Test Statistic $F = \frac{18.4^2}{17.8^2} = 1.07$	M1A1		awfw1.06 ~ 1.07 (1.0681)
	upper 2.5% value of $F_{9,7} = 4.823$	B1B1		B1 df , B1 cv
	$1.07 < 4.823$ accept H_0			
	there is no difference in the variability of the yields from plants grown using *Moretom* or from plants grown using *Tomsplus* at the 5% level of significance.	E1	8	conclusion in context dep A1 and cv B1
3(b)	$H_0 : \mu_M = \mu_T$ $H_1 : \mu_M > \mu_T$	B1		Both
	${S_p}^2 = \frac{7(17.8)^2 + 9(18.4)^2}{18-2} = 329$	M1A1		M1 A1 (awfw 328 ~ 330)
	$\bar{x}_M = 1377 \quad \bar{x}_T = 1342$	B1		B1 both means (awfw 1377 ~ 1378, 1342 ~ 1343)
	Test statistic $t = \frac{1377 - 1342}{\sqrt{329\left(\frac{1}{8} + \frac{1}{10}\right)}} = 4.101$	M1 M1 A1		M1 (numerator) accept 1342–1377 M1 (denominator - ft on their ${S_p}^2$ but must have 1/8 + 1/10) A1 awfw 4.10 ~ 4.11
	Critical value $t_{16} = \pm 2.583$	B1		c.v. ignore sign
	$4.10 > 2.583$ or $-4.10 < -2.583 \rightarrow$ reject H_0	A1		A1 – dependent on A1 for ts (sign consistent with hypotheses) and B1 for c.v
	Evidence at 1% level that **average yield** from cherry tomato plants is **increased** by using the *Moretom* fertiliser.	E1	10	Conclusion in context dependent on previous A1

Q	Solution	Marks	Total	Comments
3(c)	Cost per plant : using *Tomsplus* £1.25 using *Moretom* £1.50	B1		B1: calculating the cost per plant correctly.
	Average income per plant : using *Tomsplus* £3 ×1.342 = £4.026 using *Moretom* £3 × 1.377 = £4.131 (Total income : *Tomsplus*: £32.21 *Moretom*; £41.32)	M1		M1 Finding the income per plant (or total income) by multiplying the mean yield (or total yield) **in kilograms** by £3 for each type of fertiliser.
	Profit per plant: *Tomsplus* : £2.78 *Moretom*: £2.63 Thomas should continue with *Tomsplus* as this gives a greater profit per plant.	A1	3	A1 £2.78 and £2.63 seen and "continue with *Tomsplus*" recommended. sc B1 (if no calculations seen) Recommend using *Moretom* as additional expense justified by almost certain increase in average yield.
	Total		**21**	

Q	Solution	Marks	Total	Comments
4(a)	H_0 : B(15, 0.3) is an appropriate model for the data H_1 : B(15, 0.3) is not an appropriate model for the data	B1		At least H_0; must quote at least $p = 0.3$
	x | P(x) | O | E | $\frac{(O-E)^2}{E}$ ≤2 | 0.127 | 17 | 10.16 | 4.63 3 | 0.170 | 21 | 13.6 | 4.03 4 | 0.219 | 16 | 17.52 | 0.127 5 | 0.206 | 13 | 16.48 | 0.742 6 | 0.147 | 9 | 11.76 | 0.656 ≥7 | 0.131 | 4 | 10.48 | 4.01 Total | 14.195	M1 m1 m1 m1 m1		M1 probabilities m1 expected frequencies m1 combining classes m1 attempt at $(O-E)^2$ m1 dividing $(O-E)^2$ by E and summing – at least 2 values seen.
	$\upsilon = 6-1=5$	A1 B1√		A1 awfw 13.9 ~ 14.3 B1√ their no. of classes - 1
	crit. value χ_5^2 at 5% = 11.07 14.195 > 11.07 reject H_0	B1 A1		B1 awfw 11.0 ~ 11.1 dependent on previous A1 and B1 for cv
	Evidence at 5% level that **B(15, 0.3) is not an appropriate model** for the data.	E1	11	E1 conclusion in context dependent on previous A1 must have parameters.
(b)	H_0 : data may be adequately modelled by a binomial distribution H_1 : data may not be adequately modelled by a binomial distribution	B1		At least H_0, condone sight of B(15, 0.25)
	$\upsilon = 6-2=4$	B1		B1df
	crit. value χ_4^2 at 5% = 9.488	B1		B1 aqfw 9.48 ~ 9.50
	1.37 < 9.488 accept H_0	A1		dependent on B1 for cv
	Evidence at 5% level that the data **can be modelled adequately by a binomial distribution**.	E1	5	E1; conclusion in context dependent on A1, may mention B(15, 0.25)
(c)	Number of children with short-sight in a random sample of primary school children in Year 6 follow a binomial distribution.	B1		Comment on suitability of distribution
	Evidence that the proportion of these children with short-sight is less than 0.3.	B1	2	Comment on proportion – some context needed. Or Gill's original suspicion is not supported
	Total		**18**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$P(1 \le X \le 7) = \frac{7-1}{8} = 0.75$	M1 A1	2	M1: using correct rectangular distribution, allow slip eg 7/8 or 5/8
(ii)	Mean = 4 mins	B1		
	Standard deviation = $\sqrt{\frac{(8-0)^2}{12}} = 2.31$	M1A1	3	A1 awfw 2.30 ~ 2.31 s.c B1 for $\frac{64}{12}$
(iii)	Under this model it is impossible for a consultation to last longer than 8mins	B1	1	
(b)(i)	F(7) – F(1) =	M1		M1: sight of $1-e^{-\frac{7}{4}}$ or $1-e^{-\frac{1}{4}}$ or 1 – 0.1738 = 0.8262 or 1 – 0.7788 = 0.2212
	$\left(1-e^{\frac{-7}{4}}\right)-\left(1-e^{\frac{-1}{4}}\right)$	m1		m1: subtracting their F(7) – their F(1)
	= 0.605	A1	3	awfw 0.60 ~ 0.61
(ii)	$P(X = 8) = 0$	B1	1	
(iii)	$P(X \ge 8) = 1 - F(8) = 1 - \left(1-e^{\frac{-8}{4}}\right)$	M1		
	= 0.135	A1	2	awfw 0.135 ~ 0.136
(iv)	$P(X \ge 10 / X \ge 8) = P(X \ge 2)$	M1		Using "no memory"property
	$= 1 - F(2)$	M1		
	$= 0.61$	A1	3	A1 awfw 0.60 ~ 0.61 accept $e^{-0.5}$
	or			
	$P(X \ge 10 / X \ge 8) = \frac{P(X \ge 10)}{P(X \ge 8)}$			
	$= \frac{e^{\left(\frac{-10}{4}\right)}}{e^{\left(\frac{-8}{4}\right)}}$	(M1)		M1 Numerator and dividing
		(M1)		M1 denominator
	$= 0.61$	(A1)	(3)	A1 awrt 0.60 ~ 0.61 accept $e^{-0.5}$
				NB: must use correct probability distribution in all parts above

Q	Solution	Marks	Total	Comments
5(c)	Under new system 13.5% of appointments would overrun and of these approx 61% would take longer than 10 minutes. Reduction in appointment time is likely to make patients wait – doctors' wishes are supported. Note: the use of expressions such as "likely" or "most" must be supported by a numerical probability. scE1 for answers unsupported by correct numerical evidence	B2 E1	3	B1 for each distinct correct numerical comment on probabilities using the exponential model to a maximum of 2. E1 A single conclusion supported by numerical comments dependent on at least one B1. Alternatives: Approx. 60% of consultations last between 1 and 7 minutes and only 13.5% take longer than 8 minutes. Health centre's suggestion is reasonable; Margaret's wishes are supported. or unlikely almost 22% of all appointments last less than 1 minute poor model – more research needed
	Total		**18**	
	TOTAL		**75**	