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Statistics **SS04**

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Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1 (a)	No. turning up (X) ~ B(60, 0.8) which is approximately N(48, 9.6)	B1 M1		May be implied Attempt at any normal approximation
		A1		Mean = 48 cao, variance = 9.6 cao (or SD = 3.09 ~ 3.1(0))
	$P(X > 50) = P(Z > \frac{50.5-48}{\sqrt{9.6}})$	M1		Standardizing with their μ and σ (allow missing CC or 49.5); ignore sign
	$= P(Z > 0.81)$	m1		Use of 50.5 and correct sign (0.81 not required)
	= 1-0.79103			
	= 0.20897	A1	6	AWFW 0.208~0.212 (more exact value 0.2099) (Notes: (i) Use of B(60, 0.8) gives answer 0.21321 scoring 1/6) (ii) Use of 50 or 49.5 gives 0.259 or 0.314 respectively scoring 4/6
(b)	Fairly high probability (of disgruntled customers.)	B1		Sensible comment on size of prob (may say 0.21 is ‘high’ or ‘low’; if prob >> 0.21 should be ‘high; if prob << 0.21 should be ‘low’) **or** using original numbers + some attempt at a conclusion
	No, would not recommend continuing.	Bdep1	2	All correct (as bracket above) with clear, sensible conclusion (eg fairly low prob so recommend continuing gets 2/2)
	Total		**8**	

Q	Solution	Marks	Total	Comments
2 (a)(i)	Use of 2.5758	B1		Accept 2.57~2.58 (even if called *t*)
	99% CI is $6.75 \pm 2.5758 \times \dfrac{1.29}{\sqrt{1403}}$	M1		Use of $\dfrac{1.29}{\sqrt{1403}}$ (= 0.0344)
		m1		Correct interval, allow any Z
	$= 6.75 \pm 0.088(7)$	A1		**Either** for 6.75 ± (0.088 ~ 0.089)
	= (6.66, 6.84)		4	**or** AWRT 6.66 and 6.84
(ii)	Comparing upper limit of CI for males with lower limit of CI for females.	M1		Comparing limits of two CIs
	There is a difference between males and females.	A1	2	Requires implication of "difference" and clearly implied comparison of their upper limit with 6.87
(b)(i)	$H_0 : p = 0.15$ $H_1 : p > 0.15$	B1		For both
	Under H_0 , number sleeping over 8 hours ~ Bin(14, 0.15)	B1		Use of correct distribution
	Then $p(X \geq 4) = 1 - 0.8535$	M1		For finding $p(X \geq 4)$ or $p(X > 4)$ from a binomial distribution
	= 0.1465	A1		(0.146 ~ 0.147)
	Cannot reject H_0 at the 5% level	M1		Their binomial prob compared with 0.05 **and** correct ft conclusion
	No evidence of difference from 15 percent.	E1	6	Correct conclusion in context. Needs all previous marks. *Note* $p(X > 4) = 1 - 0.9533 = 0.047$ so reject H_0 gets B1B1M1A0M1E0 max 4/6
(ii)	Roughly the same age (not all teenagers) Same school/location. (not all UK) Similar social backgrounds etc	B1		Any sensible comment on the restricted nature of Rowan's sample.
	May not know/remember how long they slept. May not be truthful.	B1	2	Any sensible comment on issues such as inability to remember or truthfulness or accuracy.
	Total		**14**	

Q	Solution	Marks	Total	Comments
3 (a)	Total weight when Safeerah's riding is: 65 + 15 = 80kg on steel bike 65 + 10 = 75 kg on carbon bike Then % reduction = 5/80×100%	M1		For attempt to include Safeerah's weight
	= 6.25%.	A1		Correct calculation (Accept "approx 6"). Alternatively, something like "94% of 80 is approx 75"
	Hence Josh's figure		2	
(b)	$\bar{x} = 88.175 \qquad s = 4.6809$	B1		For 88.2 and $s_{n-1} = 4.68$ or s_n=4.48(16) (ignore labels)
	$H_0 : \mu = 90$ $H_1 : \mu < 90$	B1		Both
	$t = \dfrac{88.175 - 90}{4.6809/\sqrt{12}}$	M1		M1 for use of $\dfrac{s_{n-1}}{\sqrt{n}}$ *or* $\dfrac{s_n}{\sqrt{n-1}}$ in test statistic formula
		m1		Correct formula, ignore sign for m1. Or $t = \dfrac{88.175 - 90}{4.4816/\sqrt{11}}$
	= −1.351	A1		AWFW −1.36 to −1.33
	Critical value $t_{11} = -1.796$	B1 B1		For 11 df may be implied (eg by 1.80 or 2.20(1) For $t_{11} = (-1.80 \sim -1.79)$ **or** p = 0.10197 (AWFW 0.1005 to 0.1055)
	Accept H_0 at 5% level. No evidence that mean journey time is < 90 minutes.	E1		Their t and critical t (both negative) OR their p-value and 0.05 but still requires negative t.. Requires M1 and m1 and "acc H_0". In context **Alternatives** Full marks available for: (i) one-sided CI 88.175 + 2.427 = 90.6 then cf 90 (ii) critical region 90 – 2.427 = 87.6 then cf 88.175
			8	
	Total		**10**	

Q		Solution	Marks	Total	Comments
4	(a)	Distributed at random/independently/ at constant average rate.	B1	1	For any of these.
	(b)	$H_0 : \lambda = 12$ $H_1 : \lambda < 12$	B1		For both. Allow μ or "rate".
		Find $P(X \le 8)$ from Poisson tables	M1		Attempt to calculate $P(X \le 8)$ or $P(X < 8)$ (= 0.0895)
		= 0.155(0)	M1		Compare their Poisson prob with 0.10
		This is > 0.10 so no evidence it's reduced.	A1	4	All correct including 0.155. Allow "accept H_0" here and isw.
	(c)	Number of lob worms in 10 m^2 (Y) has a Poisson distribution with $\lambda = 142$			
		Then 95% CI for Y using Z=1.96 is	B1		For 1.96
		$142 \pm 1.96\sqrt{142}$	M1		For $c \pm z\sqrt{c}$. Allow $c \pm z\sqrt{\frac{c}{10}}$
		$= 142 \pm 23.356$	A1		$142 \pm (23.3 \sim 23.4)$ or (119, 165)
		Then average density per m^2 is			
		14.2 ± 2.34	M1		Dividing this (or limits) by 10
		(11.86, 16.54)	A1	5	For AWRT (11.9, 16.5) or $14.2 \pm (2.3 \sim 2.4)$ **Notes:** (i) **SR:** Using $14.2 \pm 1.96\sqrt{14.2}$ gets B1M1A0 then B1 for answer 14.2 ± 7.4 getting max 3/5. (ii) Using $14.2 \pm 1.96\sqrt{\frac{14.2}{10}}$ gets potentially full marks.
	(d) (i)	$M_A = 56$ and $M_B = 13$	B1		For both
	(ii)	Would expect $V_A > M_A$	B1		
		Ants 'attract' each other → tendency for small or large numbers in a sq m → larger variation (than expected from Poisson)	B1		Or equiv. Any indication of why they would expect "large" variation (more than just saying they live in colonies). Note: B1's indep in either order
	(iii)	Would expect $V_B < M_B$	B1		
		Beetles 'repel' each other → more evenly spread (eg unlikely to have high concentrations) → smaller variation (than expected from Poisson).	B1	5	Or equiv. Any indication of why they would expect "small" variation (more than just saying they are territorial). As note in (ii).
		Total		**15**	

Q	Solution	Marks	Total	Comments
5 (a)	P(dizziness) × P(swollen jnts) = 0.12 × 0.15 = 0.018 = given value for joint probability. Hence independent.	 M1 A1	 2	 For 0.12 × 0.15 On correct reasoning.
(b) (i)	No. with dry skin ~ B(90, 0.01) Which is approx Poisson with $\lambda = 90 \times 0.01 = 0.9$ Then P(X > 2) = 1 – 0.937(1) = 0.0629	 B1 B1dep M1 A1	 4	 For Poisson (may be implied) May be implied Allow for 1 – 0.7725 (may be implied by 0.227 ~ 0.228) 0.0629 ~ 0.063 **Note:**Exact binom→0.0619→(0/4)
(ii)	$H_0 : p = 0.12$ $H_1 : p \neq 0.12$	B1		Both
	$X \sim B(90, 0.12) \approx N(10.8, 9.504)$	B1 B1		B1 for mean 10.8 cao B1 for variance (9.5(0) ~ 9.51) or SD (3.08 ~ 3.09)
	TS = $\dfrac{21 - 10.8}{\sqrt{9.504}} = 3.31$	M1 A1		Their mean/SD 3.25 ~ 3.4
	or TS = $\dfrac{20.5 - 10.8}{\sqrt{9.504}} = 3.15$	(M1) (A1)		3.13 ~ 3.15 Note: wrong CC (21.5) gives 3.47 for M1A0
	or using proportions Use of N(0.12, .00117)	 (B1) (B1)		 B1 for mean 0.12 cao B1 for variance(0.0011 ~ 0.0012) or SD (0.033~0.035)
	$\hat{p} = \dfrac{21}{90} = 0.233$ TS = $\dfrac{0.233 - 0.12}{\sqrt{\dfrac{0.12 \times 0.88}{90}}} = 3.31$	 (M1) (A1)		 Their mean/SD 3.25 ~ 3.4
	Critical values are ± 2.5758	B1		2.57 ~ 2.58 **or** *p*=0.000938 (AWFW 0.00065 to 0.00115) from TS=3.31 **or** *p*=0.00163 (AWFW 0.00163 to 0.00175) from TS=3.15 Allow *p*=0.000466 or 0.000816 only if compared with 0.005
	Reject H_0 at the 1% level	A1		ft their TS and critical z value **or** their *p*-value and 0.01 **or** their ½ p and 0.005. Must correctly reject H_0
	There is evidence that the proportion of patients really does differ from 0.12.	E1	8	In context, requires all previous marks in (b)(ii).
	Total		**14**	

Q	Solution	Marks	Total	Comments
6(a) (i)	Let L be the height of cut trees Then $L = H + G - S$ E(L) = 50 + 150 – 10 = 190 V(L) = $1.49^2 + 2.67^2 + 0.51^2$ (= 9.6091) Then SD(L) = 3.10 as required.	 B1 M1 A1	3	 For 190 cao. Method must be seen and correct. Correct expression for V(L) seen AWRT 3.10 answer given.
(ii)	$P(L < 1.95) = P(Z < \frac{195-190}{3.10})$ $= P(Z < 1.61(3))$ = 0.9463(0)	M1 A1	2	Allow wrong tail but needs 195, 190 and 3.10 or their E(L)&SD(L) (0.946 ~ 0.947) **SC** (1.95-1.9)/3.1 = 0.0161 gets B1
(b) (i)	$P(97 < height < 103) = P(-1 < Z < 1)$ = 0.84134 – (1 – 0.84134) = 0.68268 Can expect 100 × 0.683 = 68.3 trees	 M1 A1 B1	3	 Correct method for z and 0.841 seen. (0.682 ~ 0.683) For their probability × 100. May be rounded up or down.
(ii)	Let P be the tree length for shredding Then E(P) = 190 – 100 = 90 and V(P) = $3.1^2 + 3^2$ = 18.61 Income (Y) = 500 + 100×0.02×P = 500 +2P Then E(Y) = 500 + 2×90 = 680 and V(Y) = 2^2×V(P) = 74.44 Thus Y ~ N(680, 74.44)	 M1 A1 AF1 B1	4	Marks in (ii) can be earned in (iii) No marks yet! Need to use these values for income. For 500 + attempt at pulp income cao (74.4 ~ 74.5) ft for 4 × their V(P) For **normal** distribution
(iii)	Since council's offer gets £680 < £750 she was NOT wise to take it up.	M1 E1	2	Their E(Y) compared with 750 (may be implied) All clear & correct, including 680 **Alternative** May use their distribution to find a probability (eg P(Y > 750) ≈ 0). Then M1 for attempt at relevant probability and E1 as above. **SC** Something like "no guarantee she will sell them all so should take what is firmly offered" B1
	Total		**14**	
	TOTAL		**75**	