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AQA

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Statistics SS04

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Statistics 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS04

Q	Solution	Marks	Total	Comments
1	H_0: $\mu = 140$ and H_1: $\mu \neq 140$	B1		Both correct
	Use SE(mean) = $22.79/\sqrt{12}$	M1		Use of SE = $S/\sqrt{n}$ allow $n = 11$ or 12
	$t = \pm(137.24 - 140)/(22.79/\sqrt{12})$	m1		Correct expression - ignore sign
	$-0.4195 = (-)\ 0.420$	A1		$(-)0.415$ to $(-)0.425$
	df, $\nu = 11$	B1		$\nu = 11 \Rightarrow$ implied by 1.363, 1.796, 2.201, 2.718 or 3.106
	FT 2.5% point: $t_{(0.025)}$ $(\pm)$ 2.201	B1√		$\pm\ t_{(0.025)}$ FT their df, (e.g. 2.179 for $\nu = 12$)
	Accept H_0 (or p-value approach)	A1√		Their TS vs recognisable t value, matching signs $P(t_{11} < -0.4195) = 0.341463 > 0.025$ Ignore missing/faulty H_0
	Accept Aaron's claim. There is no significant evidence that the mean weight of pears is not 140g.	A1	8	Completely correct, 0.420 vs 2.201, conclusion in context: Mean pears weight = 140g.
	CI approach			
	Hypotheses correct $137.24 \pm 2.201 \times (22.79/\sqrt{12})$ Using 2.201: 122.76 to 151.72 CI includes 140, accept H_0 **Accept Aaron's claim**.	(B1) (M1) (m1) (B1) (B1√) (A1) (A1√) (A1)		SE(mean) with $n = 11$ or 12, expression for CI, allow use of t or Z $\nu = 11$ seen or implied FT on t value, 2.201 Limits 122.7 – 123.0 and 151.7 to 152.0 compare 140 to limits, Accept H_0 completely correct, contextual conclusion
	$z(1.96)$: interval 124.35 to 150.13			B1 M1 m1 B0 B0√ A1 A0√ A0
	124.0 – 124.4; 150.0 – 150.2			Max 4/8
	Total		**8**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
2(a)	Sales ~ P(187) $\rightarrow$ N(μ, μ)	M1		Attempt a Normal approximation
	$\mu = \sigma^2 = 187$	A1		Mean and variance both 187 (σ= 13.7)
	$z = \pm\,(176 - 0.5 - 187)/\sqrt{187} = -0.841$	m1		Standardise 176, ignore any CCF, $\sigma = \sqrt{187}$
	$P(S < 176) = 1 - \Phi(0.841)$	m1		Attempt lower tail probability ($p < 0.5$)
	$1 - 0.800 = 0.200$	A1	5	0.199 ~ 0.201
(b)	Binomial $n = 175$ $p = 0.008$	B1		B(175,0.008) identified
	$\rightarrow$ Poisson $\mu = 175\times0.008 = 1.4$	M1		Attempt Po($\mu = n\times p$) Normal gets M0
	Use of P(1.4), tables or calculation	m1		P(0) = 0.2466, P(1) = 0.3452, P(2) = 0.2417
	Attempt P(2 +) = 1 – P(0,1) = 1 – 0.5918	m1		1 – P(0, 1) or 1– P(0,1,2) = 1 – 0.8335 = 0.1665
	= 0.408	A1	5	0.40 ~ 0.41
	B(175, 0.008) $\Rightarrow$ 1 – 0.5913 = 0.409			B used: P(0) = 0.2452,P(1) = 0.3461, P(2) = 0.2428
(c)	During Elani's first week,			
	Allow generous first E1for reasonable comment about sales/faults			Sales below average and faults above average
	Observed sales below average, but $P(S \leq 176)$ not too small (0.2)	E1$\surd$		176 sales below average but not significantly so (quite likely)
	Observed faults above average, but P(2+) not small (0.4)	E1$\surd$		2 faults slightly more than expected but not significantly so (quite likely) Faults are responsibility of manufacturers (not management)
	If both above not gained, allow E1 for comment on paucity of data	(E1)		Too little data, one week insufficient
	Consider **both** sales and faults, and argue claim not justified or P(both) = 0.2×0.4 = 0.08 quite small so there may be a concern	E1dep	3	Overall no evidence Elani ineffective or Poor performance in both areas may indicate a problem Requires consideration of both sales & faults ***maximum 3***
	Total		**13**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Sample mean = 20.7 and SD = 9.154 σ_n = 8.6838	B1		20.7 CAO and 9.15 (9.15 ~ 9.16) (implied by s^2 = 83.7 ~ 83.8)
	use $t_{(0.025)}$ = 2.26(2)	B1		2.262 seen
	95% CI: 20.7 ± 2.262 × 9.154/√10	M1		CI method, t or z, √10 used
	20.7 ± 2.262 × 9.154/√10	m1		correct 2.26 and √10, their mean/SD M0 for σ_n/√10, s/√9, mean = 40
	20.7 ± 6.55 or 14.2 ~ 27.2	A1	5	(14.1 ~ 14.2) to (27.2 ~ 27.3) or ± (6.50 ~ 6.55)
(b)	If average = 50% of available marks (80) target mean should be 40. IF SD = 10% of available mark (80) Target SD should be 8	E1		target mean = 40 marks seen and / or target SD = 8 marks seen
	CI for mean < 40, so evidence that the test is too difficult/ inappropriate/ target not met.	E1√		FT their values
	SD close to (>) 8, so variability criteria satisfied/ target met.	E1√	3	FT their values
	Total		**8**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	H_0: λ or $\mu = 9$ per week; H_1: $\mu < 9$	B1		Both — allow $\mu = 36$ (per 4 weeks)
	Over 4 weeks $\mu = 4\times9 = 36$	B1		mean 36
	Poisson $\rightarrow$ N(36, 36)	M1		N(μ, μ) attempt (e.g $\mu = 5.75$, 9, 23 or 36)
	$z = \pm(23.5 - 36)/\sqrt{36} = -2.08$ or $z = \pm(23 - 36)/\sqrt{36} = -2.17$	m1		Standardise 23 — ignore sign and CC $\mu = 36$ and $\sigma = 6$ correct
	Either $z = -2.08$ or -2.17 implied by $\Phi(z) = 0.0188$ or 0.0150	A1		$(-)2.08 \sim (-)2.09$; $(-)2.16 \sim (-)2.17$ $0.0183 \sim 0.0188$ $0.0150 \sim 0.0154$
	CV = $z_{(0.10)} = (-)1.2816$ OR $\Phi(z)$ vs 0.1	B1 (M1)		$(-)1.28$ to $(-)1.282$, ignore sign Their p-value vs 0.1
	TS vs 1.282; p vs 0.1 Reject H_0	A1$\surd$		Conclusion , FT their TS vs 1.282 consistent signs/(valid) p-value vs 0.10
	Evidence that the mean number of complaints of anti-social behaviour in 2011 is likely to be less than 9 per week.	A1	8	Completely correct, correct conclusion in context (ignore faulty H_0 H_1)
(b)	Complaints may not be independent Mean number of complaints may change due to weather, seasonal events, time of year so not constant etc	E1	1	Any valid contextual reason why Poisson invalid But "Complaints not constant" — E0
	Total		**9**	

Q	Solution	Marks	Total	Comments
5(a)(i)	10 Pubs raise £P ~ Normal, $\mu = 9000$	B1		9000
	$\sigma = \sqrt{(10\times185^2)} = \sqrt{342\,250} = 585.02$	B1	2	$\sigma = 585$ validly **shown**
(ii)	$z = (10\,000 - 9\,000) / 585.02 = 1.709$ $P(P > 10\,000) = P(Z > 1.709)$	M1		Standardise 10 000 and attempt $\Phi(\pm z)$ FT on μ, $\pm z$
	$= 1 - 0.956 = 0.044$	A1	2	0.044 (0.043 ~ 0.044)
(b) (i)	Total raised ~ $T = 3P$ $\mu = 27000$	B1		27 000 CAO
	$\sigma^2 = \mathbf{9}(585^2)$ or $\sigma = \mathbf{3}\times585.02$	M1		method for σ or σ^2
	$\sigma^2 = 3\,080\,025$ or $\sigma = 1755$	A1	3	σ^2: 3080 000 ~ 3 080 500 σ: 1 750 ~ 1 760
(ii)	Distribution of Surplus = T – Cost,	M1		Attempt **distribution** of difference, neither treated as a constant
	$\mu = 27000 - 29000 = (-)2000$	A1		$\pm$ 2000
	$\sigma^2 = 1755^2 + 500^2 = 3330025$ or $\sigma = 1825$	M1		$500^2 +$ (b)(i) variance
	$P(S > 0) = P(Z > (0 + 2000)/1825)$	m1		Standardise 0, attempt $\Phi(\pm \mu/\sigma)$
	$= P(Z > 1.096) = 1 - 0.863 = 0.137$	A1	5	0.135 ~ 0.140
(c) (i)	Amount raised by each pub may be affected by time of year/recession which will affect all pubs/pubs close together may affect each other's fund-raising.	E1		Reason why money raised by each pub is not independent of other pubs
(ii)	Final specification may be arranged so that the machine may be bought with the money raised/extra effort may be made to raise funds if total falls slightly short of final price.	E1	2	Reason why final cost of equipment may not be independent of money raised
	Total		**14**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	H_0: $p = 0.03$ **and** H_1: $p > 0.03$	B1		H_0 and H_1 — may be earned in (a)(ii)
	B(30, 0.03)	M1		attempted use of B(30, 0.03)
	attempt P(2+) = 1 – P(0,1) = 1 – 0.7731	m1		1 – P(0,1) or 1 – P(0,1,2) = 1 – 0.9399 = 0.0601
	= 0.2269	A1		0.226 ~ 0.227
	Accept H_0, as 0.2269 > 0.05	A1√		FT Conclusion (Ignore H_0), their (valid) p- value vs 0.05
	No evidence scheme was effective	A1	6	Completely correct and conclusion in context
(ii)	H_0: $p = 0.03$ H_1: $p > 0.03$			Can recover B1 above if not gained in a(i)
	B(583,0.03)	B1		attempt B(583, 0.03) use NB P(2) = 0.167
	Normal N(np, npq)	M1		N(np, npq) attempt, n = 583 their p
	$\mu = 583\times0.03 = 17.49$ $\sigma^2 = 583\times0.03\times0.97 = 16.97$	m1		Attempt np and npq with $n = 583$, $p = 0.03$ $\mu = 17.5$ and $\sigma^2 = 17.0$ ($\sigma = 4.119 = 4.12$)
	P(28+): $z = (27.5 - 17.49)/4.119 = 2.43$ Or $(28.0 - 17.49)/4.119 = 2.55$	m1		Standardise 27 FT μ and σ, ignore CC
	Either $z = 2.43$ or 2.55	A1		2.42 ~ 2.44 ; 2.54 ~ 2.56
	CV: $z_{(0.05)} = 1.6449$ OR $\Phi(z)$ vs 0.05	B1 (M1)		1.64 ~ 1.65, allow if p-value vs 0.05 Appropriate tail probability vs 0.05
	$z > 1.64/1.65$, Reject H_0 $\Phi(z) = 0.00755$ or $0.00539 < 0.05$	A1√		FT conclusion TS vs 1.65, or p-value 0.0075 ~ 0.0076; 0.0053 ~ 0.0054 < 0.05
	Evidence show scheme has been effective	A1	8	Completely correct, in context
	POISSON approx, $\mu = 17.5$	(B1) (M1A1)		B(583,0.03) Po(17.49 = 17.5)
	$P(X \geq 28) = 1 - P(27) = 1 - 0.9875 = 0.0125$	(M1A1)		$P(X \geq 28) =$ 0.012 ~ 0.013
	0.0125 < 0.05 reject H_0	(M1A1)		p value < 0.05, Reject H_0
	Conclusion in context	(A1)		context
	Binomial ⇒ Poisson ⇒ Normal			Allow M's and B's only for consistent working, max 5/8
a(iii)	Accept (a)(ii) conclusion since it is based on a larger sample	E1	1	(a)(ii) because larger sample allow mark for comments casting doubt on appropriateness of binomial model

SS04 (cont)

Q	Solution	Marks	Total	Comments
6(b)	$p = 10/583 = 0.01715$	B1		10/583 ; 0.017 ~ 0.0172
	Use z = 1.96	B1		1.96 seen
	$p \pm z\times\sqrt{(pq/n)}$	M1		attempt CI, $n = 583$, their p, any z
	$0.01715 \pm 1.96\times\sqrt{(0.01715\times0.98285/583)}$	m1		correct expression using 1.96
	0.01715 ± 0.01054 = 0.0066 to 0.0277	A1	5	0.017 ± 0.011 (0.006 ~ 0.007) to (0.027 ~ 0.028)
(c)	Some customers (did) do spend more than £30 without a voucher, so Jarrald could be correct	E1		Argument that Jarrald could be correct
	Confidence interval in (b) suggests the proportion of customers spending more than £30 without voucher is below 3%.	E1		Upper CI below 3%, customers spending more than £30 without voucher is below previous level. Or comment that majority (18/28) spending over £30 used voucher.
	Before vouchers 3% spent more than £30. Evidence from (a)(ii) indicates promotion increases proportion above 3% so extra revenue might make cost of promotion worthwhile.	E1	3	Argument that increased revenue might justify cost, or argue data presented makes cost benefit uncertain. *Also allow marks for comments on appropriateness of model if not already awarded in (a)(iii)/possible increase in number of customers etc*
	Total		**23**	
	TOTAL		**75**	