AQA↗

Statistics   SS04

Unit Statistics 4

Wednesday 20 January 2010   1.30 pm to 3.00 pm

For this paper you must have:

• an 8-page answer book
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed
• 1 hour 30 minutes

Instructions
• Use black ink or black ball-point pen. Pencil should only be used for drawing.
• Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is SS04.
• Answer **all** questions.
• Show all necessary working; otherwise marks for method may be lost.
• The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 75.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

SS04

Answer **all** questions.

1 A crossroads in Chorlton is controlled by traffic lights. There are green-surfaced areas before the lights for the exclusive use of cyclists when the light is red.

A recent survey observed that, on 34 out of 50 occasions when traffic had halted at a red light, cars encroached on to these areas.

(a) Making any necessary assumptions, calculate an approximate 95% confidence interval for the proportion of times that, when traffic halts at a red light, cars encroach on to these areas. *(6 marks)*

(b) Comment on a cyclist's claim that on more than 90% of occasions, when traffic halts at a red light, cars encroach on to these areas. *(2 marks)*

2 (a) In a population, 1 person in 10 000 is known to suffer from a rare disease. A medical team screened a sample of 20 000 people from this population for the disease.

Use a suitable distributional approximation to find the probability that the sample, which may be assumed to be random, included:

(i) no one suffering from this disease;

(ii) more than 5 people suffering from this disease. *(6 marks)*

(b) Symptoms of the disease include discoloured skin and shortage of breath. People who exhibit these symptoms have a probability of 0.32 of suffering from the disease. The medical team examined a sample of 80 people who exhibited these symptoms.

Use a suitable distributional approximation to find the probability that the sample included more than 20 people suffering from the disease. Assume that the sample was random. *(6 marks)*

3 Marek owns a filling station which also sells newspapers. He is concerned about the effect on his business of a recent increase in fuel prices. Before the increase took place, customers purchasing fuel bought an average of 37.5 litres per visit.

Following the increase in prices, the amounts of fuel, in litres, bought by a random sample of customers were

$$24.2 \quad 38.9 \quad 22.3 \quad 42.3 \quad 30.6 \quad 18.2 \quad 35.3 \quad 24.3$$

(a) Carry out a suitable test, using the 5% significance level, to examine whether there is evidence that the mean amount of fuel bought by customers is now less than 37.5 litres per visit. Assume that the sample is from a normal distribution. *(9 marks)*

(b) Before the price increase, the number of customers on a Friday afternoon could be modelled by a Poisson distribution with mean 84 per hour. Following the increase, Marek decided to count the customers. During a **two-hour** period on Friday afternoon, there were 142 customers.

Using a suitable approximation, test, at the 1% significance level, whether there has been a reduction in the number of customers following the price increase. Assume that the number of customers on Friday afternoon continues to be modelled by a Poisson distribution. *(7 marks)*

(c) Before the increase in fuel prices, 20 per cent of customers bought newspapers but not fuel.

Following the increase in fuel prices, the next 20 customers were

$$\text{f, f, f, n, f, n, f, f, f, f, n, n, f, f, f, f, f, n, f, f}$$

where f represents a customer who bought fuel and n represents a customer who bought a newspaper but not fuel.

Using an exact test and the 10% significance level, examine whether the proportion of customers who buy a newspaper but not fuel has increased. Assume that the customers acted independently and that they may be regarded as a random sample. *(5 marks)*

(d) Summarise your findings as to whether or not Marek's concerns about the effects of the increase in fuel prices are justified. *(4 marks)*

Turn over ▶

4 A tribunal hears appeals against rating valuations. The duration, T minutes, of an appeal may be modelled by a normal distribution with mean 110 and standard deviation 25.

On a particular day, two appeals are to be completed. The first appeal starts at 9.00 am.

(a) Find the probability that it is completed before 11.00 am. *(3 marks)*

(b) The first appeal will be completed after T minutes. Immediately after this appeal, Debbie, a member of the tribunal, will telephone for a taxi to collect her in a further T minutes' time. Thus the taxi should arrive $2T$ minutes after 9.00 am.

Find the probability that Debbie will ask the taxi to collect her before 12 noon. *(4 marks)*

(c) The tribunal takes a break between the two appeals. The duration, B minutes, of this break may be modelled by a normal distribution with mean 12 and standard deviation 4.

Assuming that the durations of the two appeals, T_1 and T_2, and the break, B, are all independent of each other, find the probability that the second appeal will **not** be completed by 12 noon. *(5 marks)*

(d) (i) Find the probability that

$$T_2 + B - T_1 < 0$$

where T_1 is the duration of the first appeal and T_2 is the duration of the second appeal. *(3 marks)*

(ii) Explain why this probability represents the probability that the second appeal will be completed before the taxi, ordered in part (b), is due to arrive. *(2 marks)*

5 The lengths, in millimetres, of a random sample of components produced by a particular machine are

135.1 136.9 134.2 133.2 133.7 134.1 133.8 134.7 134.6 134.8

(a) (i) Given that the sample comes from a normal distribution, show that a 95% confidence interval for the mean length of components produced by this machine has limits of 133.78 and 135.24, correct to five significant figures.

(6 marks)

(ii) Explain why more than three significant figures are required for the limits of this confidence interval. *(1 mark)*

(b) Read the following three statements, which refer to the population of lengths of components produced by the machine. Ignore the rounding error in the confidence interval quoted in part (a)(i). Indicate, for **each** statement, whether it is:

A definitely true;
B almost certainly true;
C almost certainly untrue; or
D definitely untrue.

Justify each answer.

Statement 1 Exactly 95% of the population lies between 133.78 and 135.24.

Statement 2 A random sample of size 10 is to be taken from the population. The probability that the mean of this sample will lie in the interval 133.78 to 135.24 is 0.95.

Statement 3 A random sample is to be taken from the population and a 95% confidence interval for the population mean calculated. There is a probability of 0.95 that this confidence interval will contain the mean of this sample. *(6 marks)*

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END OF QUESTIONS

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