Teacher Support Materials 2009

Statistics GCE

Paper Reference SS05

Question 1

Sadia arranges to meet Arlene at a coffee bar on Saturday evenings at 8.00 pm. Past experience suggests that Arlene will arrive at the coffee bar X minutes after 8.00 pm, where X may be modelled by an exponential distribution with parameter 0.05.

(a) Find the mean and standard deviation of the number of minutes after 8.00 pm that Arlene will arrive at the coffee bar. *(2 marks)*

(b) If Sadia arrives at 8.20 pm, find the probability that Arlene will already have arrived. *(3 marks)*

(c) Sadia arrives at 8.20 pm and finds that Arlene has not yet arrived. Find the probability that Arlene will arrive after 8.30 pm. *(3 marks)*

Student Response

1a) mean $= \frac{1}{0.05} = 20$ ✓

standard deviation $= 20$ ✓ 2

b) $\left[1 - e^{-0.05x}\right]_0^{20}$

$1 - e^{-0.05 \times 20} - 0$

$= 1 - 0.368$

$= 0.632$ ✓ 3

c) Prob hasn't arrived by 8.20 $= 1 - 0.632$

$= 0.368$ ✗ (5)

$\left[1 - e^{-0.05x}\right]_{30}^{\infty}$

$1 - (1 - e^{-0.05 \times 30})$

$= 1 - (1 - 0.223)$

$= 0.223$ ✗

Commentary

This candidate has correctly answered parts (a) and (b). In part (c) the probability that Arlene will arrive after 8.30pm is calculated ignoring the fact that it is already known that she has not arrived by 8.20pm. This was a common error.

Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	mean 1/0.05 = 20 s.d. 1/0.05 = 20	M1		Method for both
		A1	2	20 both, CAO
(b)	$1 - e^{-0.05 \times 20}$	B1		0.05×20
	$= 1 - e^{-1}$	M1		Method - allow wrong tail
	$= 0.632$	A1	3	0.6315 ~ 0.6325
(c)	$e^{-0.05 \times 10}$	M1		Attempt to find > or < 10 from exponential parameter 0.05 or equivalent
	$= e^{-0.5}$	m1		Method - allow wrong tail
	$= 0.607$	A1	3	0.606 ~ 0.607
	Total		**8**	

Question 2

Helen sells machines for filling jars and claims that the weights of the contents of jars will be distributed with a standard deviation of 1.4 grams. One of her machines was used to fill jars with mustard.

The weights, in grams, of the contents of the first eight jars filled were

212 227 216 224 217 216 218 220

Leonidas, a statistician, glanced at the data and stated that he thought the standard deviation of the weights of the contents of jars was greater than 1.4 grams.

(a) Suggest a possible reason for Leonidas's statement. *(2 marks)*

(b) Assuming that the data may be regarded as a random sample from a normal distribution, calculate a 90% confidence interval for the standard deviation of the weights of the contents of jars. *(7 marks)*

(c) State, giving a reason, whether or not your calculation in part (b) supports Leonidas's statement. *(2 marks)*

(d) Using a method which is appropriate in the light of your conclusion in part (c), calculate a 95% confidence interval for the mean weight of the contents of jars. *(5 marks)*

(e) The target was for each jar to contain 212 grams of mustard. Helen stated that the confidence interval calculated in part (d) indicated that the mean could be reduced and all jars would still contain at least 212 grams of mustard. Comment on this statement. *(3 marks)*

Student Response

2. (a) The spread of the data is larger than 3 times of the 1.4. So, the total distribution width would be inaccurate due to the variability of the data. E1

(b) $n = 8$ $S = 4.80327$ $\sigma = 1.4$

$$\left(\frac{(n-1)S^2}{\chi^2 \text{ upper}(0.975)}, \frac{(n-1)S^2}{\chi^2 \text{ lower}(0.025)}\right) \quad v = n-1 = 7$$

$$= \left(\frac{7(4.80327)^2}{16.013}, \frac{7(4.80327)^2}{1.69}\right) \quad 95\%$$

$$= (10.086, 95.562)$$

$$= (\sqrt{10.1}, \sqrt{95.6}) \quad (\text{Var})$$

$$(3.18, 9.78) \quad (\text{S.D}) \quad \times$$

5

(c) The Calculation I made in part b supports the claim. The interval lies completely above the 1.4 given by helen, supporting Leonida's Claim. 2

Leave blank

(d) ~~TTEST~~ T distribution (As we disagree with the 1.4 σ)

(214.73, 222.76)

5

(e) The Confidence interval Calculated lies above 212. This means that if they lower the mean, the Confidence interval values will be lower. As the interval is above 212, this supports helen's claim, ✗ making her statement valid.

13

Commentary

This candidate has the right idea in comparing the range (although called spread) with the claimed standard deviation. However there is nothing surprising in the range of a sample of eight exceeding three standard deviations. Four standard deviations is more unlikely but six would have made Leonidas's statement clearly appropriate.

In part (b) the candidate has calculated a 95% confidence interval instead of 90% as requested but is otherwise correct.

Part (c) is correct.

In Part (d) the candidate has correctly chosen to use the t-distribution and has obtained the answer directly from a calculator. This received full marks but some candidates who showed no working wrote down an incorrect answer and received no marks.

In part (e) the candidate ignores the fact that the confidence interval refers to the mean and Helen's statement refers to individual jars.

Mark Scheme

Q	Solution	Marks	Total	Comments
2(a)	Range 15g $> 10 \times 1.4$ or $6 \times 1.4 = 8.4 < 15$	E1		Comparison of range and s.d.
	Range is too large if $\sigma = 1.4$	E1	2	Full explanation
(b)	$s = 4.8033$	B1		4.8033 (4.80 ~ 4.81) or 23.07 (23 ~ 23.1) or 161.5 (161 ~ 162)
	$2.167 < 7 \times 4.8033^2/\sigma^2 < 14.067$	M1		Any correct expression; allow small slip, incorrect χ^2
		m1		Correct expression, allow incorrect χ^2
		B1		7 df
		B1		14.067 (14 ~14.1) and 2.167 (2.16 ~ 2.17)
	$161.5/14.067 < \sigma^2 < 161.5/2.167$	m1		Correct method for interval for σ, or σ^2 provided it is clearly called σ^2 or variance
	$11.481 < \sigma^2 < 74.527$			
	$3.39 < \sigma < 8.63$	A1	7	3.39 (3.385 ~3.395) and 8.63 (8.63 ~8.64)
	or using F $4.8033^2/\sigma^2 < 2.010$ *and* $\sigma^2/4.8033^2 < 3.230$			
(c)	Statement supported since 1.4 is below	B1√		Statement supported - their c.i.
	lower bound of confidence interval	E1	2	Explanation
(d)	$\bar{x} = 218.75$ 95% confidence interval			
	$218.75 \pm 2.365 \times 4.8033 / \sqrt{8}$	M1		Use of their s.d. $/ \sqrt{8}$
		M1		Attempt at c.i. using *t*
		m1		Method - allow incorrect *t*-value
		B1		2.365 (2.36 ~ 2.37)
	218.75 ± 4.02			
	214.7 ~ 222.8	A1	5	214.7 (214.7 ~ 214.8) and 222.8 (222.7 ~ 222.8); allow 215 and 223
(e)	Confidence interval indicates that mean is above 212. Hence mean could be reduced	E1		Statement incorrect
	and the mean could still be greater than 212. However, the fact that one member of the sample only contains 212g indicates	E1		Mean could be reduced and still be greater than 212
	that if the mean were reduced some individual jars would contain less than 212g.	E1	3	If mean were reduced some individual jars would contain less than 212g
	Total		**19**	

Question 3

Each new employee joining a construction company is issued with a safety helmet. The helmets come in five sizes: 1, 2, 3, 4 and 5.

The data below represent the sizes of the helmets issued to the 40 most recent new employees.

3	3	1	2	5	4	4	2	2	3	5	4	5	4	3	4	4	3	4	3
3	2	4	5	2	5	3	5	5	4	3	3	5	4	5	4	4	5	1	5

(a) Form the data into a frequency distribution. *(2 marks)*

(b) Olan, who is in charge of ordering the helmets, believes that the distribution of helmet sizes required by new employees may be modelled by the following probability distribution.

Helmet size	Probability
1	0.15
2	0.20
3	0.30
4	0.20
5	0.15

Using the 5% significance level, examine whether this distribution adequately models the helmet sizes required by new employees. Assume that the 40 most recent new employees may be regarded as a random sample of all new employees. *(7 marks)*

(c) Olan is about to order 1000 new helmets with the proportion of each size as indicated by the probability distribution.

Advise Olan as to whether he should modify the proportions of the sizes in the order. If you advise him to modify the order, state, in general terms, how you believe the proportions should be changed. *(2 marks)*

Student Response

3)a)

Helmet size	1	2	3	4	5	Total
Freq	2	~~4~~ 5	10	12	11	40

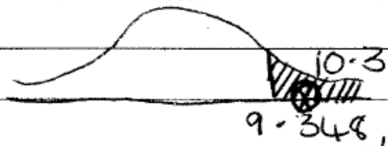
2

3b)

	1	2	3	4	5	Total
Observed	2	5	10	12	11	40
Expected	6	8	12	8	6	40
$\frac{(O-E)^2}{E}$	2·67	1·13	0·33	2	4·17	10·3

H₀: The distribution does fit the model
H₁: The distribution does not fit the model

$V = n - 2$
$V = 5 - 2$
$V = 3$



3 dp.
0.474

T·S = 10·3
C·V = 9·348 ✗

Reject H₀, there is significant evidence to suggest at the 5% level the distribution used to model the new employees helmet size does not fit the model.

5

3C) Olan should alter the proportion in the sizes in the order. This is because the amount in the expected varies

E1

Commentary

The frequency distribution is correct as is the calculation. However the candidate has used the wrong degrees of freedom and the wrong significance level for their critical value.

In part (c) it is correctly suggested that Olan should modify the order but no suggestion is made as to how it should be modified.

Mark Scheme

<table>
<tr><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>3(a)</td><td><table><tr><th>Size</th><th>Frequency</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>10</td></tr><tr><td>4</td><td>12</td></tr><tr><td>5</td><td>11</td></tr></table></td><td>M1
A1</td><td>
2</td><td>Method for frequency distribution
Frequencies CAO</td></tr>
<tr><td>(b)</td><td><table><tr><th>Size</th><th>O</th><th>E</th></tr><tr><td>1</td><td>2</td><td>6</td></tr><tr><td>2</td><td>5</td><td>8</td></tr><tr><td>3</td><td>10</td><td>12</td></tr><tr><td>4</td><td>12</td><td>8</td></tr><tr><td>5</td><td>11</td><td>6</td></tr></table></td><td>B1</td><td></td><td>Correct values for E</td></tr>
<tr><td></td><td>H_0: Probability distribution is adequate model
H_1: Probability distribution is not adequate model</td><td>B1</td><td></td><td>Hypotheses - may be earned in conclusion</td></tr>
<tr><td></td><td>$\Sigma(O - E)^2/E$
$= 4^2/6 + 3^2/8 + 2^2/12 + 4^2/8 + 5^2/6$</td><td>M1</td><td></td><td>Attempt at $\Sigma(O - E)^2/E$ - their Es and Os</td></tr>
<tr><td></td><td>$= 10.3$</td><td>A1</td><td></td><td>10.25 ~ 10.35</td></tr>
<tr><td></td><td>c.v. χ_4^2 is 9.488</td><td>B1
B1√</td><td></td><td>4 df
9.488 - their df</td></tr>
<tr><td></td><td>Significant evidence that the probability distribution does not adequately model the distribution of required helmet sizes</td><td>A1√</td><td>7</td><td>Conclusion - needs correct method for Es and Os and comparison with upper tail of χ^2</td></tr>
<tr><td>(c)</td><td>Modify order - more large helmets, less small helmets than suggested by probability distribution</td><td>E1√
E1</td><td>
2</td><td>Modify order
More large, less small</td></tr>
<tr><td></td><td></td><td></td><td>11</td><td></td></tr>
</table>

Question 4

Sopphira spends the summer in a large European city. She frequently catches an underground train to and from the centre of the city. The trains run every six minutes and her arrivals at the station are independent of when a train is due. The length of time, in minutes, that she has to wait for a train may be modelled by a rectangular distribution on the interval (0, 6).

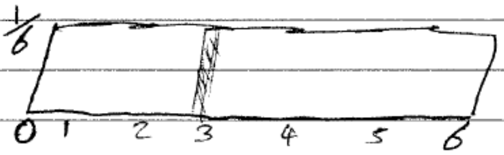
(a) Calculate the probability that when she goes to catch a train she has to wait for between 2.9 and 3.1 minutes. *(2 marks)*

(b) Find the mean and standard deviation of the length of time that she has to wait for a train. *(3 marks)*

(c) During her holiday, Sopphira catches the train on 46 occasions. Find, approximately, the probability that the mean length of time that she has to wait for a train is between 2.9 and 3.1 minutes. *(5 marks)*

Student Response

4. (a)



$P(2.9 < x < 3.1) = 0.2 \times \frac{1}{6}$

$= 0.0\dot{3}$

$= 0.0333$

2

(b) $\bar{x} = \frac{1}{2}(a+b)$

$= \frac{1}{2}(6) = 3.$

$\sigma = \sqrt{\text{var}\,x}$

$= \sqrt{\frac{1}{12}(6)^2}$

$= \sqrt{\frac{36}{12}} = \sqrt{3} = 1.7321$

$= 1.73$

3

Question number

4(c) $x \sim N(3, 1.73)$ ~~or~~ $\bar{x} \sim N(3, \frac{\sigma}{\sqrt{n}})$ not used

$P(2.9 < x < 3.1) =$ ~~0.0846~~ 0.31

Leave blank

5

Commentary

The candidate has correctly answered parts (a) and (b). In part (c) despite realising that a normal approximation with standard deviation $\sigma/\sqrt{n}$ is needed there is no progress towards using it. This part totally defeated many candidates.

Mark Scheme

Q	Solution	Marks	Total	Comments
4(a)	0.2/6 = 0.0333	M1 A1	 2	Method 0.0333 (0.033 ~ 0.034) or 1/30
(b)	mean 3	B1		CAO
	s.d. $6/\sqrt{12} = 1.73$	M1 A1	 3	Correct method 1.73 (1.73 ~ 1.735) or $\sqrt{3}$ **SC** allow B1 instead of M1A1 for variance = 3
(c)	$z_1 = (3.1-3)/(1.732/\sqrt{46}) = 0.392$ $z_2 = (2.9-3)/(1.732/\sqrt{46}) = -0.392$	M1		Use of their s.d./$\sqrt{46}$
		m1 m1		z-values - their mean and s.d. method for z-values - requires correct method for mean and s.d.
	Probability between 2.9 and 3.1 = 0.6525 – (1 – 0.6525)	m1		Method
	= 0.305	A1	5	0.3 ~ 0.31
			10	

Question 5

Fidel owns a shop selling fishing tackle. He obtains fishing line from Raoul, a manufacturer. Raoul states that his standard fishing line has a mean breaking strength of 15.3 kg with a standard deviation of 0.65 kg.

Fidel would also like to stock stronger fishing line which he could sell at a higher price. Raoul states that he is able to supply premium fishing line with a mean breaking strength at least 5 kg greater than the standard fishing line. However, the standard deviation would also be increased to 0.95 kg.

Fidel decides to measure the breaking strengths of samples of each type of fishing line with the following results, in kg.

Standard fishing line	15.9	16.4	14.8	15.2	14.3	14.9	15.0
Premium fishing line	18.8	20.4	22.1	19.1	19.3	18.7	

(a) By carrying out tests at the 5% significance level, verify that it is reasonable to assume that the standard deviation of the breaking strength of:

(i) the standard fishing line is 0.65 kg;

(ii) the premium fishing line is 0.95 kg.

Regard each sample as a random sample from a normal distribution. *(11 marks)*

(b) Assuming that the standard deviations of the breaking strengths of the two types of fishing line are as stated by Raoul, test whether the data are consistent with the mean breaking strength of the premium fishing line being at least 5 kg greater than the mean breaking strength of the standard fishing line. Use the 10% significance level. *(7 marks)*

(c) By using the data above and the 5% significance level, verify that the hypothesis that the standard deviations of the breaking strengths of the two types of fishing line are equal is accepted. *(6 marks)*

(d) Compare the results of part (a) with those of part (c) and comment. *(3 marks)*

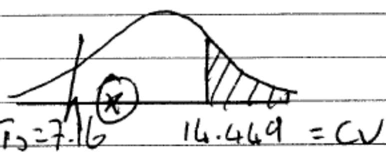
Student Response

5ai) $\sigma = 0.65$ $\sigma^2 = 0.4225$ $s = .71$ $s^2 = .504$

$H_0: \sigma^2_{standard} = 0.4225$ 5%
$H_1: \sigma^2_{standard} \neq 0.4225$ B1

$\frac{(n-1)s^2}{\sigma^2} = \frac{6 \times (0.71^2)}{0.4225} = 7.16$ 2

$Ts = 7.16$ $v = 7 - 1 = 6$ $cv = 14.449$ B1



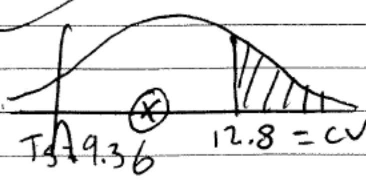
Accept Ho, there is significant evidence to suggest the standard deviation of breaking strength of the standard line is equal to 0.65. A1

5aii) $\sigma = .95$ $\sigma^2 = .903$ $s = 1.3$ $s^2 = 1.69$

$H_0: \sigma^2_{premium} = .903$ 5% $n = 6$
$H_1: \sigma^2_{premium} \neq .903$ B1

$\frac{(n-1)s^2}{\sigma^2} = \frac{5 \times 1.69}{.903} = 9.36$

$Ts = 9.36$ $v = 6 - 1 = 5$ $cv = 12.833$



Accept Ho, there is significant evidence to suggest the σ of breaking strength of the premium line is 0.95.

5b) ~~Ho: Ms = M~~ x = Standard y = Premium

$H_0: \mu x + 5 = \mu y$ 10% Sig level.
$H_1: \mu x + 5 < \mu y$

TS = −10.89 ✗

~~n = 13 − 2 = 11~~
CV = ± 1.2815

TS = −10.89 −1.2815 = CV

B1

Reject Ho, accept H₁, there is significant evidence to suggest the premium fishing line is atleast 5kg greater than the mean breaking strength

Cwestiwn yn wag

.504

5c) ~~θ^2~~ x = Standard y = Premium 1.69
$H_0: \theta^2 x = \theta^2 y$
$H_1: \theta^2 x \neq \theta y$ 5% level

$T_s = \frac{1.69}{.504} = 3.35$ CV = 5.988
$V_1 = n_y - 1$ $V_2 = n_x - 1$

$\frac{\theta^2 y}{\theta^2 x} = T_s$

Ts = 3.35 5.988 = CV

Accept Ho, there is significant evidence to suggest the variance of bothe lines are equal.

5

d) Part a states that the premium line has a θ of .95 and standard fishing line 0.65 as θ.
This conflicts with the answers in part c, as the hypothesis saids bothe lines are equal.
Part ~~is~~ C is probably more reliable as it compares results directly.

E1

(13)

Commentary

This candidate has attempted part (b) first but has incorrectly identified the alternative hypothesis as being that the mean breaking strength of premium fishing line is more than 5kg greater than that of standard fishing line. No working is shown for the test statistic which is incorrect - hence no marks. One mark has been picked up for a numerically correct critical value.

In part (a) accuracy marks have been lost due to premature approximation. Marks are also lost due to only considering the upper tail of a two-sided test.

A good answer to part (c) but accuracy marks have again been lost. In part (d) the apparent contradiction between the conclusions to parts (a) and (c) is identified but possible reasons are not identified.

Mark Scheme

Q	Solution	Marks	Total	Comments
5(a)(i)	H_0: $\sigma_s = 0.65$ H_1: $\sigma_s \neq 0.65$	B1		Both hypotheses
	$s_s = 0.710466 \quad (\bar{x}_s = 15.2143)$ $\sum(x-\bar{x})^2/\sigma^2 = 6\times0.710466^2/0.65^2$	M1		Method for test statistic - allow small slip, eg 7×0.710466^2
		m1		Correct method for test statistic
	$= 7.17$	A1		$7.165 \sim 7.175$
	c.v. χ_6^2 are 1.237 and 14.449	B1		6 df
		B1		1.237 and 14.449 - allow 1.24 and 14.4
	Accept H_0: $\sigma_s = 0.65$, ie accept standard deviation of breaking strength of standard line is 0.65kg	A1√		Conclusion - must be compared with least one χ^2 value
	Using F, compare 1.19 (1.19 ~ 1.2) with 2.408 (or reciprocals)			
(ii)	H_0: $\sigma_p = 0.95$ H_1: $\sigma_p \neq 0.95$	B1		Both hypotheses
	$s_p = 1.30945 \quad (\bar{x}_p = 19.7333)$ $\sum(x-\bar{x})^2/\sigma^2 = 5\times1.30945^2/0.95^2$			
	$= 9.50$	A1		$9.49 \sim 9.505$
	c.v. χ_5^2 are 0.831 and 12.833	B1		Allow 0.83 and 12.8
	Accept H_0: $\sigma_p = 0.95$, ie accept standard deviation of breaking strength of premium line is 0.95kg	A1	11	Conclusion in context. Needs both c.v. – must mention standard deviation / variance and fishing line or breaking strength
	Using F, compare 1.90 (1.895 ~ 1.905) with 6.02 (or reciprocals)			*Use mark scheme for (i) in (ii) and for (ii) in (i) if more favourable to candidate*

Q	Solution	Marks	Total	Comments
5(cont) **(b)**	H_0: $\mu_p = \mu_s + 5$ H_1: $\mu_p < \mu_s + 5$	B1		Hypotheses
	$z = \dfrac{19.7333 - 15.2143 - 5}{\sqrt{(0.95^2/6\ +\ 0.65^2/7)}}$	M1 M1		Method for variance Method for z - their variance
	$= -1.05$	A1		$-1.04 \sim -1.06$ - ignore sign
	c.v. -1.2816	B1		$-1.28 \sim -1.282$ - ignore sign
	Accept H_0 , ie accept mean breaking strength of premium line is at least 5kg greater than that of standard line	A1√ A1√	 7	Accept H_0 - must be compared with correct tail of z – needs both M marks Conclusion in context – needs previous A1√
	p-value 0.148 (0.146 ~ 0.149) compare with 0.1			*If t used, maximum B1M0M1A0B1(for 1.363)A0A0*
(c)	H_0: $\sigma_p = \sigma_s$ H_1: $\sigma_p \neq \sigma_s$	B1		Hypotheses
	$F = 1.30945^2 / 0.710466^2 = 3.40$	M1 A1		Method for F 3.40 (3.39 ~ 3.4)
	c.v. $F_{[5,6]}$ is 5.988 (or compare 0.294 with 0.167)	B1 B1		5,6 df (or 6,5 if $0.710^2/1.81^2$ calculated) 5.988 (5.98 ~ 6)
	Accept H_0: accept standard deviations of breaking strengths of two types of line are equal	A1√	6	Conclusion in context - must be compared with correct tail of F
(d)	Not all null hypotheses can be true. At least one Type II error (accepting a false null hypothesis must have been made). Accepting the null hypothesis only shows that any evidence against it is not significant, not that it is true. In this case the samples are small, so accepting the null hypothesis is quite a weak result.	E1 E1 E1	 3	Not all null hypotheses can be true Type II error must have been made Accepting null hypothesis does not prove it is true Allow any other sensible comment, eg small samples (max 3)
	Total		**27**	