Version 1.0: 0609

General Certificate of Education

Statistics 6380

SS04 Statistics 4

Mark Scheme

2009 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director Genera*

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS04

Q	Solution	Marks	Total	Comments
1	X = number of days out of 200 when delay occurs.			
(a)	$X \sim \text{B}(200, 0.005)$	B1	1	
(b)	$\text{B}(200,\ 0.005) \approx \text{Po}(1)$	B1		
	$\text{P}(X \geq 3) = 1 - \text{P}(X \leq 2)$	M1		$\text{P}(X \geq 3) = 1 - \text{P}(X \leq 2)$ using Poisson or binominal
	$= 1 - 0.9197 = 0.0803$	A1	3	AWRT
(c)	Poisson approximation to binomial with large n ...	E1		
	...and very small p.	E1	2	
	Total		**6**	
2(a)	$\bar{x} = 32.75 \quad s = 3.059$	B1		Allow 3.06 for sd
	$\text{H}_0 : \mu = 30$			
	$\text{H}_1 : \mu > 30$	B1		Both
	Test statistic = $\dfrac{32.75 - 30}{\frac{3.059}{\sqrt{8}}}$	M1		$(x - 30)$ / their sd
		m1		Correct sd.
	$= 2.543$	A1		2.54 to 2.55 (B3 by calculator)
	$\nu = 8 - 1 = 7$	B1		Allow B1 total for $t = 2.896$
	$t = 2.998$	B1		Using $\nu = 8$
	$2.543 < 2.998$ so cannot reject H_0. There is not enough evidence at the 1% significance level to say that the level of nitrate pollution is a cause for environmental concern.	A1√	8	ft on ts and critical value Must be 1-tailed test
(b)(i)	$\text{H}_0 : p = 0.6$			
	$\text{H}_1 : p < 0.6$	B1		Both
	Under H_0, $X \sim \text{B}(42, 0.6)$			
	$\approx \text{N}(25.2,\ 10.08)$	B2		B1 mean; B1 variance
				M1 if correct method with wrong n
	Test statistic = $\dfrac{16.5 - 25.2}{\sqrt{10.08}} = -2.74$	M1		
		A1		–2.75 to –2.73
	or $\dfrac{16 - 25.2}{\sqrt{10.08}} = -2.90$			–2.91 to –2.89
	Or using proportions,			
	$Y \sim \text{N}(0.6,\ 0.005714)$	(B2)		B1 if $\hat{p}$ used instead of p
	Test statistic = $\dfrac{\frac{16}{42} - 0.6}{\sqrt{0.005714}} = -2.90$	(M1)		
		(A1√)		ft on use of $\hat{p}$
	$z = -2.3263$	B1		Accept 2.33; ignore sign Allow t = 2.423
	ts < critical value so H_0 can be rejected. There is evidence at the 1% level that nitrate pollution is a cause for concern in less than 60% of the length of rivers in the region	A1√	7	ft on ts and z Exact binomial: $\text{P}(X \leq 16) = 0.00338 < 0.01$ full marks
(ii)	Assume that the sections tested are chosen at random or independent of each other	B1	1	
	Total		**16**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Y = Number of people using the machine in a 4-hour period.			
	$Y \sim \text{Po}(\lambda) \approx \text{N}(\lambda,\ \lambda)$			
	z = 1.6449	B1		Accept 1.645, 1.64
	standard error = $\sqrt{47}$			
	90% confidence limits for λ are:			
	$47 \pm 1.6449 \times \sqrt{47}$	M1m1		M1 for sample value + z × sd ml for correct sd Allow M1 if 11.75 used
	giving (35.7, 58.3)	A1	4	(35.7 to 35.8, 58.2 to 58.3)
(b)	Mean uses per 4 hours when outside = 54 or 90% CI for mean uses per hour when inside is $\left(\frac{35.7}{4},\ \frac{58.3}{4}\right)$			
	Mean when outside lies within CI for mean when inside.	B1		Comparison of CI with previous mean.
	Not enough evidence to say that fewer people use the machine in its new location while the store is open. There may still be a reduction in use because it is not available outside opening hours.			
	Conclusion from confidence interval may be suspect if model used is poor.	B2	3	Any two valid points. Clear explanation based on CI gets 2 Allow B1 if CI based on 54 (or 13.5) and 47 (or 11.75) compared with it
	Total		**7**	
4(a)(i)	$\hat{p} = \frac{188}{320} = 0.5875$	B1		Allow 0.587, 0.588
	z = 1.96	B1		Here or in (ii)
	95% confidence limits for p are:			
	$0.5875 \pm 1.96 \times \sqrt{\frac{0.5875 \times 0.4125}{320}}$	M1 M1		Form of CI Here or in (ii) Standard error
	giving (0.534, 0.641)	A1		(0.533 to 0.534, 0.64 to 0.642)
(ii)	$\hat{q} = \frac{117}{260} = 0.45$			
	95% confidence limits for q are:			
	$0.45 \pm 1.96 \times \sqrt{\frac{0.45 \times 0.55}{260}}$	M1		
	giving (0.390, 0.510)	A1	7	(0.389 to 0.39, 0.51 to 0.511)
(b)	The lower bound of the CI for p is greater than the upper bound of the CI for q.	E1		
	It seems likely that females were more likely than males to vote for Yvonne.	B1	2	B1 allowed for correct conclusion with some comparison of CIs

SS04 (cont)

Q	Solution	Marks	Total	Comments
4(c)	Two candidates so Yvonne needs > 50% of votes.	E1		May be implied
	Lower bound for $q > 0.5$ so it is likely that a majority of females voted for her, but	E1		Indication that more than half of females vote for Yvonne
	the proportion of males could be well under a half.	E1		Doubtful whether more than half of males will vote for Yvonne
	Can't be sure that Yvonne will win. (Result may depend on numbers of males and females who vote)	E1	4	Conclusion based on evidence of CIs
	Total		**13**	
5	$X \sim \mathrm{N}(110, 5^2)$			
	$Y \sim \mathrm{N}(370, 12^2)$			
(a)	$X+Y \sim \mathrm{N}(110+370, 5^2+12^2)$	M1		Means and variances added.
	$= \mathrm{N}(480, 169)$	A1		B1 if mean wrong but variance correct
	$\mathrm{P}(X+Y<500) = \Phi\left(\frac{500-480}{13}\right)$	M1		
	$= \Phi(1.538)$			1.538 to 1.54
	$= 0.938$	A1	4	AWRT 0.937 to 0.938
(b)	$3X \sim \mathrm{N}(3\times110,\ 9\times5^2)$			
	$Y-3X \sim \mathrm{N}(370-330, 12^2+225)$	B1		B1 for mean
		M2		M1 for $3^2 \times \mathrm{Var}(X)$
				M1 for adding variances
	$= \mathrm{N}(40, 369)$	A1		CAO
	$\mathrm{P}(Y>3X) = P(Y-3X>0)$	M1		
	$= 1-\Phi\left(\frac{0-40}{\sqrt{369}}\right)$	m1		or equivalent
	$= \Phi(2.082)$			
	$= 0.981$	A1	7	AWRT
	Total		**11**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$H_0 : p = 0.5$			
	$H_1 : p > 0.5$	B1		Both
	Under H_0, $X \sim B(14, 0.5)$	B1		
	$P(X \geq 11) = 1 - P(X \leq 10)$	M1		
	$= 1 - 0.9713$			
	$= 0.0287$	A1		Accept 0.029
	$0.0287 < 5\%$ so result is significant at the 5% level.	E1		Can be gained without previous M mark
	Evidence suggests that more than half of car park users think it will be too small.	A1√	6	ft on probability
(b)(i)	$\nu = 15$; $t = 2.131$	B1		
	95% confidence limits for μ are:			
	$59.9 \pm 2.131 \times \frac{7.83}{4}$	M1		
		m1		sd divided by 4
	giving (55.7, 64.1)	A1	4	(AWRT, 64 to 64.1)
(ii)	65 is above upper confidence limit.	E1		
	Seems likely that the claim is true.	B1	2	
(c)(i)	Parent population is (discrete) Poisson but with large mean so closely approximated by normal distribution.	E1		
	Reasonable to accept that CI is valid.	B1	2	
(ii)	$Y \sim \text{Po}(65) \approx \text{N}(65, 65)$	B1		
	$P(Y > 78) = 1 - \Phi\left(\frac{78.5 - 65}{\sqrt{65}}\right)$	M1		B1 for 0.0534 using calculator (no cc)
		m1		Continuity correction attempted.
	$= 1 - \Phi(1.674)$			
	$= 1 - 0.9529 = 0.0471$	A1	4	0.047 to 0.0475; CAO B1 for exact Poisson (0.0504)
(d)	$P(Y > 78)$ = probability that smaller car park is full.	E1		
	As μ was taken at the highest reasonable value, this is a small probability.	E1		Significance of μ used
		E1		Assessment of probability
	It seems likely that the car park will not be too small.	B1	4	Based on argument from $P(X > 78)$, not CI for mean $P(X > 78)$ small so car park will not be too small gets E1 B1
	Total		**22**	
	TOTAL		**75**	