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General Certificate of Education

Statistics 6380

SS04 Statistics 4

Mark Scheme

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.
Otherwise we require evidence of a correct method for any marks to be awarded.

SS04

Q	Solution	Marks	Total	Comments
1	$H_0 : \lambda = 11$ $H_1 : \lambda < 11$	B2		B1 for $\lambda = 11$, here or later B1 for hypotheses with 11 or 2.2
	$P(X \leq 7 / \lambda = 11)$	M1		Attempt to find $P(X \leq 7)$ using Poisson
	= 0.1432	A1		Accept 0.143
	0.1432 > 10% so accept H_0	E1		
	Not enough evidence to claim a reduction in the mean number of complaints per day.	A1	6	
	Total		**6**	
2(a)(i)	$\hat{p} = \frac{36}{55} (= 0.6545)$	B1		
	$z = 1.96$	B1		
	95% CLs for p are			
	$\frac{36}{55} \pm 1.96 \times \sqrt{\frac{\frac{36}{55} \times \frac{19}{55}}{55}}$	M1		$\hat{p} \pm z \times$ their s.e.
		M1		correct form of s.e.
	giving (0.529, 0.780)	A1		(0.528 to 0.530, 0.779 to 0.781)
(ii)	$\frac{32}{60} \pm 1.96 \times \sqrt{\frac{\frac{32}{60} \times \frac{28}{60}}{60}}$	m1		Correct form of expression
	giving (0.407, 0.660)	A1	7	(0.406 to 0.408, 0.658 to 0.661)
(b)(i)	Lower limit of CI in (a)(i) > 0.5.	E1		
	Supports claim that a majority of eight-year-old children think the joke is funny.	B1	2	
(ii)	Considerable overlap between confidence intervals.	E1		
	Not enough evidence to claim that proportion is higher for eight-year-olds than for eleven-year-olds.	B1	2	
	Total		**11**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$H_0 : \mu = 15$ $H_1 : \mu < 15$	B1		Both
	$\nu = 14 - 1 = 13$	B1		
	$t_{1\%} = -2.65$	B1		Ignore sign
	test statistic $= \dfrac{13.2 - 15}{\dfrac{2.4}{\sqrt{14}}}$	M1		13.2 – 15 divided by their s.d.
		m1		Correct method for s.d. M1m1 if they use $\mu = 17$
	$= -2.81$	A1		
	$-2.81 < -2.65$ so reject H_0. There is sufficient evidence at the 1% significance level to claim that the mean queuing time is less than 15 minutes.	A1F	7	ft on ts and CV
(ii)	10% below previous mean = 15.3 mins. (or % reduction = (2/17) × 100 = 11.8% > 10%)	E1		
	Seems likely that mean queuing time has been reduced by more than 10%	B1	2	
(b)	Less time queuing on approach, but slower speed through single file section.	E1		E2 if they refer to balance between queuing time and speed and say can't be sure of overall effect.
	Effect on total time depends on length of single file section.	E1	2	
	Total		**11**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$X \sim \text{Po}(27) \approx \text{N}(27\ ,\ 27)$	B1		
	$\text{P}(X \leq 20) \approx \Phi\left(\frac{20.5-27}{\sqrt{27}}\right)$	M1 m1 A1		Use of formula – ignore cc Attempt at cc Completely correct B1 if correct cc but over 27
	$= \Phi(-1.251)$			
	$= 1 - 0.8945$			
	$= 0.105$	A1	5	(0.104 to 0.106)
	B1 for exact Poisson (0.101 to 0.102)			Max: 3 with no cc; 4 using 19.5
(b)	$\text{P}(X > 30)$	M1		
	$\approx 1 - \Phi\left(\frac{30.5-27}{\sqrt{27}}\right)$	m1		Uses completely correct formula
	$= 1 - \Phi(0.674)$			
	=0.250	A1		CAO (0.248 to 0.252)
	or 0.245 using exact Poisson	(B2)	3	(0.244 to 0.245)
(c)	Upper limit of 30	E1		
	with large probability (>0.245)	E1	2	Indication that probabilities have not tailed off. E1 for sensible reason that would also affect model for number of scones requested.
	Total		**10**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\bar{x} = 74.26$	B1		74.2 to 74.3
	$s = 2.550$	B1		2.54 to 2.55
	$\nu = 10;\quad t_{1\%} = 2.764$	B1		Accept 2.76
	98% confidence limits for μ_L are			
	$74.26 \pm 2.764 \times \dfrac{2.550}{\sqrt{11}}$	M1		$\bar{x} \pm$ their t $\times$ their s.e.
		m1		$s/\sqrt{11}$
	giving (72.1, 76.4)	A1	6	(72.0 to 72.2, 76.3 to 76.5)
(b)	$S = 100 - L$			
	$\therefore \mu_S = 100 - \mu_L$	E1		
	98% confidence interval for μ_S is			Full marks if correctly argued from
	(23.6, 27.9)	B1	2	incorrect (a)
				M1A1 if complete recalculation
(c)	Prediction 1 is not true.	B1		
	Some sample values are more than 2.5 cm from 75 cm – eg first value is 3.2 cm away.	E1		Clear use of data
	CI contains values above and below 74.	E1		
	Prediction 2 may or may not be true.	B1	4	
	Total		**12**	
6(a)	$X + Y \sim \mathrm{N}(855,\ 6480)$	B2		B1 for mean, B1 for variance
	$\mathrm{P}(X + Y < 1000)$			
	$= \Phi\left(\dfrac{1000 - 855}{\sqrt{6480}}\right)$	M1		
	$= \Phi(1.801)$			
	= 0.964	A1	4	0.964 to 0.965
(b)(i)	$\mathrm{E}(X - 2Y) = 550 - (2 \times 305) = -60$	B1		
	$\mathrm{V}(X - 2Y) = \mathrm{V}(X) + 4\mathrm{V}(Y)$			
	$= 72^2 + (4 \times 36^2)$	M1		V(Y) multiplied by 4
	= 10368	M1	3	Addition of variances
(ii)	$\mathrm{P}(X > 2Y) = \mathrm{P}(X - 2Y > 0)$	M1		
	$= 1 - \Phi\left(\dfrac{0 - (-60)}{\sqrt{10368}}\right)$	M1		Correct form for $\Phi(\ )$.
		m1		Completely correct formula.
	$= 1 - \Phi(0.589)$			
	= 0.278	A1	4	CAO. Allow M1 if they state $\mathrm{P}(X < 2Y)$ and find 0.722
	Total		**11**	

SS04 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$H_0 : p = 0.5$			
	$H_1 : p > 0.5$	B1		Both
	Under $H_0, X \sim B(30, 0.5)$	B1		Use of B(30, 0.5)
	$P(X \geq 21) = 1 - P(X \leq 20)$	M1		
	$= 0.0214$	A1		CAO
	$0.0214 < 5\%$ so reject H_0.	E1F	5	ft on probability
	It is reasonable to proceed with the main trial.			
(b)	$H_0 : p = 0.6$			
	$H_1 : p > 0.6$	B1		Both. Accept $\mu = 192$; $\mu > 192$
	Under H_0, $Y \sim B(320, 0.6)$			
	$\approx N(192, 76.8)$	B2		B1 for mean; B1 for variance
	test statistic $= \dfrac{208 - 192}{\sqrt{76.8}}$	M1		
	$= 1.826$ (1.769 with cc)	A1		AWRT 1.83 (1.77)
	critical value = 1.6449	B1		Accept 1.65, 1.64.
	ts > cv so reject H_0. Evidence at the 5% level supports adoption of the new recipe.	A1		Using *p*-values: $p = 0.034$ (AWRT) < 5%
	Or, using proportions: $\hat{p} = \dfrac{208}{320} = 0.65$	(B1)		
	Under H_0, $\hat{P} \sim N\left(0.6, \dfrac{0.6 \times 0.4}{320}\right)$	(B1)		Correct formula for variance
	$= N(0.6, 0.00075)$			
	test statistic $= \dfrac{0.65 - 0.6}{\sqrt{0.00075}}$	(M1)		
	$= 1.826$	(A1)	7	
(c)	Main trial is costly and time-consuming. If there is no interest in the new recipe, this will be detected in the preliminary trial, saving time and money.	E1 E1	2	
	Total		**14**	
	TOTAL		**75**	