Teacher Support Materials 2008

Maths GCE

Paper Reference SS02

Question 1

1 [**Figure 1**, printed on the insert, is provided for use in this question.]

A family-owned firm makes ice cream. The following table shows the quantity, in tonnes, sold during each quarter from 2004 to 2006, together with values of a suitable moving average.

	2004				2005				2006			
Quarter	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Ice cream sales	7.2	11.3	14.2	8.9	9.1	13.6	16.8	9.4	11.5	15.7	17.5	12.2
Moving average		10.40	10.88	11.45	12.10	*m*	12.83	13.35	13.53	14.23		

(a) Calculate the value of m. *(2 marks)*

(b) (i) Plot the values of the moving average on **Figure 1**.

(ii) Draw a trend line. *(3 marks)*

(c) (i) Estimate the seasonal effects for each of quarters 1 and 4.

(ii) Predict the quantity of ice cream that the firm will sell during quarter 1 of 2007 and during quarter 4 of 2007 if current trends continue. *(6 marks)*

(d) The owners planned to hand over the running of the firm to one of their four children at the start of 2008. They decided to allow each of their children to run the firm for one quarter during 2007 and to choose the most successful to run the firm from 2008 onwards.

Harry ran the firm during quarter 1 and sold 13.5 tonnes of ice cream.
Charlie ran the firm during quarter 2 and sold 14.6 tonnes of ice cream.
Eddie ran the firm during quarter 3 and sold 13.9 tonnes of ice cream.
Annie ran the firm during quarter 4 and sold 15.5 tonnes of ice cream.

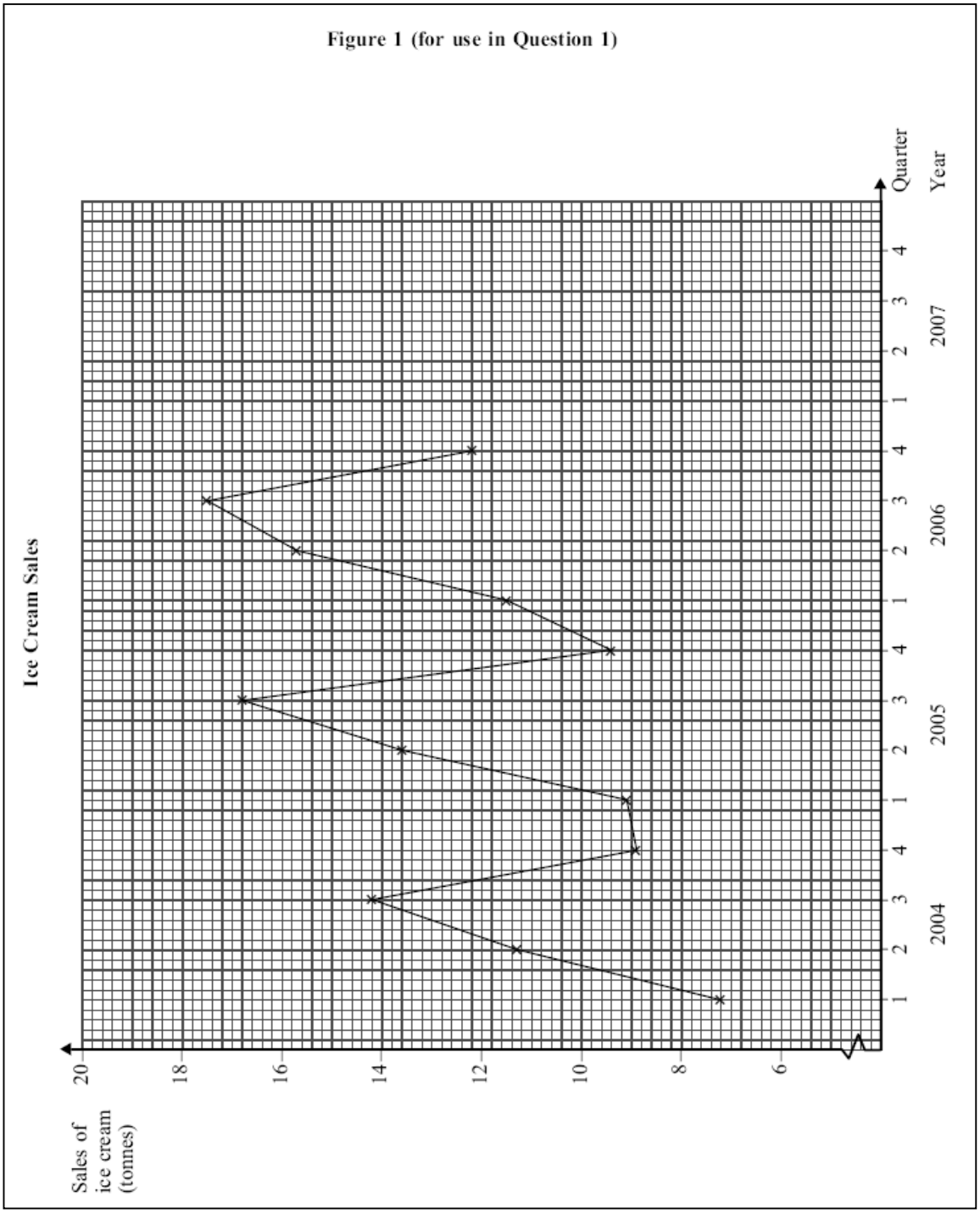
(i) Plot these values on your graph.

(ii) By examining your graph, but without carrying out further calculations, explain why the owners decided **not** to choose either Charlie or Eddie to run the firm.

(iii) Advise the owners, giving a reason, as to which of Harry or Annie should be chosen to run the firm. *(5 marks)*

Continued on next page

Figure 1 (for use in Question 1)



Student Response

1

(a) Value of $m = 12.225 \Rightarrow 12.23$ ✓ 2

(b)

(i) Plotted values of moving averages on Figure 1

(ii) Trend line drawn on Figure 1. 3

(c) blank

(i)

2004	2005	2006
	✓	✓
Q1 → 7.2 − 9.8 = −2.6	9.1 − 11.6 = −2.5	11.5 − 13.6 = −2.1

Average for Quarter 1 → $(-2.6)+(-2.5)+(-2.1)/3 = -2.4$ ✓

so 2.4 below the trend line.

	✓	✓
Q4 → 8.9 − 11.2 = −2.3	9.4 − 13.1 = −3.7	12.2 − 15.0 = −2.8.

Average for Quarter 4 → $(-2.3)+(-3.7)+(-2.8)/3 = -2.93$ ✓

so 2.93 below the trend line. 6

(ii) Quarter 1 of 2007 → 15.4 − 2.4 = 13 [tonnes of] icecreams. ✓

Quarter 4 of 2007 → 16.8 − 2.93 = 13.87 so 14 [tonnes of] icecreams.

(d)

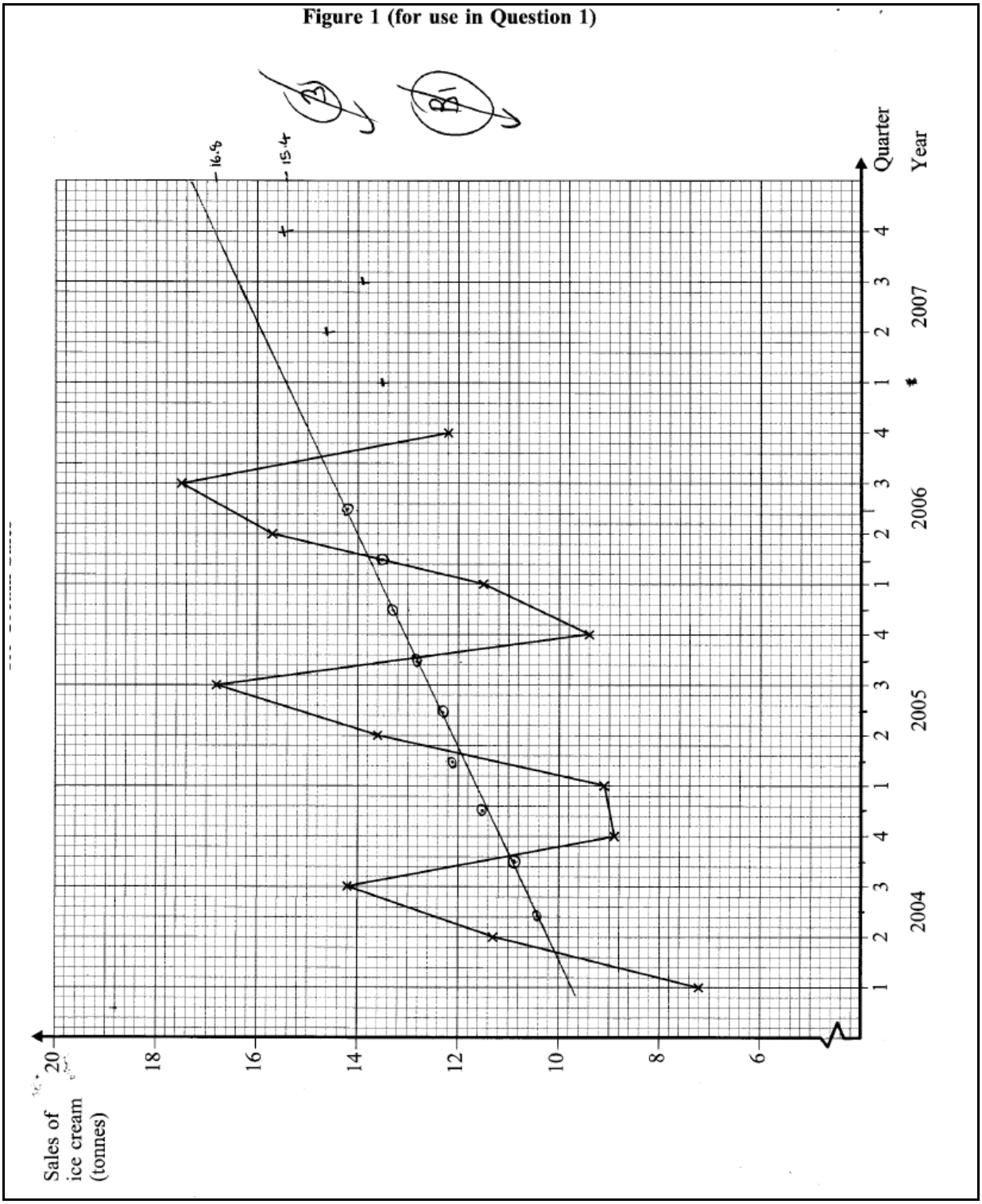
(i) Values plotted on graph. B1

(ii) Owners decided not to choose Eddie and Charlie to run the firm, this was because the trend line shows that during Quarters 2 and 3 the sales of tonnes of icecream should be above the trend line but they haven't sold as much as should be according to trend line. E2

(iii) Harry or Annie should be chosen to run the firm because even though according to the trend line not much icecream is sold in Quarters 1 and 4, they have sold a bit more than predicted. E1 B0

(x5)

Figure 1 (for use in Question 1)



Commentary

This is an excellent answer. As the question requires, the seasonal effects (including the negative sign) are clearly calculated in part (c)(i) and applied in part (c)(ii). Many candidates jumbled these two parts together. The forecasts were then rounded to a sensible number of significant figures (2 or 3 sf were acceptable but more than 3 is excessive)

In part (d)(ii) the answer took account of both the seasonal effect and the trend - many candidates only considered the seasonal effect. A mark was lost in part (d)(iii) as no choice was made between Harry and Annie.

Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	$\frac{9.1 + 13.6 + 16.8 + 9.4}{4}$	M1		
	= 12.225	A1	2	12.23 (12.22 ~ 12.23); allow 12.2
(b)(i)	Moving averages plotted in correct position – at least 3	M1		
	Accurate plot – by eye	A1		
(ii)	Trend line	B1	3	Allow reasonable line even if moving averages incorrect
(c)(i)	Q1 effect: $\frac{(9.1-11.6)+(11.5-13.5)}{2}$	M1		Method for seasonal effect – either – ignore sign, allow use of 3 Qs
	= –2.25			
	Q4 effect: $\frac{(8.9-11.2)+(9.4-13.0)}{2}$	m1		Method for both – ignore sign
	= –2.95	A1		(–2.2 ~ –2.5) and (–2.6 ~ –3.1)
(ii)	Prediction for Q1, 2007: 15.4 – 2.25 = 13 Q4, 2007: 16.8 – 2.95 = 14	M1 m1 A1	6	Prediction of moving average from their (reasonable) trend line 13 (12.9 ~ 13.3) and 14 (13.5 ~ 14.1) disallow if more than 3sf given NMS: one answer in range B1 both answers in range B3
(d)(i)	Accurate plot – by eye	B1		
(ii)	Q2 (Charlie) and Q3 (Eddie) should be well above trend line, but both are below trend line. (Harry and Annie are below trend line as expected.)	E1 E1		Comment based on seasonal variation Correct explanation
(iii)	Harry slightly (0.5 tonnes) above prediction, Annie above (1.5 tonnes) prediction. Choose Annie.	E1 B1	5	Explanation Choose Annie
	Total		**16**	

Question 2

2 A county cricket club has different categories of membership. The following table shows the categories of membership, the annual subscription for each category of membership and the probability that a new member will join that category.

Category of membership	Annual subscription, £	Probability
Full	120	0.22
Senior	80	0.28
Country	75	0.12
Junior	30	0.38

(a) (i) Show that the mean subscription paid by new members is £69.20.

(ii) Find the standard deviation of the subscription paid by new members. *(5 marks)*

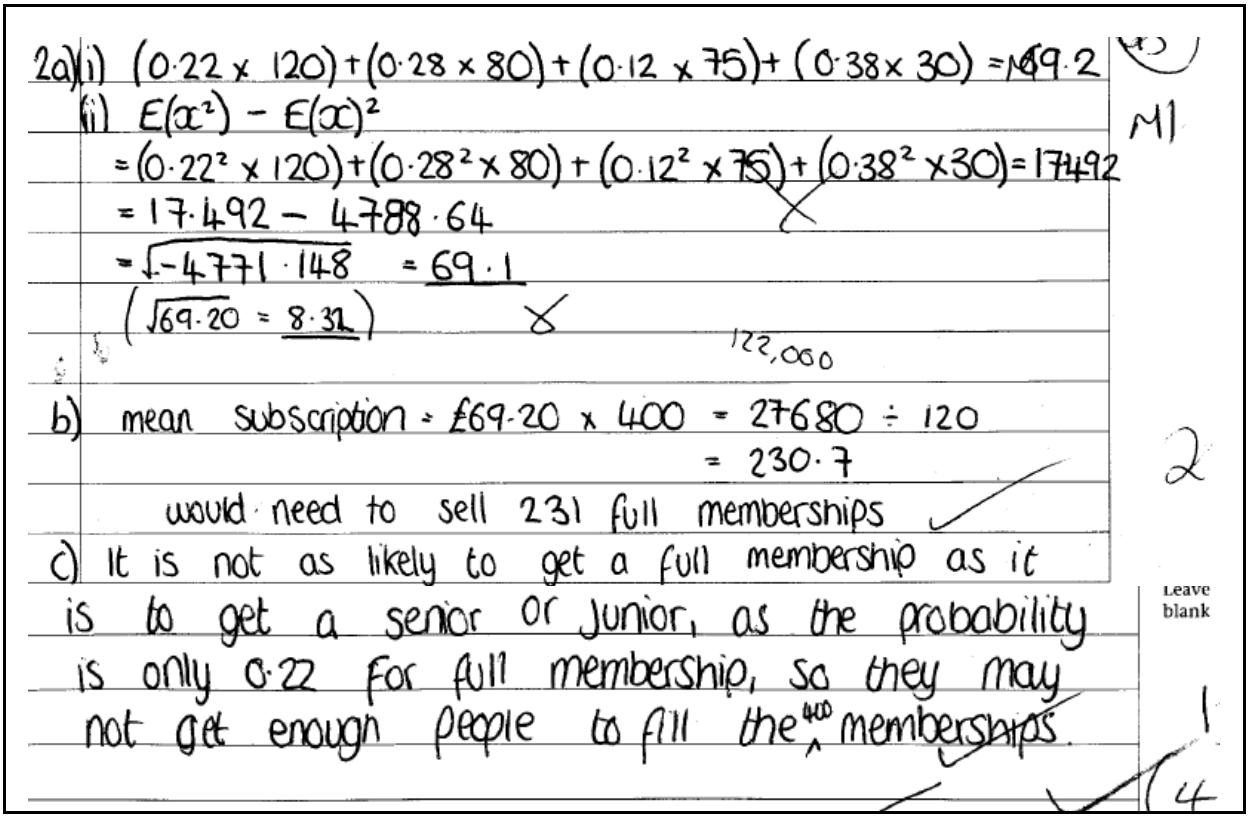
(b) As the ground capacity is limited, only 400 new members can be accepted during the coming year. The club is to stage an international match and so expects more than 400 applications for membership.

It is decided that only applications for full membership will be accepted.

How many full members would the club need to accept in order to receive more money in subscriptions than would have been provided by 400 members distributed as in the table above? *(2 marks)*

(c) Give one disadvantage of the decision in part (b). *(1 mark)*

Student response



Commentary

In part (a) the candidate has successfully demonstrated that the mean is 69.2 but in attempting to calculate $E(X^2)$ has squared the probability instead of X. The negative variance might have been a signal that something was wrong. However she has recovered well and given good answers to parts (b) and (c).

Mark Scheme

Q	Solution	Marks	Total	Comments
2(a)(i)	$E(X) = 120 \times 0.22 + 80 \times 0.28 + 75 \times 0.12 + 30 \times 0.38 = 69.2$	M1		Method for $E(X)$; AG
(ii)	$E(X^2) = 120^2 \times 0.22 + 80^2 \times 0.28 + 75^2 \times 0.12 + 30^2 \times 0.38 = 5977$	M1		Method for $E(X^2)$ – may be implied
	$V(X) = 5977 - 69.2^2 = 1188.36$	m1		Method for variance
		m1		Method for s.d. – dependent on previous 3 marks
	s.d. = £34.50	A1	5	34.50 (34.45 ~ 35.5) – ignore units
(b)	$\frac{69.2 \times 400}{120} = 230.7$	M1		
	231 full members needed	A1	2	CAO
(c)	No junior members bad for future of club. May be less than 231 applications for full membership.	E1	1	Any sensible reason
	Total		**8**	

Question 3

3 The following tables give details of the elections for the Welsh Assembly in 1999 and 2003, for the Scottish Parliament in 1999 and 2003 and for the Northern Ireland Assembly in 1998 and 2003.

Devolved assembly elections

	6 May 1999	1 May 2003
Welsh Assembly		
Electorate (thousands)	2205	2230
Valid votes counted (thousands)	1023	850
As percentage of electorate	46.4	38.1
Number of members elected (by party)		
Conservative	9	11
Labour	28	30
Liberal Democrat	6	6
Plaid Cymru	17	12
Other	0	1
Scottish Parliament		
Electorate (thousands)	4024	3879
Valid votes counted (thousands)	2342	1916
As percentage of electorate	58.2	49.4
Number of members elected (by party)		
Conservative	18	18
Labour	56	50
Liberal Democrat	17	17
Scottish National Party	35	27
Other	3	17

	25 June 1998	26 Nov 2003
Northern Ireland Assembly		
Electorate (thousands)	1179	1098
Valid votes counted (thousands)	810	702
As percentage of electorate	68.7	64.0
Number of members elected (by party)		
Alliance Party	6	6
SDLP	24	18
Sinn Fein	18	24
Democratic Unionist Party	20	30
UK Unionist Party	5	1
Ulster Unionist Party	28	27
Other	7	2

Source: University of Plymouth for the Electoral Commission and *Annual Abstract of Statistics*, Office for National Statistics, 2006

(a) How many valid votes were counted in the election for the Welsh Assembly in 1999?
(2 marks)

(b) Compare the percentages of the electorate who cast valid votes in the six elections.
(3 marks)

(c) In one of the six elections, half of the members elected came from the same party.

Identify:

(i) the election;

(ii) the party. *(3 marks)*

(d) For the 2003 elections, calculate the average electorate per member elected for each of the three countries. Comment briefly on your results. *(3 marks)*

Student Response

Leave blank

3 (a) Number of valid votes = 1023 ~~thousands~~ (ans) ✓2

(b). We observe that for the a categories of elections the percentage of ~~electorate~~ in May 1999 was greater than that of May 2003. The same as above is for the 3rd category between Jun-Nov. The percentage of electorates in both May 1999 and May 208 was much greater for Northern Ireland Assembly ~~then for~~ less for Scottish Parliament and much ~~smaller~~ less for Welsh assembly. Finally the difference between the percentage of electorate between June 1998 and November 2003 was small and about the half than the difference between the percentages of Welsh Assembly between May 1999 and May 2003 and ~~for~~ of Scottish Parliament between May 1999 and May 2003. E1 E1 E1

(c) i The election was in 1st May 2003 ✓

ii The party was of Labour's from Welsh Assembly ✓ 3

(d) ~~Average~~ Electorate / Number of members = Average

W. A: Average = 2230/60 = 37.167 = 37.2 (3sf)

S. P: Average = 3879/129 = 30.07 = 30.1 (3sf) ✓

A0 M1

N. I: Average = 1098/108 = 10.167 = 10.2 (3sf)

As it is observed the o welsh assembly has the greatest average of electorates per member contrary to the scottish parliament and northern Ireland which has the smallest average E1

(10)

Commentary

The candidate has covered all the relevant points, but although she correctly included 'thousands' in part (a) she forgot to include 'thousands' in part (d).

Mark Scheme

3(a)	1023000	B2	2	B1 for 1023
(b)	N.Ireland > Scotland > Wales at each election	E1		Any valid comparison of % in different countries
	2003 election less than 1998/9 (about 5% less in N.I., 8 or 9% less in Wales and Scotland)	E1		Any valid comparison of % in different years
	All less than 70%	E1	3	Complete answer
(c)(i)	Welsh assembly 2003	M1 B1		Any valid calculation – may be implied Welsh assembly 2003
(ii)	Labour	B1	3	
(d)	Welsh assembly $\frac{2230000}{60}=37200$ Scottish parliament $\frac{3879000}{129}=30100$	M1		Method of calculation
	N.I. assembly $\frac{1098000}{108}=10200$	A1		All correct 3sf
	N.Ireland has many less electors per member than Wales or Scotland.	E1	3	Any sensible comment – method mark not essential
	Total		**11**	

Question 4

4 On a map, the symbol 🅿 indicates a car park. A geography student divided a map into 66 squares, each representing an area of $9\,km^2$.

(a) If the number of 🅿 symbols in a square could be modelled by a Poisson distribution with mean 0.6, find the probability of a square containing:

(i) no 🅿 symbols;

(ii) 3 or more 🅿 symbols. *(3 marks)*

(b) The student counted the number of 🅿 symbols in each of the 66 squares. The results are shown in the table.

Number of 🅿 symbols	Number of squares
0	46
1	8
2	4
3	8

Calculate the mean and variance of the number of 🅿 symbols in a square. *(2 marks)*

(c) Give a reason why the Poisson distribution does **not** provide a good model for the number of 🅿 symbols in a square based on:

(i) your calculations in part (a);

(ii) your calculations in part (b);

(iii) the likely distribution of car parks. *(4 marks)*

Student Response

4a) $X \sim Po(0.6)$.

i) $P(X=0)$ ✓
$= 0.5488$.

ii) $P(X \geq 3)$.
$1 - P(X \leq 2)$.
$1 - 0.9769$.
$= 0.0231$ ✓

3

b) mean: 1.5
variance: 1.25. ✗

B0
B0

c) i) ~~My calc~~ 0.5488 shows that just over half of data would be equal to 0 when in fact nearly 70% does.

E1

ii) The mean of answer in (b) does not match the mean in part (a).

E0

iii) The distribution is likely to be a 'P' sign is put in a square away from another 'P' sign therefore ~~it~~ it is not independent from square to square.

E1
E0

(5)

Commentary

The candidate has answered part (a) correctly but has offered incorrect answers with no supporting working in part (b). Incorrect answers to part (b) were common possibly because candidates did not expect to be asked to calculate the mean and variance of grouped data in this paper.

In part (c) this candidate has attempted to match her answers to the questions. Many candidates failed to do this. A good, relevant answer to part (c)(i) but not to part (c)(ii). A little more context - e.g. a car park is unlikely to be sighted immediately next door to an existing car park - would have gained a further mark in part (c)(iii), existing car park would have gained an additional mark in part (c)(iii)

Mark Scheme

4(a)(i)	0.5488	B1		0.5488 (0.5485 ~ 0.5495)
(ii)	$P(\geq 3) = 1 - P(\leq 2)$ $= 1 - 0.9769$ $= 0.0231$	M1 A1	 3	 0.0231 (0.023 ~ 0.0232)
(b)	$\bar{x} = 0.606$ $s^2 = 1.104$	B1 B1	 2	0.606 (0.606 ~ 0.6061) 1.104 (1.08 ~ 1.11)
(c)(i)	Observed 3 or more = 8/66 = 0.12 Predicted by Po(0.6) is 0.023 – not similar (or observed zero 0.7, predicted 0.55)	 E1		 Observed probabilities not similar to those expected from Poisson
(ii)	Mean (0.606) not similar to variance (1.10)	E1		
(iii)	Car parks not likely to be distributed at random. Likely to be near shopping centres, country parks etc, not in housing estates.	E1 E1	 4	Allow not constant average rate, not independent Award for explanation in (iii) or for accurate numerical illustration in (i)
	Total		**9**	

Question 5

5 A train travelling between two major cities has ten carriages. Four of the carriages are for first-class passengers only and each contains 48 seats, numbered from 1 to 48. Six of the carriages are for standard-class passengers only and each contains 72 seats, numbered from 1 to 72.

The railway company wishes to survey passengers with a view to making the catering facilities more profitable. Interviews are to be carried out during the middle section of the journey when it can be assumed that **all seats will be occupied**. For practical reasons, only seated passengers will be interviewed.

It is planned that between 25 and 30 interviews in total will be carried out.

Owen suggests that 4 carriages should be selected at random and that 7 passengers selected at random from those seated in each of these carriages should be interviewed.

Xavier suggests that 18 passengers selected at random from the seated standard-class passengers and 8 passengers selected at random from the seated first-class passengers should be interviewed.

Jada suggests that three numbers between 1 and 48 should be selected at random and that the passengers in these seats in each carriage should be interviewed.

(a) (i) Name the type of sampling suggested by **Owen**.

(ii) Describe how random numbers could be used to select 7 passengers at random from the 72 passengers seated in a standard-class carriage. *(5 marks)*

(b) (i) Name the type of sampling suggested by **Xavier**.

(ii) Suggest reasons why **Xavier** chose the numbers 18 and 8 for his sample sizes. *(4 marks)*

(c) For **Jada's** method of sampling, state, giving a reason, whether or not:

(i) all seated passengers have an equal chance of being included in the sample;

(ii) all seated first-class passengers have an equal chance of being included in the sample. *(3 marks)*

(d) The railway company requires an estimate of the mean income of all passengers on the train.

State, giving a reason, whether the mean income of a random sample of size 26 from all seated passengers or the mean income of the sample suggested by **Xavier** would be a preferable estimate. *(2 marks)*

Student Response

5a) i) cluster

ii) Give every seat a number between 01 - 72 (~~like~~ already done). Use the random button on the calculator/~~or~~ computer or random number tables to generate 2 digit random numbers. Select ~~the first~~ 7 seats ~~which~~ with the first 7 numbers selected, ignoring 00, repeations and any numbers that ~~exte~~ are over 72. If the passenger isn't in there seat and there number was selected, select a ~~new~~ new random number.

blank B1 E4

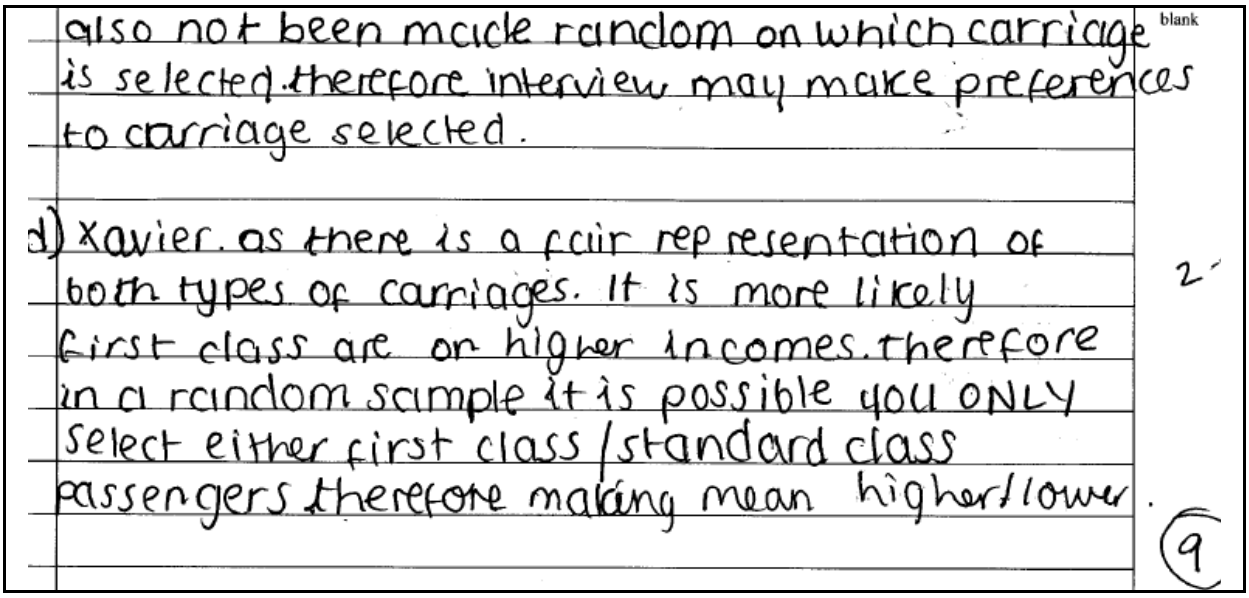
b) i) Stratified. B1

ii) There is a total of 120 passengers within the two different carriages. To get 8 she did (~~7~~ 48 ÷ 120) x 20 (sample size wanted) to obtain 8.

For the seat she did 0.

c) i) NO, because she is only selecting numbers between 1-48. This implies she is doing her invewer in the first class passenger seats therefore standard class don't get a chance. ~~Also there 3 carriages of secret participants therefore only 1 carriage will be selected.~~ E1 E0

ii) ~~Yes. As each of the 4 carrieges~~ we were No because only 1 carriage is interview. It was B0



Commentary

A good answer to parts (a) and (d). This candidate spotted that the seats were already numbered and (unlike some other candidates did not attempt to renumber them - a pointless and impractical task on a crowded train)

In part (b)(ii) and (c) she appears to be answering on the mistaken basis that there is only one first-class and one standard class coach.

Mark Scheme

5(a)(i)	Cluster	B1		
(ii)	Select 2-digit random numbers Ignore 00 and > 72 Ignore repeats Continue until 7 numbers obtained and choose passengers sitting in corresponding seats	E1 E1 E1 E1	5	If renumbered 00 to 71, max E1 E0 E1 E1
(b)(i)	Stratified	B1		Stratified / stratified random
(ii)	18:8 is ratio of number of seated standard class passengers (432) to seated first-class passengers (192) 18 + 8 = 26 in range of likely number of interviews	E1 E1 E1	4	Ratio of standard to first-class Total in right range Numerical support for ratio or demonstration that 18 and 8 is only possibility giving total in desired range
(c)(i)	No Passengers in seats numbered 49–72 have no chance of being selected	B1 E1		
(ii)	Yes, all have a chance of 3/48 of being selected	B1	3	
(d)	Xavier's sample preferred First-class and standard-class passengers fairly represented in sample	B1 E1	2	
	Total		**14**	

Question 6

6 A large chain of pharmacies introduces a training programme for counter staff to enable them to deal more quickly and effectively with queries from customers.

The managing director believes that the mean time for dealing with queries should be 40 seconds. A larger mean suggests an inefficient use of time while a smaller mean suggests that queries may not have been dealt with adequately.

Before the training programme, the times, in seconds, to deal with 9 queries were recorded as follows:

67 43 19 34 45 62 48 51 59

(a) Test the hypothesis that the mean time taken to deal with a query was 40 seconds. Use the 5% significance level and assume that the data may be regarded as a random sample from a normal distribution with standard deviation 17 seconds. *(8 marks)*

(b) (i) After the training programme, the times to deal with 120 queries were recorded and found to have a mean of 35 seconds and a standard deviation of 12 seconds.

Examine, using the 5% significance level, whether, after the training programme, the mean time to deal with a query was less than 40 seconds. *(5 marks)*

(ii) State one assumption that it is necessary to make in order for the test in part (b)(i) to be valid. *(1 mark)*

(c) Summarise the effects of the training programme. *(3 marks)*

Student Response

6. a) $H_0 : \mu = 40$ ✓

$H_1 : \mu \neq 40$ ✓

$\mu = 40 \quad \bar{x} = 47.6 \quad \sigma = 17 \quad n = 9$

$z = \frac{47.6 - 40}{17/\sqrt{9}}$ ✓

$= 1.34$ ✓

The CV is 1.9600. ✓

1.34 is not in the critical region. ✓

Accept H_0. ✓

No evidence that the mean time to deal with a query isn't 40 seconds. ✓

8

b) (i) $H_0 : \mu = 40$ ✓

$H_1 : \mu < 40$ ✓

$\mu = 40 \quad \bar{x} = 35 \quad \sigma = 12 \quad n = 120$

$z = \frac{35 - 40}{12/\sqrt{120}}$ ✓

$= -4.56$ ✓

The CV is -1.6449. ✓

-4.56 is in the critical region.

Reject H_0.

Evidence that the mean time is less than 40 seconds.

5

(ii) That the 120 queries recorded were from a random sample.

Leave blank

B1

c) The mean time to deal with a query has gone from being 40 seconds (on average) to being less than 40 seconds (on average). The staff have become quicker and more effective at dealing with queries.

E1

15

Commentary

A good, concise answer to part (a). Many candidates find it helpful to draw a diagram. This also helps examiners to understand what the candidate is doing. However, this answer is sufficient. To have gained full marks in part (c) the candidate needed to refer to the reduction in variability and the managing director's belief that a mean time of less than 40 seconds may indicate that queries have not been dealt with adequately.

Mark Scheme

6(a)	H_0: $\mu = 40$ H_1: $\mu \neq 40$	B1 B1		One hypothesis correct Both hypotheses correct – must use μ or state 'population'
	$\overline{x} = 47.56$			
	$z = \frac{47.56 - 40}{17/\sqrt{9}} = 1.33$	M1 m1 A1		Use of (their s.d.)/$\sqrt{9}$ Correct method for z – ignore sign 1.33 (1.33 ~ 1.34)
	Critical values ±1.96 Accept H_0 – no significant evidence that mean time to deal with queries differs from 40 seconds	B1 A1√ A1√	8	Ignore sign ft conclusion – must be compared with upper tail of z ft conclusion in context – requires M1m1A1√
(b)(i)	H_0: $\mu = 40$ H_1: $\mu < 40$	B1		Both – don't penalise same mistake twice
	$z = \frac{35 - 40}{12/\sqrt{120}} = -4.56$	M1 A1		Method for z – ignore sign −4.56 (−4.54 ~ −4.57)
	c.v. −1.6449 Reject H_0 – significant evidence that mean time to deal with queries is less than 40 seconds	B1 A1√	5	Ignore sign; −1.64 ~ −1.65 Conclusion in context – must compare lower tail of z
(b)(ii)	Queries were a random sample	B1	1	Random
(c)	Training appears to have reduced mean time to deal with queries and also to have reduced variability. Mean may now be too small to deal with queries adequately.	E1 E1 E1	3	Mean reduced Variability reduced Mean may now be too small – context required for full marks
	Total		**17**	
	TOTAL		**75**	