Teacher Support Materials

Maths GCE

Paper Reference SS06

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Question 1

1 A pharmaceutical company manufactures tablets with a nominal potency of 6.0. The units are $mg\,cm^{-3}$. The process is to be controlled by taking samples of size 5 at regular intervals and measuring the potency of the tablets. The potency may be assumed to be normally distributed.

The means and ranges of the last eight samples are given in the table.

Sample	1	2	3	4	5	6	7	8
Mean	5.95	5.99	5.96	6.03	5.99	6.05	6.01	6.03
Range	0.12	0.04	0.17	0.09	0.11	0.05	0.14	0.12

(a) Use the ranges to show that an estimate of the current standard deviation of the process is 0.045, correct to three decimal places. *(3 marks)*

(b) Using 0.045 as the standard deviation, calculate upper and lower warning (95%) and action (99.8%) control limits for charts for:

(i) means;

(ii) ranges.

You are not required to draw the charts. *(6 marks)*

(c) The potencies of tablets in the next sample are:

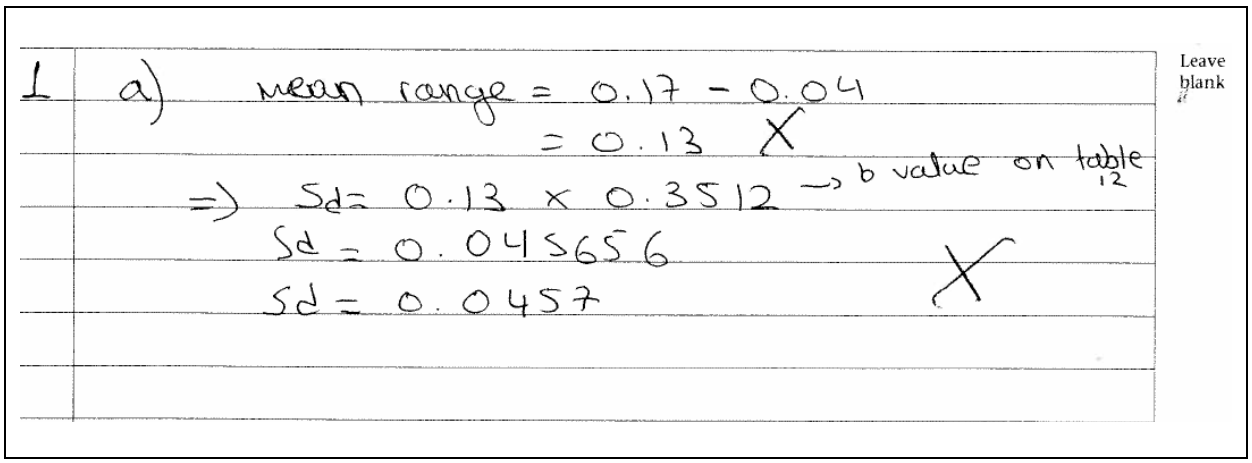
5.96 6.09 5.99 6.16 6.08

State what action, if any, you would advise as a result of this sample. *(3 marks)*

(d) A customer specifies tolerances of 6.00 ± 0.15.

Find the proportion of tablets outside these tolerances if the current mean is 6.06 and the current standard deviation is 0.045. *(2 marks)*

Student Response



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b) i) $\mu = \frac{5.95 + 5.99 + \ldots + 6.03}{8}$

$\mu = 6.001$

$\Rightarrow \mu \pm 1.96 \frac{s}{\sqrt{n}}$

$6.001 \pm 1.96 \times \frac{0.045}{\sqrt{8}}$

(5.967, 6.03) Lower and upper warnings

$\Rightarrow \mu \pm 3.09 \frac{s}{\sqrt{n}}$

$6.001 \pm 3.09 \times \frac{0.045}{\sqrt{8}}$ B1

(5.95, 6.05) Lower and upper action.

ii) Lower action = $0.045 \times 0.835 = 0.0376$
Lower warning = $0.045 \times 1.410 = 0.0635$ M1
Upper warning = $0.045 \times 4.605 = 0.207$
Upper action = $0.045 \times 5.823 = 0.262$

c) $x = 5.96$ between Lower action and warning limit so take another sample
$x = 6.09$ greater than action limit take action
$x = 5.99$ no action inside warning
$x = 6.16$ take action
$x = 6.03$ take action

number

Leave blank

d) $\frac{1}{5}$ outside tolerance

$\frac{1}{5} \times 100 = 20\%$

$p = 0.2$ (2)

Commentary

This candidate has made several common errors -
(i) Using the mean of recent samples instead of the target value as the centre line in the chart for means.
(ii) Using n=8 instead of n=5 in both the chart for means and the factors for the chart for ranges.
(iii) Basing actions on the potency of single tablets instead of on the sample mean and range.

Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	mean range $=\frac{0.84}{8}=0.105$	M1		attempt to find mean range
	estimated s.d. $=0.4299\times0.105$	B1		0.4299
	$=0.045$	A1	3	0.045 ag by any correct method $(0.045\sim0.05)$
(b)(i)	chart for means warning limits			
	$6.00\pm1.96\times\frac{0.045}{\sqrt{5}}$	B1 M1		1.96 and 3.09 – allow 2 and 3 use of $\frac{0.045}{\sqrt{5}}$
	$5.961\sim6.039$	M1		method – both limits, allow incorrect z-value, use of $\sqrt{8}$, disallow if not centred on 6.00
	action limits $6.00\pm3.09\times\frac{0.045}{\sqrt{5}}$			
	$5.938\sim6.062$	A1	4	$5.96(5.959\sim5.961)$ $6.04(6.039\sim6.041)$ $5.94(5.937\sim5.94)$ $6.06(6.06\sim6.063)$
(ii)	chart for ranges LA $0.367\times0.045=0.017$ LW $0.850\times0.045=0.038$ UW $4.197\times0.045=0.189$	M1		$D\times0.045$ allow upper limits only allow any D
	UA $5.484\times0.045=0.247$	A1	2	$0.017(0.016\sim0.017)$ $0.038(0.038\sim0.039)$ $0.189(0.1885\sim0.1895)$ $0.247(0.246\sim0.247)$ } allow one small slip
(c)	mean 6.056 range 0.20 both between warning and action limits take another sample immediately – if mean or range on new sample outside warning limits take action	B1 E1√ E1	3	$6.056(6.05\sim6.06)$ and 0.2 CAO correct conclusion – their figures take another sample immediately - based on all correct working
(d)	$z_1=\frac{(6.15-6.06)}{0.045}=2$ $z_2=\frac{(5.85-6.06)}{0.045}=-4.67$ proportion outside tolerances $=1-0.97725$	M1		method – allow z_1 only, allow proportion inside tolerances
	$=0.02275$	A1	2	$0.02275(0.022\sim0.023)$
	Total		**14**	

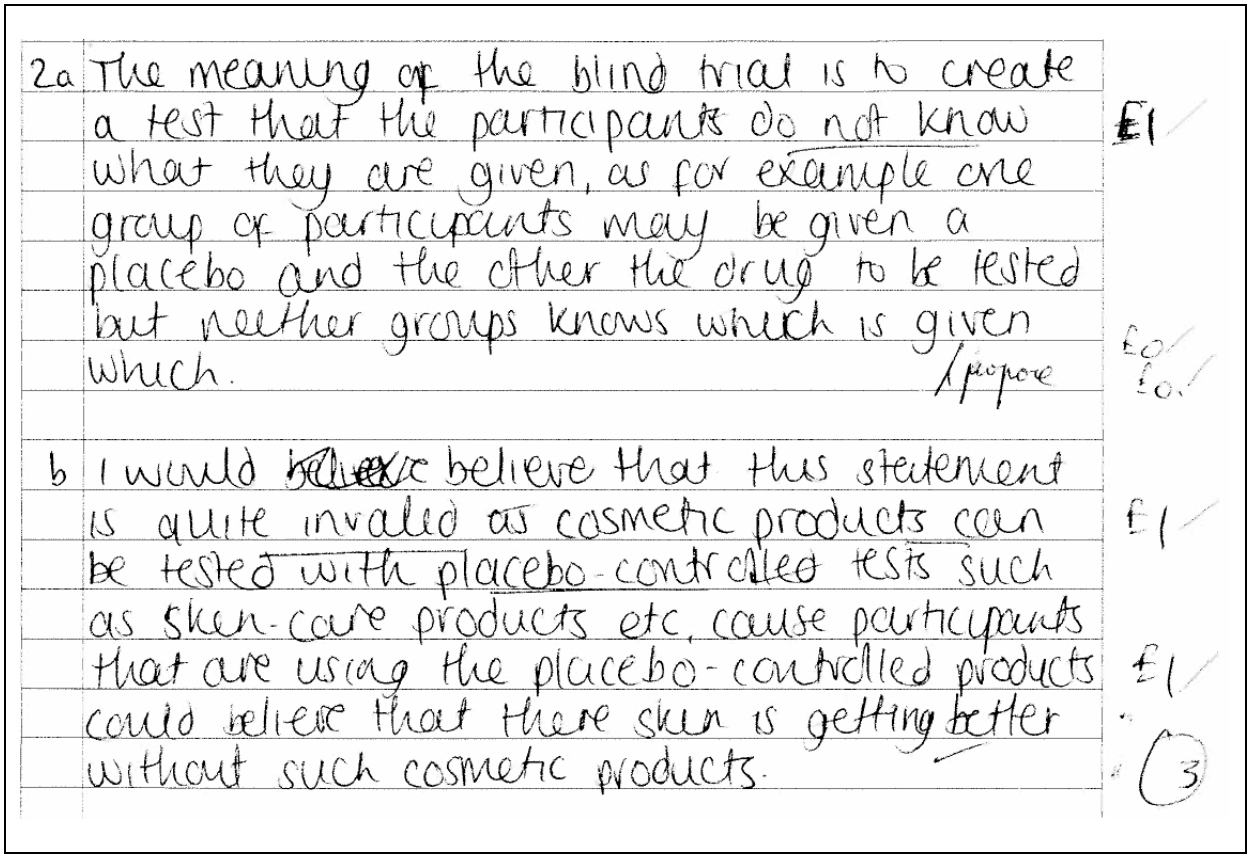
Question 2

2 (a) Explain the meaning and purpose of blind trials. *(3 marks)*

(b) A spokesman for a cosmetics company was quoted in an interview as saying, "Carrying out a placebo-controlled test does not make much sense in our industry. A cosmetic product is a balanced and precise mixture of cosmetic ingredients and its effectiveness relies on this specific combination of ingredients."

Comment on the validity of this statement. *(2 marks)*

Student response



Commentary

A good explanation of the meaning of blind trials but the candidate has omitted to explain their purpose. A good answer to part (b)

Mark Scheme

S300 (cont)

Q	Solution	Marks	Total	Comments
2(a)	In a blind trial the subject does not know whether they are being treated	E1		subject does not know
	with an active ingredient or a placebo – which looks similar but contains no active ingredient.	E1		purpose
	Purpose is to prevent outcome of the trial being affected by subjects' expectations.	E1	3	complete answer
(b)	If any measurable benefit is claimed for a product it can be tested using a placebo.			
	The reason for the product's effectiveness is irrelevant	E1		nonsense
	Statement nonsense.	E1	2	explanation
	Total		**5**	

Question 3

3 To make it easier for customers to find the items that they require, a supermarket chain employed a consultant to redesign the layout of its stores. The new layout was introduced in the Guildford store. Ahmed was asked to evaluate the effectiveness of the new layout in enabling customers to collect items more quickly.

He compiled 12 lists, each of 15 items available in all branches of the supermarket. He then obtained 12 volunteers and took them in a minibus to the Woking store, which still had the old layout. Each volunteer was given one of the lists and asked to collect their 15 items from the shelves. Ahmed then drove the volunteers to the Guildford store. He asked each volunteer to collect their 15 items from the shelves of the Guildford store. The times, in seconds, taken by the volunteers to collect their items were as follows.

Volunteer	1	2	3	4	5	6	7	8	9	10	11	12
Woking	344	390	205	399	240	422	399	189	402	354	278	349
Guildford	312	288	198	345	244	378	308	183	355	306	260	390

(a) Use a paired *t*-test and the 5% significance level to examine whether items can be collected more quickly at the Guildford store than at the Woking store. *(10 marks)*

(b) Identify one source of possible bias in Ahmed's design and suggest how it could have been removed. *(2 marks)*

Student Response

Leave blank

3 $H_0: \mu_G = \mu_W$

$H_1: \mu_G < \mu_W$

Difference	1	2	3	4	5	6	7	8	9	10
	32	102	7	54	-4	44	91	6	47	48
	11	12								
	18	-41								

$D_T = 404$

$D_T^2 = 31180$

$\bar{D} = 33.66$

$s^2 = \frac{1}{11}\left[31180 - 12 \times 33.66^2\right]$

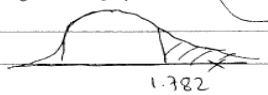
$s = 39.98$

$t = \frac{\bar{D} - (\mu_G - \mu_W)}{\frac{s}{\sqrt{n}}}$

$t = \frac{33.66}{\frac{39.98}{\sqrt{12}}} = 2.9165$

6

Critical value = 1.782



Reject H_0

There is significant evidence to suggest that the μ_G is less than μ_W

A1

b) The volunteen 12 is very slower in Guildford than the working

7

Commentary

The candidate has carried out the calculation correctly but has lost marks by failing to state his conclusion in the context of the question and by using an incorrect critical value. He has not stated his degrees of freedom and so it is not clear whether he has used incorrect degrees of freedom or has misread the table.

Part (b) was generally poorly answered and this candidate has focused on the performance of one volunteer instead of pointing out that bias could have been introduced by all volunteers collecting first in the Woking store and then in the Guildford store.

Mark Scheme

3(a)	vol 1 2 3 4 5 6 7 W – G 32 102 7 54 –4 44 91 8 9 10 11 12 6 47 48 18 –41	M1		method for differences – disallow all same sign (W – G or G – W)
	$\bar{d}$=33.6667 s=39.97575	B1		33.67(33.6~33.7) and 39.98(39.9~40.0)
	$H_0: \mu_d = 0$ $H_1: \mu_d > 0$ allow $H_0: \mu_G = \mu_w$ $H_1: \mu_G < \mu_w$	B1		both hypothesis consistent with their differences – needs population or μ
	$t = \frac{(33.6667-0)}{\frac{(39.97575)}{\sqrt{12}}} = 2.92$	M1 m1 A1		use of $\frac{\text{their s.d.}}{\sqrt{12}}$ method for t – ignore sign – needs both previous M marks 2.92(2.91~2.92) or –2.92 if G–W used
	c.v. $t_{11} = 1.1796$	B1 B1√		11 df 1.796(1.79~1.8) ignore sign
	reject H_0, significant evidence that items can be collected more quickly, on average, at Guildford than at Woking	A1√ A1√	10	conclusion – must be compared with correct tail of t conclusion in context – needs previous A mark **For sign test/Wilcoxon** allow maximum M1 B0 B1
(b)	All volunteers collected from Woking first then Guildford – possible learning effect. Could have 6 collect at Guildford first and the other 6 collect at Woking first.	E1 E1	2	source of possible bias – allow familiarity with store / particular items included in lists etc. method of removal
	Total		**12**	

Question 4

4 A garden centre sells bags of compost. They are delivered to the garden centre in large batches. When a batch of bags is delivered, the following acceptance sampling scheme is used.

Select a random sample of 10 bags.
Accept the batch if the mean weight of these bags exceeds 25.2 kg.
Otherwise reject the batch.

The weights of the bags may be assumed to be normally distributed with a standard deviation of 0.65 kg.

(a) By carrying out suitable calculations, determine whether or not this scheme will satisfy the requirement that there is:

(i) a probability of at least 0.9 of **rejecting** a batch with mean weight 24.6 kg;

(ii) a probability of at least 0.95 of **accepting** a batch with mean weight 25.7 kg.
(6 marks)

(b) Comment on the suggestion that it would be possible to reduce the sample size and still meet both the requirements in part (a). Further calculation is not required. *(2 marks)*

Student Response

4. $n = 10 \quad \bar{X} \sim N(\mu, 0.65^2)$
accept the batch if: $\bar{x} > 25.2$
Reject the batch if: $\bar{x} < 25.2$

a) (i) $\bar{X} \sim N(24.6, 0.65^2)$

$P(R) = P(\bar{x} < 25.2)$
$= P\left(z < \frac{25.2 - 24.6}{0.65/\sqrt{10}}\right)$
$= P(z < 2.92)$
$= 0.99825 \Rightarrow 0.998 > 0.9$
$\Rightarrow$ the requirement is satisfied.

(ii) $X \sim N(25.7, 0.65^2)$

$P(A) = P(\bar{x} > 25.2)$
$= P\left(z > \frac{25.2 - 25.7}{0.65/\sqrt{10}}\right)$
$= P(z > -2.43)$
$= P(z < 2.43) = 0.99245 \Rightarrow 0.992 > 0.95$
$\Rightarrow$ the requirement is satisfied.

6

Commentary

A correct answer, clearly presented.

Mark Scheme

SS06 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$z = \frac{(25.2-24.6)}{\left(\frac{0.65}{\sqrt{10}}\right)}$	M1		method for z – ignore sign
	= 2.919			
		m1		method for P(reject) – both method marks may be earned in (a)(ii)
	P (reject) = 0.998	A1		0.998(0.998~0.9985)
	>0.9			
	or 2.92 > 1.2816			
	condition met	A1√	4	condition met
(ii)	$z = \frac{(25.2-25.7)}{\left(\frac{0.65}{\sqrt{10}}\right)}$			
	= – 2.433			
	P (accept) = 0.993	A1		0.993(0.992~0.993)
	> 0.95			
	or – 2.433 < – 1.6449			
	condition met	A1√	2	condition met
(b)	Since both conditions are easily met, it is likely that the sample size could be reduced and the conditions still met. **can imply A1√ A1√ in (a)**	E1 E1	2	likely sample size could be reduced because conditions easily met allow answers based on further calculation
	Total		**8**	

Question 5

5 [**Figure 1**, printed on the insert, is provided for use in this question.]

A firm, which assembles personal computers, buys components in large batches.

A random sample of 50 components is taken from each batch and the batch is accepted if the sample contains 2 or fewer non-conforming components.

(a) (i) Find the probability of accepting batches containing 1%, 3%, 5%, 7%, 10% and 15% non-conforming components. *(3 marks)*

(ii) Hence draw the operating characteristic on **Figure 1**. *(2 marks)*

(b) Frank, the managing director, complains that some batches with a low percentage of non-conforming components are being rejected and some batches with a high percentage of non-conforming components are being accepted.

He asks Sally, the quality control manager, to change the sampling plan. She introduces the following double sampling plan.

> Take a random sample of size 40 and accept the batch if 1 or fewer non-conforming components are found; reject the batch if 4 or more non-conforming components are found.
>
> If 2 or 3 non-conforming components are found, take a further random sample of size 40 and accept the batch if a total of 3 or fewer (out of 80) non-conforming components are found; otherwise reject the batch.

The following table shows the probability of accepting batches containing various percentages of non-conforming components.

% non-conforming	1	3	5	7	10	15
P (accept)	0.994	0.833	x	0.286	0.095	0.013

(i) Calculate the value of x. *(5 marks)*

(ii) Add the operating characteristic for this double sampling plan to **Figure 1**. *(2 marks)*

(c) Compare the two sampling plans. Include in your answer a comment on Frank's complaint in part (b). *(3 marks)*

Student Response

$X \sim Bin(50, p)$

5 (a)(i)

non-conforming components/%	0.01	0.03	0.05	0.07	0.1	0.15
probability	0.9864	0.8108	0.5405	0.3108	0.1117	0.0162

3

(ii) See graph

(b) (i) $P(\text{acceptance}) = P(X \leq 1) + P(X=2) \times P(Y \leq 1) + P(X=3) \times P(Y \leq 0)$

$= 0.3991 + (0.6767 - 0.3991) \times 0.3991 + (0.8619 - 0.6767) \times 0.1285$

$= 0.534$

$x = 0.534$

5

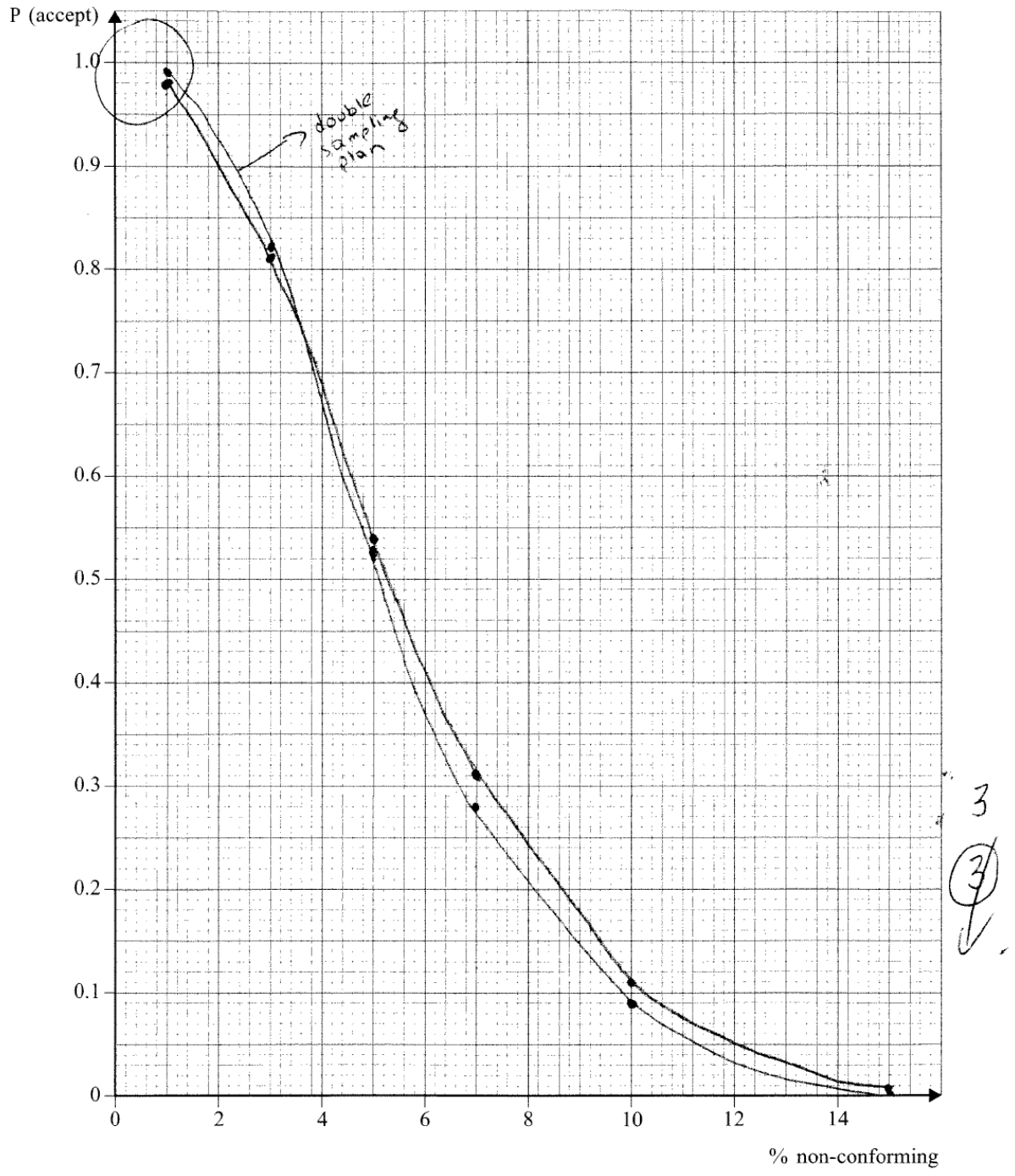
(ii) See graph

(c) In the ~~second,~~ double sampling plan when the % non-conforming is ~~the~~ low, we have ~~bigger~~ bigger probability of acceptance than in the first sampling plan. And when the % non conforming is high, we have ~~smaller~~ smaller probability of acceptance than in the first sampling plan.

E1

12

Figure 1 (for use in Question 5)



Commentary

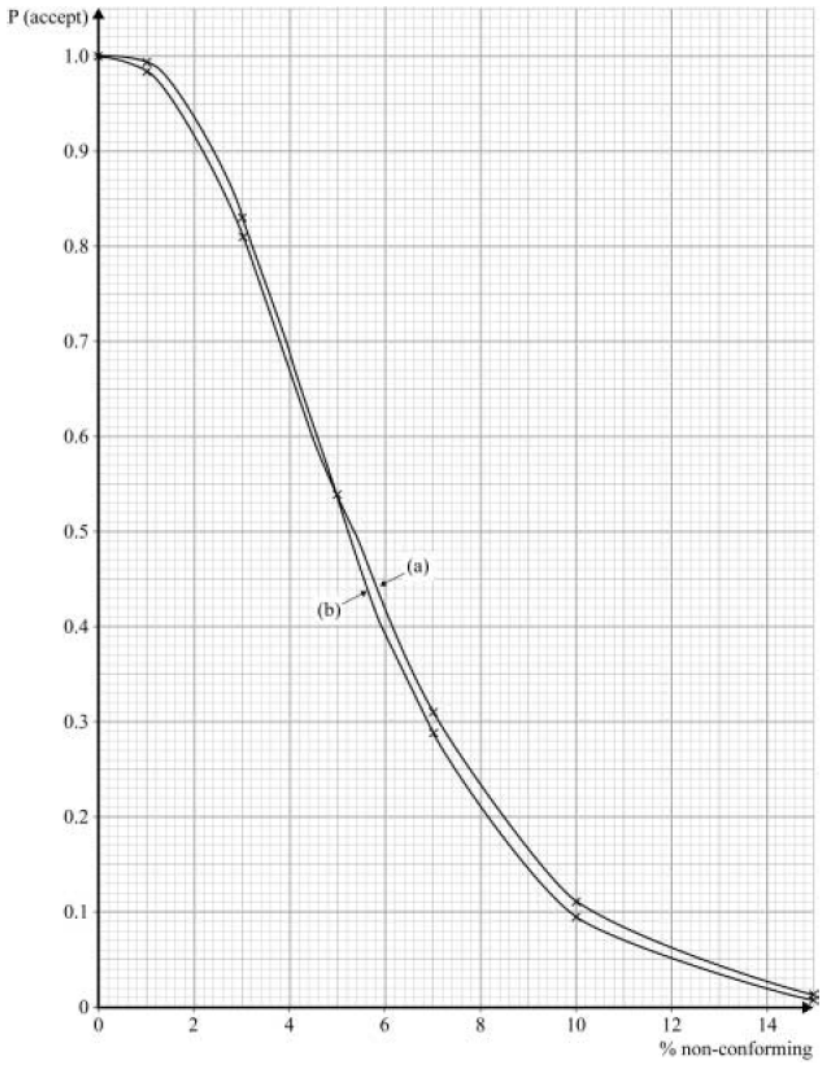
A good answer but the candidate lost a mark by failing to include the point (0,1) in the operating characteristics. For full marks in part (c) comments on double sampling plans being more complicated than single sampling plans and on Frank demanding the impossible were required. Any decision based on samples will occasionally be wrong and so Frank's demand cannot be met.

Mark Scheme

SS06 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	% n-c 1 3 5 7 10 15 P(accept) 0.986 0.811 0.541 0.311 0.112 0.014	B1 M1 A1	3	use of binomial n = 50 method all values ±0.001
(ii)	on next page	M1 A1	2	method – points must be joined accurate plot – allow 1 small slip – must go through (0,1)
(b)(i)	accept 1st 0 1 2 2 3 2nd 0 1 0 B (40,0.05) P(accept) = P(0 or 1)+P(2)×P(0 or 1)+P(3) × P(0) =0.3991+0.2776×0.3991+0.1852×0.1285 = 0.534	M1 m1 B1 m1 A1	5	reasonable attempt to enumerate ways of accepting or rejecting correct enumeration use of B (40,0.05) correct method 0.534(0.533~0.534)
(ii)	on next page	M1 A1	2	method for given data – points must be joined accurate plot – allow one small slip – don't penalise omission of (0,1) twice
(c)	Double sampling plans more likely to accept good (low % n–c) batches and to reject bad (high % n–c) batches. More complicated to operate. All acceptance sampling plans will reject some good batches and accept some bad batches.	E1 E1 E1	3	double sampling plan 'better' double sampling plan more complicated all acceptance sampling plans will reject some good batches
	Total		**15**	

SS06 (cont)



Question 6

6 (a) A researcher, investigating the effect of drinking alcohol on mental dexterity, obtained 12 volunteers and divided them randomly into three groups of four. Thirty minutes before solving a simple Sudoku puzzle, the volunteers in Group 2 each drank one measure of whisky and those in Group 3 each drank three measures of whisky. The volunteers in Group 1 had no alcohol. The time, in seconds, that it took each volunteer to solve the puzzle was recorded.

Group 1 (no alcohol)	Group 2 (1 measure)	Group 3 (3 measures)
184	196	262
126	98	168
108	222	240
204	144	190

Carry out a one-factor analysis of variance to test for differences between the effects of different amounts of alcohol. Use the 5% significance level. *(11 marks)*

(b) It was pointed out that the time taken to solve a Sudoku puzzle may also depend on the weight and the sex of the subject. Twelve new female volunteers were obtained. They were ranked by weight and divided into four groups.

Volunteers ranked 1, 2, 3 formed the first group;
4, 5, 6 formed the second group;
7, 8, 9 formed the third group;
10, 11, 12 formed the fourth group.

One member of each group was randomly chosen to drink no alcohol, one to drink one measure of whisky and one to drink three measures of whisky. They were timed to solve the same simple Sudoku puzzle.

Copy and complete the following analysis of variance table which arose from this second experiment. Test for differences between the effects of different weights and of different amounts of alcohol, using the 5% significance level.

Source	Sum of squares	Degrees of freedom	Mean square
Between amounts of alcohol	9348		
Between weights	7980		
Residual			
Total	20542		

(6 marks)

(c) In the light of your results, comment on the effectiveness of the experiment in part (b) compared to that in part (a). *(2 marks)*

(d) The analysis of variance undertaken in part (b) assumes that there is no interaction between weight and amount of alcohol consumed. Explain, in the context of this experiment, the meaning of this assumption. *(2 marks)*

Student Response

number

Leave blank

⑥ a) Ho: No difference between the effects of different amounts of alcohol

Hi: Difference between the effects of different amounts of alcohol.

	Group 1	Group 2	Group 3	
	184	196	262	
	126	98	168	
	108	222	240	
	204	144	190	
Total	622	660	860	2142

$SS_T = \sum x^2 - \frac{G^2}{n} = 411620 - 382347 = 29273$

$SS_B = \sum \frac{T^2}{n} - \frac{G^2}{n} = 390521 - 382347 = 8174$

ANOVA	S.Squares	D.F	M.Squares	F-ratio
Between Groups	8174	2	4087	1.74
Within Groups	21099	9	2344.33	
Total	29273	11		

C.V. $F(2,9) = 4.256 > 1.74$

⇒ Do not Reject Ho

There is no evidence to suggest a difference between the effects of different amounts of alcohol.

10

b)

Source	S. Squares	D.F.	Mean Square	F-ratio
Between alcohol	9348	2	4674	10.18
Between weights	7980	3	2660	5.79
Residual	3214	(7)	459.14	
Total	20542	12		

M1
m1

Ho: No effect due to weights
H1: Effect due to weights

5.79
C.V: $F(3,7) = 4.347 < 5.79$
⇒ Reject Ho
Evidence for an effect due to different weights

Ho: No effect due to alcohol
H1: Effect due to alcohol

10.18
C.V: $F(2,7) = 4.737 < 10.18$
⇒ Reject Ho
Evidence for an effect due to different amounts of alcohol.

A1

c) The two ~~factor~~ way analysis was more effective that the one in part one, which was one way analysis

E1

d) It means that the amount of alcohol drunk by a volunteer will not affect the volunteer according to his weight. In other words, ~~a related~~ the damage made to mental dexterity be the alcohol to be independend of the weight of every person.

2

16

Commentary

Part (a) was well answered but for full marks a slightly fuller context for the conclusion was needed. Some mention of the time to solve Sudoku puzzles should have been included. In common with many other candidates the answer to part (b) has been spoilt by incorrect degrees of freedom.

Mark Scheme

SS06 (cont)

Q	Solution	Marks	Total	Comments
6(a)	group 1 2 3 total 622 660 860 $\sum x = 2142 \quad \sum x^2 = 411620$			
	total SS $= 411620 - \frac{2142^2}{12} = 29273$	M1		method for total SS disallow negative SS
	between groups SS $= \frac{622^2}{4} + \frac{660^2}{4} + \frac{860^2}{4} - \frac{2142^2}{12} = 8174$	M1		method for between groups SS
	source / SS / DF / MS between groups / 8174 / 2 / 4087 residual / 21099 / 9 / 2344.3 total / 29273 / 11 /	B1 M1 m1		df 2, 9 method for residual SS $MS = \frac{SS}{\text{their df}}$
	H_0: no difference between groups H_1: not all group means equal	B1		hypotheses – population not essential
	$F = \frac{4087}{2344.3} = 1.74$	M1 A1		method for F – their figures 1.74(1.73~1.75)
	c.v. $F_{[2,9]} = 4.256$	B1		4.256(4.25~4.26)
	accept H_0: no significant evidence of differences in mean times to complete Sudoku for groups drinking different quantities of alcohol	A1√ A1√	11	their figures – must be compared with upper tail of F – needs previous M only in context – requires previous A mark
(b)	source / SS / DF / MS alcohol / 9348 / 2 / 4674 weights / 7980 / 3 / 2660 residual / 3214 / 6 / 535.67 total / 20542 / 11 /	B1 M1		2, 3, 6 df method for all MS (including method for residual SS), their df
	H_0: no difference between amounts of alcohol			
	$F = \frac{4674}{535.67} = 8.73$	m1		method for F (either) – their figures
	reject H_0: significant evidence differences in mean times to do Sudoku between groups drinking different amounts of alcohol			
	H_0: no difference between weights			
	$F = \frac{2660}{535.67} = 4.97$	A1		8.73(8.72~8.73) and 4.97(4.96~4.97)
	c.v. $F_{[3,6]} = 4.757$	B1		5.143(5.14~5.15) and 4.757(4.75~4.76)
	reject H_0: significant evidence differences in mean times to do Sudoku between groups of different weights	A1√	6	both conclusions – their figures – must be compared with upper tail of F

SS06 (cont)

Q	Solution	Marks	Total	Comments
6(c)	The design in (b) has greatly reduced the residual MS, thus making it more likely to detect a difference if one exists. Design successful.	E1 E1	2	design effective reason
(d)	No interaction means that drinking alcohol has the same effect (in terms of time to do Sudoku) on a light person as on a heavy person.	E1 E1	2	meaning of interaction in context
	Total		**21**	
	TOTAL		**75**	