Version 1.0: 0606

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Statistics 6380

SS03 Statistics 3

Mark Scheme

2006 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS03

Q	Solution	Marks	Total	Comments
1(a)	Ranks Rank x | Rank y 10 (1) | 10 (1) 5 (6) | 9 (2) 1 (10) | 8 (3) 8 (3) | 7 (4) 7 (4) | 6 (5) 6 (5) | 5 (6) 3 (8) | 4 (7) 4 (7) | 3 (8) 9 (2) | 2 (9) 2 (9) | 1 (10)	M1 M1		Ranks effort made for x Ranks effort made for y (consistent)
	r_s = 0.273 (3 sig figs)	B3	5	**Alternative** $\lvert d \rvert = 0,4,7,1,1,1,1,1,7,1$ $\sum d^2 = 120$ $r_s = 1 - \frac{6 \times 120}{10 \times 99} = 0.273$ M1 M1 A1 0.272 or 0.274 M1 M1 A0 **sc** r = 0.27, no method 3/5
(b)	H_0: ranks are independent in population H_1 : ranks are not independent – an association does exist between x and y 2 tail 10 % sig level test stat r_s = 0.273 $\lvert$cv$\rvert$ = 0.5636 since $\lvert$ts$\rvert$ < 0.5636 Accept H_0 . No significant evidence at 10% level to suggest an association between municipal waste and CO_2 emissions per capita – ranks are independent in the population.	B1 B1 M1 A1 E1	5	Allow $\rho_s = 0$ $\rho_s \neq 0$ For cv For comparison ts/cv Allow small slip in r_s In context
	Total		**10**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)	H_0: Age is independent of home location H_1: Age is not independent of home location 1 tail 5%	B1		
	Expected(U20 and N) = $\frac{47\times12}{100}$ etc Expected values			
	U 20 | **20-29** | **30-39** | **40 +** **North** 5.64 | 14.1 | 21.62 | 5.64 **South** 6.36 | 15.9 | 24.38 | 6.36	M1 m1		*E* method for 4 correct, can be integers All *E* correct, 1 dp, not integers
	χ^2 test stat = $\sum\frac{(O-E)^2}{E}$			
	$=\frac{3.36^2}{5.64}+\frac{3.9^2}{14.1}+\frac{3.62^2}{21.62}+\frac{3.64^2}{5.64}$	m1		**sc** Pool max 4 M1 m0 m1 A0 5.991 B0 B1 M1 A0
	$+\frac{3.36^2}{6.36}+\frac{3.9^2}{15.9}+\frac{3.62^2}{24.38}+\frac{3.64^2}{6.36}$			**sc** Yates max 4 M1 m1 m0 A0 B1 B1 m0 A0
	$= 11.4 \qquad \nu = 3$	A1 B1		Test stat 11.3 – 11.5 Allow M1 m1 m1 A1 if no method For $\nu = 3$
	5% cv = 7.815 ts > 7.815 so sig evidence to reject H_o and conclude that age is not independent of home location.	B1 M1 A1	9	For cv
(b)	Women aged 40 and over are much more likely to live in the South of the UK and those aged under 20 are more likely to live in the North of the UK.	E1 E1	2	If test muddled and used common sense E1,0 If test ok, but wrong conclusion E1,0 dep on (a)
(c)(i)	No change as the test is still the same	E1		For 'staying the same'
(ii)	No change as $\nu = 3$ still	E1		For 'staying the same'
(iii)	Test statistic would also be doubled ts = 22.8 as all expected/observed values are doubled.	B1 E1		For saying there will be a change/increase For identification of new ts doubling
(iv)	Conclusion is the same because the new test statistic is still greater than the critical value (22.8 > 7.815)	E2√	6	For 'staying the same' conclusion For explanation in context Bigger test stat., cv same and explained in context E2
	Total		**17**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	H_0: pop median, η = £148.50	B1		Correct pop median used Or pop average/pop mean
	H_1: pop median, η > £148.50	B1		Correct tail/direction
	1 tail 5%			
	diff £ 8.95 -1.85 3.80 3.10 8.60 **rank** 8 -2 6 3½ 7	M1 M1 m1		For differences X - £148.50 For ties, however ranked For ranks, rank 1 = smallest
	3.65 -3.10 -0.90 16.25 10.10 5 -3½ -1 10 9			
	$T_+ = 8 + 6 + \ldots + 9 = 48½$ $T_- = 2 + 3½ + 1 = 6½$	m1√		For totals
	Test stat $T = 6½$	A1		For one correct total
	cv = 11	B1		For cv
	$T < 11$	M1√		Comparison cv/ts; ft however ranked
	Significant evidence at 5% level to reject H_0	A1	10	
(b)	There is significant evidence to suggest that the median weekly wage for full-time workers in company cafeterias has increased from £148.50	E1	1	In context (might be in (a))
(c)(i)	Wilcoxon signed-test is preferred because the magnitudes of the differences are taken into account whereas, with the sign test, only the signs of the differences are used.	B1	1	Not 'more accurate' More powerful More likely to detect a difference if one exists
(ii)	Data not symmetrically distributed therefore Wilcoxon signed-rank cannot be carried out Data given only as signs/preferences so only sign test possible	B1 E1	2	Correct reasoning and explained well
	Total		**14**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	H_0: Samples from identical populations	B1		Or H_0: $\eta_A = \eta_B = \eta_C = \eta_D$
	H_1 : Samples not from identical populations 1% sig level	B1		H_1: at least two of $\eta_A, \eta_B, \eta_C, \eta_D$ do differ Or pop average B1 B0 Or H_0: no difference in pop medians H_1: one pop median does differ B1 B1 H_0: $\mu_A = \mu_B = ...$ B1 B0
	Totals of ranks $T_A = 29$ $T_B = 55$ $T_C = 92$ $T_D = 77$ $n_A = 6$ $n_B = 6$ $n_C = 5$ $n_D = 5$	M1		Totals of ranks in each depth
	$\sum_{i=1}^{m} \frac{T_i^2}{n_i} = \frac{29^2}{6} + \frac{55^2}{6} + \frac{92^2}{5} + \frac{77^2}{5}$	m1		Correct numerators ft
	$= 3522.933$	m1		Correct denominators
	$H = \frac{12}{22 \times 23} \times 3522.933 - (3 \times 23)$	m1		
	$= 14.5$	A1		H 14.4 – 14.6
	Critical value from $\chi^2_3 = 11.345$	B1		
	$H > 11.345$	m1		Dep on M1 m1 m1
	Sig evidence to reject H_0 and conclude that samples are not from identical populations	A1		
	There is significant evidence that at least two of the average alfalfa yields (from methods A, B, C or D) do differ.	E1 E1	12	Difference in context Mention of 'at least two'
(b)	It would appear that the average yield for method C is significantly higher (as there is significant evidence of a difference detected in part (a) and C has the highest ranked yields) and this would be the recommended method for greatest alfalfa yield.	B1 E1 E1	3	Identification of method C with some reason Significant evidence to suggest that the average yield for C is certainly greater than the average yield for method A With reference to average ranks and clearly understood. Need at least 2 different, or refer to conclusion in (a)
	Total		**15**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	H_0: pop median difference $\eta_d = 0$	B1		Not difference
	H_1: pop median difference $\eta_d \neq 0$			Not μ/average
	2 tail 10%			
	Signs: + + + + – – + + +	M1		Signs or WSR signed differences
	n = 9			
	test stat $= 7^+ / 2^-$	A1		Test stat correct
	Model B(9. 0.5)	M1		Bin model seen to be used with $n = 9$
	0.0898 > 0.05 or 0.1796 > 0.10 seen	M1		Comparison of correct B(9, 0.5) prob with 0.05 or 0.10 Or use of identified cr $\{0,1\}$ or $\{8,9\}$ with prob 0.195
	Accept H_0 No significant evidence at 10% level to doubt H_0	A1	6	
				sc Wilcoxon 4/6 max M1 for diffs and signs: 4, 6, 5, 23, -5, -5, 19, 24, 20 (ranks 1, 5, 3, 8, -3, -3, 6, 9, 7) A1 total correct 6 or 39 M1 comparing total with cv: cv = 8, $T = 6$
				sc one-tail max 4/6 H_0 $\eta = 0$ H_1 $\eta > 0$ B1 M1 A1 M1 M0 A0
(b)	There is no significant evidence to suggest that either aerosol A or B is preferable as there is no significant difference in average effectiveness	E1√	1	Explanation in context
	Total		**7**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
6	H_0: Samples are taken from identical populations	B1		Hypotheses referring to population averages also acceptable
	H_1: Samples are not taken from identical populations (population average weight differs) 2 tail 5%	B1		Refer to μ: B1 B0
	Ranks	M1		For ranks as one group
	E Side 14 18 7½ 17 9 5 16 7½ 15 **W Side** 2 4 10 12 1 3 11 13 6	M1 A1		At least 12 correct. (Ties not necessary 7,8 OK) Other alternative methods acceptable
	$T_E = 14 + 18 + \ldots + 15 = 109$ $T_W = 2 + 5 + \ldots + 6 = 62$	m1		For totals of ranks in each group
	$U_E = 109 - \frac{9\times10}{2} = 64$ $U_W = 62 - \frac{9\times10}{2} = 17$	m1		For U attempted
	Test stat $U = 17$	A1		For U correct – either Method not seen, award full marks
	cv = 18	B1		For consistent cv with U cv = 63 acceptable
	$U = 17 < 18$	M1		For comparison U/cv
	Reject H_0	A1		
	Significant evidence at the 5% level to suggest that the population average weight for plants from the East Side differs from the population average weight for plants from the West Side	E1	12	In context
	Total		**12**	
	TOTAL		**75**	