MEI EXAMINATION FORMULAE AND TABLES
(MF2)

For use with:

Advanced General Certificate of Education

Advanced Subsidiary General Certificate of Education

MEI STRUCTURED MATHEMATICS

and

Advanced Subsidiary GCE

QUANTITATIVE METHODS (MEI)

MF2
CST251

MEI STRUCTURED MATHEMATICS

and

QUANTITATIVE METHODS (MEI)

EXAMINATION FORMULAE AND TABLES

Arithmetic series

General (kth) term, $u_k = a + (k-1)d$

last (nth) term, $l =$ $u_n = a + (n-1)d$

Sum to n terms, $S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n-1)d]$

Geometric series

General (kth) term, $u_k = a\,r^{k-1}$

Sum to n terms, $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a(r^n-1)}{r-1}$

Sum to infinity $S_\infty = \dfrac{a}{1-r}, \; -1 < r < 1$

Binomial expansions

When n is a positive integer

$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots b^n \;, n \in \mathbb{N}$

where

$\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$ $\qquad \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

General case

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!} x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{1.2\ldots r} x^r + \ldots, \; |x| < 1, \; n \in \mathbb{R}$

Logarithms and exponentials

$e^{x\ln a} = a^x$ $\qquad \log_a x = \dfrac{\log_b x}{\log_b a}$

Numerical solution of equations

Newton-Raphson iterative formula for solving $f(x) = 0$, $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Complex Numbers

$\{r(\cos\theta + j\sin\theta)\}^n = r^n(\cos n\theta + j\sin n\theta)$

$e^{j\theta} = \cos\theta + j\sin\theta$

The roots of $z^n = 1$ are given by $z = \exp(\frac{2\pi k}{n} j)$ for $k = 0, 1, 2, \ldots, n-1$

Finite series

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ $\qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

Infinite series

$f(x) = f(0) + xf'(0) + \dfrac{x^2}{2!} f''(0) + \ldots + \dfrac{x^r}{r!} f^{(r)}(0) + \ldots$

$f(x) = f(a) + (x-a)f'(a) + \dfrac{(x-a)^2}{2!} f''(a) + \ldots + \dfrac{(x-a)^r f^{(r)}(a)}{r!} + \ldots$

$f(a+x) = f(a) + xf'(a) + \dfrac{x^2}{2!} f''(a) + \ldots + \dfrac{x^r}{r!} f^{(r)}(a) + \ldots$

$e^x = \exp(x) = 1 + x + \dfrac{x^2}{2!} + \ldots + \dfrac{x^r}{r!} + \ldots, \text{ all } x$

$\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \ldots + (-1)^{r+1}\dfrac{x^r}{r} + \ldots, \; -1 < x \leqslant 1$

$\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots + (-1)^r \dfrac{x^{2r+1}}{(2r+1)!} + \ldots, \text{ all } x$

$\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \ldots + (-1)^r \dfrac{x^{2r}}{(2r)!} + \ldots, \text{ all } x$

$\arctan x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \ldots + (-1)^r \dfrac{x^{2r+1}}{2r+1} + \ldots, \; -1 \leqslant x \leqslant 1$

$\sinh x = x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \ldots + \dfrac{x^{2r+1}}{(2r+1)!} + \ldots, \text{ all } x$

$\cosh x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \ldots + \dfrac{x^{2r}}{(2r)!} + \ldots, \text{ all } x$

$\text{artanh}\, x = x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \ldots + \dfrac{x^{2r+1}}{(2r+1)} + \ldots, \; -1 < x < 1$

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$, $\sinh 2x = 2\sinh x \cosh x$, $\cosh 2x = \cosh^2 x + \sinh^2 x$

$\text{arsinh}\, x = \ln(x + \sqrt{x^2+1})$, $\qquad \text{arcosh}\, x = \ln(x + \sqrt{x^2-1}), \; x \geqslant 1$

$\text{artanh}\, x = \frac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right), \; |x| < 1$

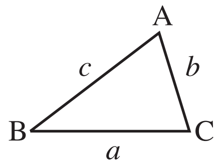
Matrices

Anticlockwise rotation through angle θ, centre O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = x\tan\theta$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Cosine rule $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ (etc.)

$a^2 = b^2 + c^2 - 2bc \cos A$ (etc.)



Trigonometry

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\tan(\theta \pm \phi) = \dfrac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$, $[(\theta \pm \phi) \neq (k + \tfrac{1}{2})\pi]$

For $t = \tan\frac{1}{2}\theta$: $\sin\theta = \dfrac{2t}{(1+t^2)}$, $\cos\theta = \dfrac{(1-t^2)}{(1+t^2)}$

$\sin\theta + \sin\phi = 2\sin\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$

$\sin\theta - \sin\phi = 2\cos\frac{1}{2}(\theta+\phi)\sin\frac{1}{2}(\theta-\phi)$

$\cos\theta + \cos\phi = 2\cos\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$

$\cos\theta - \cos\phi = -2\sin\frac{1}{2}(\theta+\phi)\sin\frac{1}{2}(\theta-\phi)$

Vectors and 3-D coordinate geometry

(The position vectors of points A, B, C are **a**, **b**, **c**.)

The position vector of the point dividing AB in the ratio λ:μ is $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{(\lambda + \mu)}$

Line: Cartesian equation of line through A in direction **u** is

$$\frac{x - a_1}{u_1} = \frac{y - a_2}{u_2} = \frac{z - a_3}{u_3} \left(= t\right)$$

The resolved part of **a** in the direction **u** is $\dfrac{\mathbf{a}\,.\,\mathbf{u}}{|\mathbf{u}|}$

Plane: Cartesian equation of plane through A with normal **n** is

$n_1 x + n_2 y + n_3 z + d = 0$ where $d = -\mathbf{a}\,.\,\mathbf{n}$

The plane through non-collinear points A, B and C has vector equation

$\mathbf{r} = \mathbf{a} + s(\mathbf{b} - \mathbf{a}) + t(\mathbf{c} - \mathbf{a}) = (1 - s - t)\,\mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The plane through A parallel to **u** and **v** has equation

$\mathbf{r} = \mathbf{a} + s\mathbf{u} + t\mathbf{v}$

Perpendicular distance of a point from a line and a plane

Line: (x_1, y_1) from $ax + by + c = 0$: $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Plane: (α, β, γ) from $n_1 x + n_2 y + n_3 z + d = 0$: $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{(n_1^2 + n_2^2 + n_3^2)}}$

Vector product

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

$$\mathbf{a}.\,(\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \mathbf{b}.\,(\mathbf{c} \times \mathbf{a}) = \mathbf{c}.\,(\mathbf{a} \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c}\,.\,\mathbf{a})\,\mathbf{b} - (\mathbf{a}\,.\,\mathbf{b})\,\mathbf{c}$$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular hyperbola**
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$	$(ct, \frac{c}{t})$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm c\sqrt{2}, \pm c\sqrt{2})$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm c\sqrt{2}$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Any of these conics can be expressed in polar coordinates (with the focus as the origin) as: $\dfrac{l}{r} = 1 + e\cos\theta$
where l is the length of the semi-latus rectum.

Mensuration

Sphere : Surface area $= 4\pi r^2$

Cone : Curved surface area $= \pi r \times$ slant height

Differentiation f(x)	f'(x)
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\arcsin x$	$\dfrac{1}{\sqrt{(1-x^2)}}$
$\arccos x$	$\dfrac{-1}{\sqrt{(1-x^2)}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\mathrm{sech}^2 x$
$\mathrm{arsinh}\, x$	$\dfrac{1}{\sqrt{(1+x^2)}}$
$\mathrm{arcosh}\, x$	$\dfrac{1}{\sqrt{(x^2-1)}}$
$\mathrm{artanh}\, x$	$\dfrac{1}{(1-x^2)}$

Integration f(x)	$\int$f(x) dx (+ a constant)
$\sec^2 kx$	$(1/k)\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\mathrm{cosec}\, x$	$-\ln\lvert\mathrm{cosec}\, x + \cot x\rvert = \ln\lvert\tan \frac{x}{2}\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\left\lvert\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\rvert$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$
$\dfrac{1}{\sqrt{(a^2-x^2)}}$	$\arcsin\left(\frac{x}{a}\right)$, $\lvert x\rvert < a$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\,\mathrm{artanh}\left(\frac{x}{a}\right)$, $\lvert x\rvert < a$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\dfrac{1}{\sqrt{(a^2+x^2)}}$	$\mathrm{arsinh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$,
$\dfrac{1}{\sqrt{(x^2-a^2)}}$	$\mathrm{arcosh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2})$, $x > a$, $a > 0$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Trapezium rule $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0+y_n) + 2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Integration by parts $\int u\dfrac{dv}{dx}\,dx = uv - \int v\dfrac{du}{dx}\,dx$

Area of a sector $A = \frac{1}{2}\int r^2\,d\theta$ (polar coordinates)

$A = \frac{1}{2}\int (x\dot{y} - y\dot{x})\,dt$ (parametric form)

Arc length $s = \int\sqrt{(\dot{x}^2+\dot{y}^2)}\,dt$ (parametric form)

$s = \int\sqrt{\left(1+\left[\dfrac{dy}{dx}\right]^2\right)}\,dx$ (cartesian coordinates)

$s = \int\sqrt{\left(r^2+\left[\dfrac{dr}{d\theta}\right]^2\right)}\,d\theta$ (polar coordinates)

Surface area of revolution $S_x = 2\pi\int y\,ds = 2\pi\int y\sqrt{(\dot{x}^2+\dot{y}^2)}\,dt$

$S_y = 2\pi\int x\,ds = 2\pi\int x\sqrt{(\dot{x}^2+\dot{y}^2)}\,dt$

Curvature

$$\kappa = \frac{d\psi}{ds} = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{(\dot{x}^2+\dot{y}^2)^{3/2}} = \frac{\dfrac{d^2y}{dx^2}}{\left(1+\left[\dfrac{dy}{dx}\right]^2\right)^{3/2}}$$

Radius of curvature $\rho = \dfrac{1}{\kappa}$, Centre of curvature $\mathbf{c} = \mathbf{r} + \rho\hat{\mathbf{n}}$

L'Hôpital's rule

If f(a) = g(a) = 0 and g'(a) ≠ 0 then $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{f'(a)}{g'(a)}$

Multi-variable calculus

$\mathbf{grad}\ g = \begin{pmatrix}\partial g/\partial x\\ \partial g/\partial y\\ \partial g/\partial z\end{pmatrix}$ For $w = g(x, y, z)$, $\delta w = \dfrac{\partial w}{\partial x}\delta x + \dfrac{\partial w}{\partial y}\delta y + \dfrac{\partial w}{\partial z}\delta z$

Centre of mass (uniform bodies)

Triangular lamina:	$\frac{2}{3}$ along median from vertex
Solid hemisphere of radius r:	$\frac{3}{8}r$ from centre
Hemispherical shell of radius r:	$\frac{1}{2}r$ from centre
Solid cone or pyramid of height h:	$\frac{1}{4}h$ above the base on the line from centre of base to vertex
Sector of circle, radius r, angle 2θ:	$\frac{2r \sin \theta}{3\theta}$ from centre
Arc of circle, radius r, angle 2θ at centre:	$\frac{r \sin \theta}{\theta}$ from centre
Conical shell, height h:	$\frac{1}{3}h$ above the base on the line from the centre of base to the vertex

Motion in polar coordinates

Motion in a circle

Transverse velocity:	$v = r\dot{\theta}$
Radial acceleration:	$-r\dot{\theta}^2 = -\frac{v^2}{r}$
Transverse acceleration:	$\dot{v} = r\ddot{\theta}$

General motion

Radial velocity:	$\dot{r}$
Transverse velocity:	$r\dot{\theta}$
Radial acceleration:	$\ddot{r} - r\dot{\theta}^2$
Transverse acceleration:	$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})$

Moments as vectors

The moment about O of $\mathbf{F}$ acting at $\mathbf{r}$ is $\mathbf{r} \times \mathbf{F}$

Moments of inertia (uniform bodies, mass M)

Thin rod, length $2l$, about perpendicular axis through centre:	$\frac{1}{3}Ml^2$
Rectangular lamina about axis in plane bisecting edges of length $2l$:	$\frac{1}{3}Ml^2$
Thin rod, length $2l$, about perpendicular axis through end:	$\frac{4}{3}Ml^2$
Rectangular lamina about edge perpendicular to edges of length $2l$:	$\frac{4}{3}Ml^2$
Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre:	$\frac{1}{3}M(a^2 + b^2)$
Hoop or cylindrical shell of radius r about perpendicular axis through centre:	Mr^2
Hoop of radius r about a diameter:	$\frac{1}{2}Mr^2$
Disc or solid cylinder of radius r about axis:	$\frac{1}{2}Mr^2$
Disc of radius r about a diameter:	$\frac{1}{4}Mr^2$
Solid sphere of radius r about a diameter:	$\frac{2}{5}Mr^2$
Spherical shell of radius r about a diameter:	$\frac{2}{3}Mr^2$

Parallel axes theorem: $I_A = I_G + M(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the (x, y) plane)

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) \,.\, P(B|A)$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

Bayes' Theorem: $P(A_j|B) = \dfrac{P(A_j)P(B|A_j)}{\sum P(A_i)P(B|A_i)}$

Populations

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $P(X = x_i) = p_i$

Expectation: $\mu = E(X) = \sum x_i p_i$

Variance: $\sigma^2 = \text{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

For a function $g(X)$: $E[g(X)] = \sum g(x_i) p_i$

Continuous distributions

X is a continuous variable with probability density function (p.d.f.) $f(x)$

Expectation: $\mu = E(X) = \int x\, f(x)\, dx$

Variance: $\sigma^2 = \text{Var}(X) = \int (x - \mu)^2 f(x)\, dx = \int x^2 f(x)\, dx - \mu^2$

For a function $g(X)$: $E[g(X)] = \int g(x) f(x)\, dx$

Cumulative distribution function $F(x) = P(X \leqslant x) = \int_{-\infty}^{x} f(t)\, dt$

Correlation and regression For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}, \quad S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n},$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

Covariance $\dfrac{S_{xy}}{n} = \dfrac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} = \dfrac{\sum x_i y_i}{n} - \bar{x}\,\bar{y}$

Product-moment correlation: Pearson's coefficient

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left[\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2\right]}} = \frac{\dfrac{\sum x_i y_i}{n} - \bar{x}\,\bar{y}}{\sqrt{\left[\left(\dfrac{\sum x_i^2}{n} - \bar{x}^2\right)\left(\dfrac{\sum y_i^2}{n} - \bar{y}^2\right)\right]}}$$

Rank correlation: Spearman's coefficient

$$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

Regression

Least squares regression line of y on x: $y - \bar{y} = b(x - \bar{x})$

$$b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{\dfrac{\sum x_i y_i}{n} - \bar{x}\,\bar{y}}{\dfrac{\sum x_i^2}{n} - \bar{x}^2}$$

Estimates

Unbiased estimates from a single sample

$\bar{X}$ for population mean μ; $\text{Var}\, \bar{X} = \dfrac{\sigma^2}{n}$

S^2 for population variance σ^2 where $S^2 = \dfrac{1}{n-1} \sum (x_i - \bar{x})^2 f_i$

Probability generating functions

For a discrete distribution

$G(t) = E(t^X)$

$E(X) = G'(1)$; $\text{Var}(X) = G''(1) + \mu - \mu^2$

$G_{X+Y}(t) = G_X(t)\, G_Y(t)$ for independent X, Y

Moment generating functions:

$M_X(\theta) = E(e^{\theta X})$

$E(X) = M'(0) = \mu$; $E(X^n) = M^{(n)}(0)$

$\text{Var}(X) = M''(0) - \{M'(0)\}^2$

$M_{X+Y}(\theta) = M_X(\theta)\, M_Y(\theta)$ for independent X, Y

Markov Chains

$\mathbf{p}_{n+1} = \mathbf{p}_n\mathbf{P}$

Long run proportion $\mathbf{p} = \mathbf{pP}$

Bivariate distributions

Covariance $\mathrm{Cov}(X, Y) = \mathrm{E}[(X - \mu_X)(Y - \mu_Y)] = \mathrm{E}(XY) - \mu_X\mu_Y$

Product-moment correlation coefficient $\rho = \dfrac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y}$

Sum and difference

$\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y) \pm 2ab\,\mathrm{Cov}(X,Y)$

If X, Y are independent: $\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$

$\mathrm{E}(XY) = \mathrm{E}(X)\,\mathrm{E}(Y)$

Coding

$$\left.\begin{array}{l} X = aX' + b \\ Y = cY' + d \end{array}\right\} \Rightarrow \mathrm{Cov}(X, Y) = ac\,\mathrm{Cov}(X', Y')$$

Analysis of variance

One-factor model: $x_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, where $\varepsilon_{ij} \sim \mathrm{N}(0,\sigma^2)$

$$SS_B = \sum_i n_i(\bar{x}_i - \bar{x})^2 = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{n}$$

$$SS_T = \sum_i \sum_j (x_{ij} - \bar{x})^2 = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{n}$$

Regression

Y_i	RSS	No. of parameters, p
$\alpha + \beta x_i + \varepsilon_i$	$\sum(y_i - a - bx_i)^2$	2
$\alpha + \beta \mathrm{f}(x_i) + \varepsilon_i$	$\sum(y_i - a - b\mathrm{f}(x_i))^2$	2
$\alpha + \beta x_i + \gamma z_i + \varepsilon_i$	$\sum(y_i - a - bx_i - cz_i)^2$	3

$\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$ a, b, c are estimates for α, β, γ. $\hat{\sigma}^2 = \dfrac{\mathrm{RSS}}{n - p}$

For the model $Y_i = \alpha + \beta x_i + \varepsilon_i$,

$$b = \frac{S_{xy}}{S_{xx}}, \quad b \sim \mathrm{N}\left(\beta, \frac{\sigma^2}{S_{xx}}\right), \quad \frac{b - \beta}{\sqrt{\hat{\sigma}^2 / S_{xx}}} \sim t_{n-2}$$

$$a = \bar{y} - b\bar{x}, \quad a \sim \mathrm{N}\left(\alpha, \frac{\sigma^2 \sum x_i^2}{n\,S_{xx}}\right)$$

$$a + bx_0 \sim \mathrm{N}\left(\alpha + \beta x_0, \sigma^2\left\{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right\}\right)$$

$$\mathrm{RSS} = S_{yy} - \frac{(S_{xy})^2}{S_{xx}} = S_{yy}(1 - r^2)$$

Randomised response technique

$$\mathrm{E}(\hat{p}) = \frac{\frac{y}{n} - (1 - \theta)}{(2\theta - 1)} \qquad \mathrm{Var}(\hat{p}) = \frac{[(2\theta - 1)\,p + (1 - \theta)][\theta - (2\theta - 1)p]}{n(2\theta - 1)^2}$$

Factorial design

Interaction between 1st and 2nd of 3 treatments

$$(-)\left\{\frac{(Abc - abc) + (AbC - abC)}{2} - \frac{(ABc - aBc) + (ABC - aBC)}{2}\right\}$$

Exponential smoothing

$$\hat{y}_{n+1} = \alpha y_n + \alpha(1 - \alpha)y_{n-1} + \alpha(1 - \alpha)^2 y_{n-2} + \ldots + \alpha(1 - \alpha)^{n-1} y_1 + (1 - \alpha)^n y_0$$

$$\hat{y}_{n+1} = \hat{y}_n + \alpha(y_n - \hat{y}_n)$$

$$\hat{y}_{n+1} = \alpha y_n + (1 - \alpha)\,\hat{y}_n$$

Description	Test statistic	Distribution
Pearson's product moment correlation test	$r = \dfrac{\dfrac{\sum x_i y_i}{n} - \bar{x}\,\bar{y}}{\sqrt{\left[\left(\dfrac{\sum x_i^2}{n} - \bar{x}^2\right)\left(\dfrac{\sum y_i^2}{n} - \bar{y}^2\right)\right]}}$	
Spearman rank correlation test	$r_s = 1 - \dfrac{6\sum d_i^2}{n(n^2-1)}$	
Normal test for a mean	$\dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	N(0, 1)
t-test for a mean	$\dfrac{\bar{x} - \mu}{s/\sqrt{n}}$	t_{n-1}
χ^2 test	$\sum \dfrac{(f_o - f_e)^2}{f_e}$	χ^2_v
t-test for paired sample	$\dfrac{(\bar{x}_1 - \bar{x}_2) - \mu}{s/\sqrt{n}}$	t with $(n-1)$ degrees of freedom
Normal test for the difference in the means of 2 samples with different variances	$\dfrac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	N(0, 1)

Description	Test statistic	Distribution
t-test for the difference in the means of 2 samples	$\dfrac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{s\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ where $s^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$	$t_{n_1+n_2-2}$
Wilcoxon single sample test	A statistic T is calculated from the ranked data.	See tables
Wilcoxon Rank-sum (or Mann-Whitney) 2-Sample test	Samples size m, n: $m \leqslant$ n Wilcoxon W = sum of ranks of sample size m Mann-Whitney $T = W - \frac{1}{2}m(m+1)$	See tables
Normal test on binomial proportion	$\dfrac{p - \theta}{\sqrt{\left(\dfrac{\theta(1-\theta)}{n}\right)}}$	N(0, 1)
χ^2 test for variance	$\dfrac{(n-1)s^2}{\sigma^2}$	χ^2_{n-1}
F-test on ratio of two variances	$\dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}$, $s_1^2 > s_2^2$	$F_{n_1-1,\,n_2-1}$

Name	Function	Mean	Variance	p.g.f. G(t) (discrete) m.g.f. M(θ) (continuous)
Binomial B(n, p) *Discrete*	$P(X = r) = {}^nC_r\, q^{n-r} p^r$, for $r = 0, 1, \dots, n$, $0 < p < 1$, $q = 1 - p$	np	npq	$G(t) = (q + pt)^n$
Poisson (λ) *Discrete*	$P(X = r) = e^{-\lambda} \dfrac{\lambda^r}{r!}$, for $r = 0, 1, \dots$, $\lambda > 0$	λ	λ	$G(t) = e^{\lambda(t-1)}$
Normal N(μ, σ^2) *Continuous*	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right)$, $-\infty < x < \infty$	μ	σ^2	$M(\theta) = \exp(\mu\theta + \frac{1}{2}\sigma^2\theta^2)$
Uniform (Rectangular) on [a, b] *Continuous*	$f(x) = \dfrac{1}{b-a}$, $a \leqslant x \leqslant b$	$\dfrac{a+b}{2}$	$\frac{1}{12}(b-a)^2$	$M(\theta) = \dfrac{e^{b\theta} - e^{a\theta}}{(b-a)\theta}$
Exponential *Continuous*	$f(x) = \lambda e^{-\lambda x}$, $x \geqslant 0$, $\lambda > 0$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$	$M(\theta) = \dfrac{\lambda}{\lambda - \theta}$
Geometric *Discrete*	$P(X = r) = q^{r-1}p$, $r = 1, 2, \dots$, $0 < p < 1$, $q = 1 - p$	$\dfrac{1}{p}$	$\dfrac{q}{p^2}$	$G(t) = \dfrac{pt}{1 - qt}$
Negative binomial *Discrete*	$P(X = r) = {}^{r-1}C_{n-1}\, q^{r-n} p^n$, $r = n, n+1, \dots$, $0 < p < 1$, $q = 1 - p$	$\dfrac{n}{p}$	$\dfrac{nq}{p^2}$	$G(t) = \left(\dfrac{pt}{1 - qt}\right)^n$

Numerical Solution of Equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Numerical integration

The trapezium rule

$$\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$

The mid-ordinate rule

$$\int_a^b y\,\mathrm{d}x \approx h(y_{\frac{1}{2}} + y_{1\frac{1}{2}} + \ldots + y_{n-1\frac{1}{2}} + y_{n-\frac{1}{2}}), \text{ where } h = \frac{b-a}{n}$$

Simpson's rule

for n even

$$\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{3}h\{(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\},$$

where $h = \dfrac{b-a}{n}$

The Gaussian 2-point integration rule

$$\int_{-h}^{h} \mathrm{f}(x)\,\mathrm{d}x \approx h\left[\mathrm{f}\left(\frac{-h}{\sqrt{3}}\right) + \mathrm{f}\left(\frac{h}{\sqrt{3}}\right)\right]$$

Interpolation/finite differences

Lagrange's polynomial : $\mathrm{P}_n(x) = \sum \mathrm{L}_r(x)\mathrm{f}(x_r)$ where $\mathrm{L}_r(x) = \prod_{\substack{i=0 \\ i \neq r}}^{n} \dfrac{x - x_i}{x_r - x_i}$

Newton's forward difference interpolation formula

$$\mathrm{f}(x) = \mathrm{f}(x_0) + \frac{(x - x_0)}{h}\Delta\mathrm{f}(x_0) + \frac{(x - x_0)(x - x_1)}{2!h^2}\Delta^2\mathrm{f}(x_0) + \ldots$$

Newton's divided difference interpolation formula

$$\mathrm{f}(x) = \mathrm{f}[x_0] + (x - x_0)\mathrm{f}[x_0, x_1] + (x - x_0)(x - x_1)\mathrm{f}[x_0, x_1, x_2] + \ldots$$

Numerical differentiation

$$\mathrm{f}''(x) \approx \frac{\mathrm{f}(x+h) - 2\mathrm{f}(x) + \mathrm{f}(x-h)}{h^2}$$

Taylor polynomials

$$\mathrm{f}(a + h) = \mathrm{f}(a) + h\mathrm{f}'(a) + \frac{h^2}{2!}\mathrm{f}''(a) + \text{error}$$

$$\mathrm{f}(a + h) = \mathrm{f}(a) + h\mathrm{f}'(a) + \frac{h^2}{2!}\mathrm{f}''(a + \xi),\ 0 < \xi < h$$

$$\mathrm{f}(x) = \mathrm{f}(a) + (x - a)\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(a) + \text{error}$$

$$\mathrm{f}(x) = \mathrm{f}(a) + (x - a)\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(\eta),\ a < \eta < x$$

Numerical solution of differential equations

For $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$:

Euler's method : $y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$; $x_{r+1} = x_r + h$

Runge-Kutta method (order 2) (modified Euler method)

$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$

where $k_1 = h\,\mathrm{f}(x_r, y_r)$, $k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$

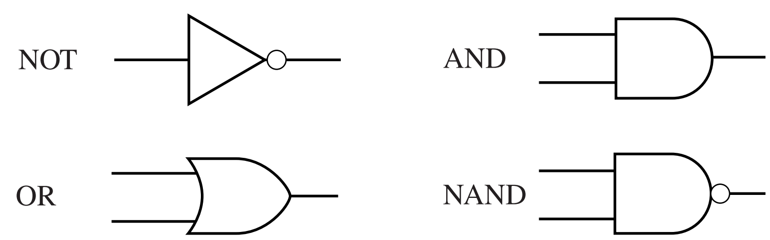
Runge-Kutta method, order 4:

$$y_{r+1} = y_r + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ $k_2 = h\mathrm{f}(x_r + \tfrac{1}{2}h, y_r + \tfrac{1}{2}k_1)$

$k_3 = h\mathrm{f}(x_r + \tfrac{1}{2}h, y_r + \tfrac{1}{2}k_2)$ $k_4 = h\mathrm{f}(x_r + h, y_r + k_3)$.

Logic gates



Statistical Tables

The Binomial distribution: cumulative probabilites

$$P(X \leqslant x) = \sum_{r=0}^{x} {}^{n}C_{r} (1-p)^{n-r} p^{r}$$

n	x \ p	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950
1	0	0.9500	0.9000	0.8500	0.8333	0.8000	0.7500	0.7000	0.6667	0.6500	0.6000	0.5500	0.5000	0.4500	0.4000	0.3500	0.3333	0.3000	0.2500	0.2000	0.1667	0.1500	0.1000	0.0500
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0	0.9025	0.8100	0.7225	0.6944	0.6400	0.5625	0.4900	0.4444	0.4225	0.3600	0.3025	0.2500	0.2025	0.1600	0.1225	0.1111	0.0900	0.0625	0.0400	0.0278	0.0225	0.0100	0.0025
	1	0.9975	0.9900	0.9775	0.9722	0.9600	0.9375	0.9100	0.8889	0.8775	0.8400	0.7975	0.7500	0.6975	0.6400	0.5775	0.5556	0.5100	0.4375	0.3600	0.3056	0.2775	0.1900	0.0975
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	0.8574	0.7290	0.6141	0.5787	0.5120	0.4219	0.3430	0.2963	0.2746	0.2160	0.1664	0.1250	0.0911	0.0640	0.0429	0.0370	0.0270	0.0156	0.0080	0.0046	0.0034	0.0010	0.0001
	1	0.9928	0.9720	0.9392	0.9259	0.8960	0.8437	0.7840	0.7407	0.7183	0.6480	0.5748	0.5000	0.4252	0.3520	0.2818	0.2593	0.2160	0.1563	0.1040	0.0741	0.0608	0.0280	0.0073
	2	0.9999	0.9990	0.9966	0.9954	0.9920	0.9844	0.9730	0.9630	0.9571	0.9360	0.9089	0.8750	0.8336	0.7840	0.7254	0.7037	0.6570	0.5781	0.4880	0.4213	0.3859	0.2710	0.1426
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	0.8145	0.6561	0.5220	0.4823	0.4096	0.3164	0.2401	0.1975	0.1785	0.1296	0.0915	0.0625	0.0410	0.0256	0.0150	0.0123	0.0081	0.0039	0.0016	0.0008	0.0005	0.0001	0.0000
	1	0.9860	0.9477	0.8905	0.8681	0.8192	0.7383	0.6517	0.5926	0.5630	0.4752	0.3910	0.3125	0.2415	0.1792	0.1265	0.1111	0.0837	0.0508	0.0272	0.0162	0.0120	0.0037	0.0005
	2	0.9995	0.9963	0.9880	0.9838	0.9728	0.9492	0.9163	0.8889	0.8735	0.8208	0.7585	0.6875	0.6090	0.5248	0.4370	0.4074	0.3483	0.2617	0.1808	0.1319	0.1095	0.0523	0.0140
	3	1.0000	0.9999	0.9995	0.9992	0.9984	0.9961	0.9919	0.9877	0.9850	0.9744	0.9590	0.9375	0.9085	0.8704	0.8215	0.8025	0.7599	0.6836	0.5904	0.5177	0.4780	0.3439	0.1855
	4		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0041	0.0024	0.0010	0.0003	0.0001	0.0001	0.0000	
	1	0.9774	0.9185	0.8352	0.8038	0.7373	0.6328	0.5282	0.4609	0.4284	0.3370	0.2562	0.1875	0.1312	0.0870	0.0540	0.0453	0.0308	0.0156	0.0067	0.0033	0.0022	0.0005	0.0000
	2	0.9988	0.9914	0.9734	0.9645	0.9421	0.8965	0.8369	0.7901	0.7648	0.6826	0.5931	0.5000	0.4069	0.3174	0.2352	0.2099	0.1631	0.1035	0.0579	0.0355	0.0266	0.0086	0.0012
	3	1.0000	0.9995	0.9978	0.9967	0.9933	0.9844	0.9692	0.9547	0.9460	0.9130	0.8688	0.8125	0.7438	0.6630	0.5716	0.5391	0.4718	0.3672	0.2627	0.1962	0.1648	0.0815	0.0226
	4		1.0000	0.9999	0.9999	0.9997	0.9990	0.9976	0.9959	0.9947	0.9898	0.9815	0.9688	0.9497	0.9222	0.8840	0.8683	0.8319	0.7627	0.6723	0.5981	0.5563	0.4095	0.2262
	5			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0014	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	
	1	0.9672	0.8857	0.7765	0.7368	0.6554	0.5339	0.4202	0.3512	0.3191	0.2333	0.1636	0.1094	0.0692	0.0410	0.0223	0.0178	0.0109	0.0046	0.0016	0.0007	0.0004	0.0001	0.0000
	2	0.9978	0.9841	0.9527	0.9377	0.9011	0.8306	0.7443	0.6804	0.6471	0.5443	0.4415	0.3438	0.2553	0.1792	0.1174	0.1001	0.0705	0.0376	0.0170	0.0087	0.0059	0.0013	0.0001
	3	0.9999	0.9987	0.9941	0.9913	0.9830	0.9624	0.9295	0.8999	0.8826	0.8208	0.7447	0.6563	0.5585	0.4557	0.3529	0.3196	0.2557	0.1694	0.0989	0.0623	0.0473	0.0159	0.0022
	4	1.0000	0.9999	0.9996	0.9993	0.9984	0.9954	0.9891	0.9822	0.9777	0.9590	0.9308	0.8906	0.8364	0.7667	0.6809	0.6488	0.5798	0.4661	0.3446	0.2632	0.2235	0.1143	0.0328
	5		1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9986	0.9982	0.9959	0.9917	0.9844	0.9723	0.9533	0.9246	0.9122	0.8824	0.8220	0.7379	0.6651	0.6229	0.4686	0.2649
	6					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000		
	1	0.9556	0.8503	0.7166	0.6698	0.5767	0.4449	0.3294	0.2634	0.2338	0.1586	0.1024	0.0625	0.0357	0.0188	0.0090	0.0069	0.0038	0.0013	0.0004	0.0001	0.0001	0.0000	
	2	0.9962	0.9743	0.9262	0.9042	0.8520	0.7564	0.6471	0.5706	0.5323	0.4199	0.3164	0.2266	0.1529	0.0963	0.0556	0.0453	0.0288	0.0129	0.0047	0.0020	0.0012	0.0002	0.0000
	3	0.9998	0.9973	0.9879	0.9824	0.9667	0.9294	0.8740	0.8267	0.8002	0.7102	0.6083	0.5000	0.3917	0.2898	0.1998	0.1733	0.1260	0.0706	0.0333	0.0176	0.0121	0.0027	0.0002
	4	1.0000	0.9998	0.9988	0.9980	0.9953	0.9871	0.9712	0.9547	0.9444	0.9037	0.8471	0.7734	0.6836	0.5801	0.4677	0.4294	0.3529	0.2436	0.1480	0.0958	0.0738	0.0257	0.0038
	5		1.0000	0.9999	0.9999	0.9996	0.9987	0.9962	0.9931	0.9910	0.9812	0.9643	0.9375	0.8976	0.8414	0.7662	0.7366	0.6706	0.5551	0.4233	0.3302	0.2834	0.1497	0.0444
	6			1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9994	0.9984	0.9963	0.9922	0.9848	0.9720	0.9510	0.9415	0.9176	0.8665	0.7903	0.7209	0.6794	0.5217	0.3017
	7				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	x \ p	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950
8	0	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0002	0.0001	0.0000	0.0000				
	1	0.9428	0.8131	0.6572	0.6047	0.5033	0.3671	0.2553	0.1951	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0026	0.0013	0.0004	0.0001	0.0000	0.0000		
	2	0.9942	0.9619	0.8948	0.8652	0.7969	0.6785	0.5518	0.4682	0.4278	0.3154	0.2201	0.1445	0.0885	0.0498	0.0253	0.0197	0.0113	0.0042	0.0012	0.0004	0.0002	0.0000	
	3	0.9996	0.9950	0.9786	0.9693	0.9437	0.8862	0.8059	0.7414	0.7064	0.5941	0.4770	0.3633	0.2604	0.1737	0.1061	0.0879	0.0580	0.0273	0.0104	0.0046	0.0029	0.0004	0.0000
	4	1.0000	0.9996	0.9971	0.9954	0.9896	0.9727	0.9420	0.9121	0.8939	0.8263	0.7396	0.6367	0.5230	0.4059	0.2936	0.2587	0.1941	0.1138	0.0563	0.0307	0.0214	0.0050	0.0004
	5		1.0000	0.9998	0.9996	0.9988	0.9958	0.9887	0.9803	0.9747	0.9502	0.9115	0.8555	0.7799	0.6846	0.5722	0.5318	0.4482	0.3215	0.2031	0.1348	0.1052	0.0381	0.0058
	6			1.0000	1.0000	0.9999	0.9996	0.9987	0.9974	0.9964	0.9915	0.9819	0.9648	0.9368	0.8936	0.8309	0.8049	0.7447	0.6329	0.4967	0.3953	0.3428	0.1869	0.0572
	7					1.0000	1.0000	0.9999	0.9998	0.9998	0.9993	0.9983	0.9961	0.9916	0.9832	0.9681	0.9610	0.9424	0.8999	0.8322	0.7674	0.7275	0.5695	0.3366
	8							1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020	0.0008	0.0003	0.0001	0.0001	0.0000	00000					
	1	0.9288	0.7748	0.5995	0.5427	0.4362	0.3003	0.1960	0.1431	0.1211	0.0705	0.0385	0.0195	0.0091	0.0038	0.0014	0.0010	0.0004	0.0001	0.0000	0.0000			
	2	0.9916	0.9470	0.8591	0.8217	0.7382	0.6007	0.4628	0.3772	0.3373	0.2318	0.1495	0.0898	0.0498	0.0250	0.0112	0.0083	0.0043	0.0013	0.0003	0.0001	0.0000	0.0000	
	3	0.9994	0.9917	0.9661	0.9520	0.9144	0.8343	0.7297	0.6503	0.6089	0.4826	0.3614	0.2539	0.1658	0.0994	0.0536	0.0424	0.0253	0.0100	0.0031	0.0011	0.0006	0.0001	
	4	1.0000	0.9991	0.9944	0.9911	0.9804	0.9511	0.9012	0.8552	0.8283	0.7334	0.6214	0.5000	0.3786	0.2666	0.1717	0.1448	0.0988	0.0489	0.0196	0.0090	0.0056	0.0009	0.0000
	5		0.9999	0.9994	0.9989	0.9969	0.9900	0.9747	0.9576	0.9464	0.9006	0.8342	0.7461	0.6386	0.5174	0.3911	0.3497	0.2703	0.1657	0.0856	0.0480	0.0339	0.0083	0.0006
	6		1.0000	1.0000	0.9999	0.9997	0.9987	0.9957	0.9917	0.9888	0.9750	0.9502	0.9102	0.8505	0.7682	0.6627	0.6228	0.5372	0.3993	0.2618	0.1783	0.1409	0.0530	0.0084
	7				1.0000	1.0000	0.9999	0.9996	0.9990	0.9986	0.9962	0.9909	0.9805	0.9615	0.9295	0.8789	0.8569	0.8040	0.6997	0.5638	0.4573	0.4005	0.2252	0.0712
	8						1.0000	1.0000	0.9999	0.9999	0.9997	0.9992	0.9980	09954	0.9899	0.9793	0.9740	0.9596	0.9249	0.8658	0.8062	0.7684	0.6126	0.3698
	9								1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000						
	1	0.9139	0.7361	0.5443	0.4845	0.3758	0.2440	0.1493	0.1040	0.0860	0.0464	0.0233	0.0107	0.0045	0.0017	0.0005	0.0004	0.0001	0.0000	0.0000				
	2	0.9885	0.9298	0.8202	0.7752	0.6778	0.5256	0.3828	0.2991	0.2616	0.1673	0.0996	0.0547	0.0274	0.0123	0.0048	0.0034	0.0016	0.0004	0.0001	0.0000	0.0000		
	3	0.9990	0.9872	0.9500	0.9303	0.8791	0.7759	0.6496	0.5593	0.5138	0.3823	0.2660	0.1719	0.1020	0.0548	0.0260	0.0197	0.0106	0.0035	0.0009	0.0003	0.0001	0.0000	
	4	0.9999	0.9984	0.9901	0.9845	0.9672	0.9219	0.8497	0.7869	0.7515	0.6331	0.5044	0.3770	0.2616	0.1662	0.0949	0.0766	0.0473	0.0197	0.0064	0.0024	0.0014	0.0001	0.0000
	5	1.0000	0.9999	0.9986	0.9976	0.9936	0.9803	0.9527	0.9234	0.9051	0.8338	0.7384	0.6230	0.4956	0.3669	0.2485	0.2131	0.1503	0.0781	0.0328	0.0155	0.0099	0.0016	0.0001
	6		1.0000	0.9999	0.9997	0.9991	0.9965	0.9894	0.9803	0.9740	0.9452	0.8980	0.8281	0.7340	0.6177	0.4862	0.4407	0.3504	0.2241	0.1209	0.0697	0.0500	0.0128	0.0010
	7			1.0000	1.0000	0.9999	0.9996	0.9984	0.9966	0.9952	0.9877	0.9726	0.9453	0.9004	0.8327	0.7384	0.7009	0.6172	0.4744	0.3222	0.2248	0.1798	0.0702	0.0115
	8					1.0000	1.0000	0.9999	0.9996	0.9995	0.9983	0.9955	0.9893	0.9767	0.9536	0.9140	0.8960	0.8507	0.7560	0.6242	0.5155	0.4557	0.2639	0.0861
	9							1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9975	0.9940	0.9865	0.9827	0.9718	0.9437	0.8926	0.8385	0.8031	0.6513	0.4013
	10										1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	0.5688	0.3138	0.1673	0.1346	0.0859	0.0422	0.0198	0.0116	0.0088	0.0036	0.0014	0.0005	0.0002	0.0000	0.0000	0.0000							
	1	0.8981	0.6974	0.4922	0.4307	0.3221	0.1971	0.1130	0.0751	0.0606	0.0302	0.0139	0.0059	0.0022	0.0007	0.0002	0.0001	0.0000	0.0000					
	2	0.9848	0.9104	0.7788	0.7268	0.6174	0.4552	0.3127	0.2341	0.2001	0.1189	0.0652	0.0327	0.0148	0.0059	0.0020	0.0014	0.0006	0.0001	0.0000	0.0000			
	3	0.9984	0.9815	0.9306	0.9044	0.8389	0.7133	0.5696	0.4726	0.4256	0.2963	0.1911	0.1133	0.0610	0.0293	0.0122	0.0088	0.0043	0.0012	0.0002	0.0001	0.0000		
	4	0.9999	0.9972	0.9841	0.9755	0.9496	0.8854	0.7897	0.7110	0.6683	0.5328	0.3971	0.2744	0.1738	0.0994	0.0501	0.0386	0.0216	0.0076	0.0020	0.0006	0.0003	0.0000	
	5	1.0000	0.9997	0.9973	0.9954	0.9883	0.9657	0.9218	0.8779	0.8513	0.7535	0.6331	0.5000	0.3669	0.2465	0.1487	0.1221	0.0782	0.0343	0.0117	0.0046	0.0027	0.0003	0.0000
	6		1.0000	0.9997	0.9994	0.9980	0.9924	0.9784	0.9614	0.9499	0.9006	0.8262	0.7256	0.6029	0.4672	0.3317	0.2890	0.2103	0.1146	0.0504	0.0245	0.0159	0.0028	0.0001
	7			1.0000	0.9999	0.9998	0.9988	0.9957	0.9912	0.9878	0.9707	0.9390	0.8867	0.8089	0.7037	0.5744	0.5274	0.4304	0.2867	0.1611	0.0956	0.0694	0.0185	0.0016
	8				1.0000	1.0000	0.9999	0.9994	0.9986	0.9980	0.9941	0.9852	0.9673	0.9348	0.8811	0.7999	0.7659	0.6873	0.5448	0.3826	0.2732	0.2212	0.0896	0.0152
	9						1.0000	1.0000	0.9999	0.9998	0.9993	0.9978	0.9941	0.9861	0.9698	0.9394	0.9249	0.8870	0.8029	0.6779	0.5693	0.5078	0.3026	0.1019
	10								1.0000	1.0000	1.0000	0.9998	0.9995	0.9986	0.9964	0.9912	0.9884	0.9802	0.9578	0.9141	0.8654	0.8327	0.6862	0.4312
	11											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITY

n	x \ p	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950
12	0	0.5404	0.2824	0.1422	0.1122	0.0687	0.0317	0.0138	0.0077	0.0057	0.0022	0.0008	0.0002	0.0001	0.0000	0.0000								
	1	0.8816	0.6590	0.4435	0.3813	0.2749	0.1584	0.0850	0.0540	0.0424	0.0196	0.0083	0.0032	0.0011	0.0003	0.0001	0.0000	0.0000						
	2	0.9804	0.8891	0.7358	0.6774	0.5583	0.3907	0.2528	0.1811	0.1513	0.0834	0.0421	0.0193	0.0079	0.0028	0.0008	0.0005	0.0002	0.0000	0.0000				
	3	0.9978	0.9744	0.9078	0.8748	0.7946	0.6488	0.4925	0.3931	0.3467	0.2253	0.1345	0.0730	0.0356	0.0153	0.0056	0.0039	0.0017	0.0004	0.0001	0.0000	0.0000		
	4	0.9998	0.9957	0.9761	0.9637	0.9274	0.8424	0.7237	0.6315	0.5833	0.4382	0.3044	0.1938	0.1117	0.0573	0.0255	0.0188	0.0095	0.0028	0.0006	0.0002	0.0001	0.0000	
	5	1.0000	0.9995	0.9954	0.9921	0.9806	0.9456	0.8822	0.8223	0.7873	0.6652	0.5269	0.3872	0.2607	0.1582	0.0846	0.0664	0.0386	0.0143	0.0039	0.0013	0.0007	0.0001	
	6		0.9999	0.9993	0.9987	0.9961	0.9857	0.9614	0.9336	0.9154	0.8418	0.7393	0.6128	0.4731	0.3348	0.2127	0.1777	0.1178	0.0544	0.0194	0.0079	0.0046	0.0005	0.0000
	7		1.0000	0.9999	0.9998	0.9994	0.9972	0.9905	0.9812	0.9745	0.9427	0.8883	0.8062	0.6956	0.5618	0.4167	0.3685	0.2763	0.1576	0.0726	0.0364	0.0239	0.0043	0.0002
	8			1.0000	1.0000	0.9999	0.9996	0.9983	0.9961	0.9944	0.9847	0.9644	0.9270	0.8655	0.7747	0.6533	0.6069	0.5075	0.3512	0.2054	0.1252	0.0922	0.0256	0.0022
	9					1.0000	1.0000	0.9998	0.9995	0.9992	0.9972	0.9921	0.9807	0.9579	0.9166	0.8487	0.8189	0.7472	0.6093	0.4417	0.3226	0.2642	0.1109	0.0196
	10							1.0000	1.0000	0.9999	0.9997	0.9989	0.9968	0.9917	0.9804	0.9576	0.9460	0.9150	0.8416	0.7251	0.6187	0.5565	0.3410	0.1184
	11									1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.9943	0.9923	0.9862	0.9683	0.9313	0.8878	0.8578	0.7176	0.4596
	12											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	0	0.5133	0.2542	0.1209	0.0935	0.0550	0.0238	0.0097	0.0051	0.0037	0.0013	0.0004	0.0001	0.0000	0.0000									
	1	0.8646	0.6213	0.3983	0.3365	0.2336	0.1267	0.0637	0.0385	0.0296	0.0126	0.0049	0.0017	0.0005	0.0001	0.0000	0.0000	0.0000						
	2	0.9755	0.8661	0.6920	0.6281	0.5017	0.3326	0.2025	0.1387	0.1132	0.0579	0.0269	0.0112	0.0041	0.0013	0.0003	0.0002	0.0001	0.0000					
	3	0.9969	0.9658	0.8820	0.8419	0.7473	0.5843	0.4206	0.3224	0.2783	0.1686	0.0929	0.0461	0.0203	0.0078	0.0025	0.0016	0.0007	0.0001	0.0000				
	4	0.9997	0.9935	0.9658	0.9488	0.9009	0.7940	0.6543	0.5520	0.5005	0.3530	0.2279	0.1334	0.0698	0.0321	0.0126	0.0088	0.0040	0.0010	0.0002	0.0000	0.0000		
	5	1.0000	0.9991	0.9925	0.9873	0.9700	0.9198	0.8346	0.7587	0.7159	0.5744	0.4268	0.2905	0.1788	0.0977	0.0462	0.0347	0.0182	0.0056	0.0012	0.0003	0.0002	0.0000	
	6		0.9999	0.9987	0.9976	0.9930	0.9757	0.9376	0.8965	0.8705	0.7712	0.6437	0.5000	0.3563	0.2288	0.1295	0.1035	0.0624	0.0243	0.0070	0.0024	0.0013	0.0001	
	7		1.0000	0.9998	0.9997	0.9988	0.9944	0.9818	0.9653	0.9538	0.9023	0.8212	0.7095	0.5732	0.4256	0.2841	0.2413	0.1654	0.0802	0.0300	0.0127	0.0075	0.0009	0.0000
	8			1.0000	1.0000	0.9998	0.9990	0.9960	0.9912	0.9874	0.9679	0.9302	0.8666	0.7721	0.6470	0.4995	0.4480	0.3457	0.2060	0.0991	0.0512	0.0342	0.0065	0.0003
	9					1.0000	0.9999	0.9993	0.9984	0.9975	0.9922	0.9797	0.9539	0.9071	0.8314	0.7217	0.6776	0.5794	0.4157	0.2527	0.1581	0.1180	0.0342	0.0031
	10						1.0000	0.9999	0.9998	0.9997	0.9987	0.9959	0.9888	0.9731	0.9421	0.8868	0.8613	0.7975	0.6674	0.4983	0.3719	0.3080	0.1339	0.0245
	11							1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9951	0.9874	0.9704	0.9615	0.9363	0.8733	0.7664	0.6635	0.6017	0.3787	0.1354
	12										1.0000	1.0000	0.9999	0.9996	0.9987	0.9963	0.9949	0.9903	0.9762	0.9450	0.9065	0.8791	0.7458	0.4867
	13												1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	0	0.4877	0.2288	0.1028	0.0779	0.0440	0.0178	0.0068	0.0034	0.0024	0.0008	0.0002	0.0001	0.0000	0.0000									
	1	0.8470	0.5846	0.3567	0.2960	0.1979	0.1010	0.0475	0.0274	0.0205	0.0081	0.0029	0.0009	0.0003	0.0001	0.0000	0.0000							
	2	0.9699	0.8416	0.6479	0.5795	0.4481	0.2811	0.1608	0.1053	0.0839	0.0398	0.0170	0.0065	0.0022	0.0006	0.0001	0.0001	0.0000						
	3	0.9958	0.9559	0.8535	0.8063	0.6982	0.5213	0.3552	0.2612	0.2205	0.1243	0.0632	0.0287	0.0114	0.0039	0.0011	0.0007	0.0002	0.0000					
	4	0.9996	0.9908	0.9533	0.9310	0.8702	0.7415	0.5842	0.4755	0.4227	0.2793	0.1672	0.0898	0.0426	0.0175	0.0060	0.0040	0.0017	0.0003	0.0000	0.0000			
	5	1.0000	0.9985	0.9885	0.9809	0.9561	0.8883	0.7805	0.6898	0.6405	0.4859	0.3373	0.2120	0.1189	0.0583	0.0243	0.0174	0.0083	0.0022	0.0004	0.0001	0.0000		
	6		0.9998	0.9978	0.9959	0.9884	0.9617	0.9067	0.8505	0.8164	0.6925	0.5461	0.3953	0.2586	0.1501	0.0753	0.0576	0.0315	0.0103	0.0024	0.0007	0.0003	0.0000	
	7		1.0000	0.9997	0.9993	0.9976	0.9897	0.9685	0.9424	0.9247	0.8499	0.7414	0.6047	0.4539	0.3075	0.1836	0.1495	0.0933	0.0383	0.0116	0.0041	0.0022	0.0002	
	8			1.0000	0.9999	0.9996	0.9978	0.9917	0.9826	0.9757	0.9417	0.8811	0.7880	0.6627	0.5141	0.3595	0.3102	0.2195	0.1117	0.0439	0.0191	0.0115	0.0015	0.0000
	9				1.0000	1.0000	0.9997	0.9983	0.9960	0.9940	0.9825	0.9574	0.9102	0.8328	0.7207	0.5773	0.5245	0.4158	0.2585	0.1298	0.0690	0.0467	0.0092	0.0004
	10						1.0000	0.9998	0.9993	0.9989	0.9961	0.9886	0.9713	0.9368	0.8757	0.7795	0.7388	0.6448	0.4787	0.3018	0.1937	0.1465	0.0441	0.0042
	11							1.0000	0.9999	0.9999	0.9994	0.9978	0.9935	0.9830	0.9602	0.9161	0.8947	0.8392	0.7189	0.5519	0.4205	0.3521	0.1584	0.0301
	12								1.0000	1.0000	0.9999	0.9997	0.9991	0.9971	0.9919	0.9795	0.9726	0.9525	0.8990	0.8021	0.7040	0.6433	0.4154	0.1530
	13										1.0000	1.0000	0.9999	0.9998	0.9992	0.9976	0.9966	0.9932	0.9822	0.9560	0.9221	0.8972	0.7712	0.5123
	14												1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITY

n	x \ p	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950
15	0	0.4633	0.2059	0.0874	0.0649	0.0352	0.0134	0.0047	0.0023	0.0016	0.0005	0.0001	0.0000	0.0000										
	1	0.8290	0.5490	0.3186	0.2596	0.1671	0.0802	0.0353	0.0194	0.0142	0.0052	0.0017	0.0005	0.0001	0.0000	0.0000								
	2	0.9638	0.8159	0.6042	0.5322	0.3980	0.2361	0.1268	0.0794	0.0617	0.0271	0.0107	0.0037	0.0011	0.0003	0.0001	0.0000	0.0000						
	3	0.9945	0.9444	0.8227	0.7685	0.6482	0.4613	0.2969	0.2092	0.1727	0.0905	0.0424	0.0176	0.0063	0.0019	0.0005	0.0003	0.0001	0.0000					
	4	0.9994	0.9873	0.9383	0.9102	0.8358	0.6865	0.5155	0.4041	0.3519	0.2173	0.1204	0.0592	0.0255	0.0093	0.0028	0.0018	0.0007	0.0001	0.0000				
	5	0.9999	0.9978	0.9832	0.9726	0.9389	0.8516	0.7216	0.6184	0.5643	0.4032	0.2608	0.1509	0.0769	0.0338	0.0124	0.0085	0.0037	0.0008	0.0001	0.0000	0.0000		
	6	1.0000	0.9997	0.9964	0.9934	0.9819	0.9434	0.8689	0.7970	0.7548	0.6098	0.4522	0.3036	0.1818	0.0950	0.0422	0.0308	0.0152	0.0042	0.0008	0.0002	0.0001		
	7		1.0000	0.9994	0.9987	0.9958	0.9827	0.9500	0.9118	0.8868	0.7869	0.6535	0.5000	0.3465	0.2131	0.1132	0.0882	0.0500	0.0173	0.0042	0.0013	0.0006	0.0000	
	8			0.9999	0.9998	0.9992	0.9958	0.9848	0.9692	0.9578	0.9050	0.8182	0.6964	0.5478	0.3902	0.2452	0.2030	0.1311	0.0566	0.0181	0.0066	0.0036	0.0003	0.0000
	9			1.0000	1.0000	0.9999	0.9992	0.9963	0.9915	0.9876	0.9662	0.9231	0.8491	0.7392	0.5968	0.4357	0.3816	0.2784	0.1484	0.0611	0.0274	0.0168	0.0022	0.0001
	10					1.0000	0.9999	0.9993	0.9982	0.9972	0.9907	0.9745	0.9408	0.8796	0.7827	0.6481	0.5959	0.4845	0.3135	0.1642	0.0898	0.0617	0.0127	0.0006
	11						1.0000	0.9999	0.9997	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7908	0.7031	0.5387	0.3518	0.2315	0.1773	0.0556	0.0055
	12							1.0000	1.0000	0.9999	0.9997	0.9989	0.9963	0.9893	0.9729	0.9383	0.9206	0.8732	0.7639	0.6020	0.4678	0.3958	0.1841	0.0362
	13									1.0000	1.0000	0.9999	0.9995	0.9983	0.9948	0.9858	0.9806	0.9647	0.9198	0.8329	0.7404	0.6814	0.4510	0.1710
	14											1.0000	1.0000	0.9999	0.9995	0.9984	0.9977	0.9953	0.9866	0.9648	0.9351	0.9126	0.7941	0.5367
	15													1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	0	0.4401	0.1853	0.0743	0.0541	0.0281	0.0100	0.0033	0.0015	0.0010	0.0003	0.0001	0.0000	0.0000										
	1	0.8108	0.5147	0.2839	0.2272	0.1407	0.0635	0.0261	0.0137	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000									
	2	0.9571	0.7892	0.5614	0.4868	0.3518	0.1971	0.0994	0.0594	0.0451	0.0183	0.0066	0.0021	0.0006	0.0001	0.0000	0.0000							
	3	0.9930	0.9316	0.7899	0.7291	0.5981	0.4050	0.2459	0.1659	0.1339	0.0651	0.0281	0.0106	0.0035	0.0009	0.0002	0.0001	0.0000						
	4	0.9991	0.9830	0.9209	0.8866	0.7982	0.6302	0.4499	0.3391	0.2892	0.1666	0.0853	0.0384	0.0149	0.0049	0.0013	0.0008	0.0003	0.0000					
	5	0.9999	0.9967	0.9765	0.9622	0.9183	0.8103	0.6598	0.5469	0.4900	0.3288	0.1976	0.1051	0.0486	0.0191	0.0062	0.0040	0.0016	0.0003	0.0000				
	6	1.0000	0.9995	0.9944	0.9899	0.9733	0.9204	0.8247	0.7374	0.6881	0.5272	0.3660	0.2272	0.1241	0.0583	0.0229	0.0159	0.0071	0.0016	0.0002	0.0000	0.0000		
	7		0.9999	0.9989	0.9979	0.9930	0.9729	0.9256	0.8735	0.8406	0.7161	0.5629	0.4018	0.2559	0.1423	0.0671	0.0500	0.0257	0.0075	0.0015	0.0004	0.0002	0.0000	
	8		1.0000	0.9998	0.9996	0.9985	0.9925	0.9743	0.9500	0.9329	0.8577	0.7441	0.5982	0.4371	0.2839	0.1594	0.1265	0.0744	0.0271	0.0070	0.0021	0.0011	0.0001	
	9			1.0000	1.0000	0.9998	0.9984	0.9929	0.9841	0.9771	0.9417	0.8759	0.7728	0.6340	0.4728	0.3119	0.2626	0.1753	0.0796	0.0267	0.0101	0.0056	0.0005	0.0000
	10					1.0000	0.9997	0.9984	0.9960	0.9938	0.9809	0.9514	0.8949	0.8024	0.6712	0.5100	0.4531	0.3402	0.1897	0.0817	0.0378	0.0235	0.0033	0.0001
	11						1.0000	0.9997	0.9992	0.9987	0.9951	0.9851	0.9616	0.9147	0.8334	0.7108	0.6609	0.5501	0.3698	0.2018	0.1134	0.0791	0.0170	0.0009
	12							1.0000	0.9999	0.9998	0.9991	0.9965	0.9894	0.9719	0.9349	0.8661	0.8341	0.7541	0.5950	0.4019	0.2709	0.2101	0.0684	0.0070
	13								1.0000	1.0000	0.9999	0.9994	0.9979	0.9934	0.9817	0.9549	0.9406	0.9006	0.8029	0.6482	0.5132	0.4386	0.2108	0.0429
	14										1.0000	0.9999	0.9997	0.9990	0.9967	0.9902	0.9863	0.9739	0.9365	0.8593	0.7728	0.7161	0.4853	0.1892
	15											1.0000	1.0000	0.9999	0.9997	0.9990	0.9985	0.9967	0.9900	0.9719	0.9459	0.9257	0.8147	0.5599
	16													1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	0	0.4181	0.1668	0.0631	0.0451	0.0225	0.0075	0.0023	0.0010	0.0007	0.0002	0.0000	0.0000											
	1	0.7922	0.4818	0.2525	0.1983	0.1182	0.0501	0.0193	0.0096	0.0067	0.0021	0.0006	0.0001	0.0000	0.0000									
	2	0.9497	0.7618	0.5198	0.4435	0.3096	0.1637	0.0774	0.0442	0.0327	0.0123	0.0041	0.0012	0.0003	0.0001	0.0000								
	3	0.9912	0.9174	0.7556	0.6887	0.5489	0.3530	0.2019	0.1304	0.1028	0.0464	0.0184	0.0064	0.0019	0.0005	0.0001	0.0000	0.0000						
	4	0.9988	0.9779	0.9013	0.8604	0.7582	0.5739	0.3887	0.2814	0.2348	0.1260	0.0596	0.0245	0.0086	0.0025	0.0006	0.0003	0.0001	0.0000					
	5	0.9999	0.9953	0.9681	0.9496	0.8943	0.7653	0.5968	0.4777	0.4197	0.2639	0.1471	0.0717	0.0301	0.0106	0.0030	0.0019	0.0007	0.0001	0.0000				
	6	1.0000	0.9992	0.9917	0.9853	0.9623	0.8929	0.7752	0.6739	0.6188	0.4478	0.2902	0.1662	0.0826	0.0348	0.0120	0.0080	0.0032	0.0006	0.0001	0.0000			
	7		0.9999	0.9983	0.9965	0.9891	0.9598	0.8954	0.8281	0.7872	0.6405	0.4743	0.3145	0.1834	0.0919	0.0383	0.0273	0.0127	0.0031	0.0005	0.0001	0.0000		
	8		1.0000	0.9997	0.9993	0.9974	0.9876	0.9597	0.9245	0.9006	0.8011	0.6626	0.5000	0.3374	0.1989	0.0994	0.0755	0.0403	0.0124	0.0026	0.0007	0.0003	0.0000	
	9			1.0000	0.9999	0.9995	0.9969	0.9873	0.9727	0.9617	0.9081	0.8166	0.6855	0.5257	0.3595	0.2128	0.1719	0.1046	0.0402	0.0109	0.0035	0.0017	0.0001	
	10				1.0000	0.9999	0.9994	0.9968	0.9920	0.9880	0.9652	0.9174	0.8338	0.7098	0.5522	0.3812	0.3261	0.2248	0.1071	0.0377	0.0147	0.0083	0.0008	0.0000
	11					1.0000	0.9999	0.9993	0.9981	0.9970	0.9894	0.9699	0.9283	0.8529	0.7361	0.5803	0.5223	0.4032	0.2347	0.1057	0.0504	0.0319	0.0047	0.0001
	12						1.0000	0.9999	0.9997	0.9994	0.9975	0.9914	0.9755	0.9404	0.8740	0.7652	0.7186	0.6113	0.4261	0.2418	0.1396	0.0987	0.0221	0.0012
	13							1.0000	1.0000	0.9999	0.9995	0.9981	0.9936	0.9816	0.9536	0.8972	0.8696	0.7981	0.6470	0.4511	0.3113	0.2444	0.0826	0.0088
	14									1.0000	0.9999	0.9997	0.9988	0.9959	0.9877	0.9673	0.9558	0.9226	0.8363	0.6904	0.5565	0.4802	0.2382	0.0503
	15										1.0000	1.0000	0.9999	0.9994	0.9979	0.9933	0.9904	0.9807	0.9499	0.8818	0.8017	0.7475	0.5182	0.2078
	16												1.0000	1.0000	0.9998	0.9993	0.9990	0.9977	0.9925	0.9775	0.9549	0.9369	0.8332	0.5819
	17														1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITY

n	x \ p	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950
18	0	0.3972	0.1501	0.0536	0.0376	0.0180	0.0056	0.0016	0.0007	0.0004	0.0001	0.0000	0.0000											
	1	0.7735	0.4503	0.2241	0.1728	0.0991	0.0395	0.0142	0.0068	0.0046	0.0013	0.0003	0.0001	0.0000										
	2	0.9419	0.7338	0.4794	0.4027	0.2713	0.1353	0.0600	0.0326	0.0236	0.0082	0.0025	0.0007	0.0001	0.0000									
	3	0.9891	0.9018	0.7202	0.6479	0.5010	0.3057	0.1646	0.1017	0.0783	0.0328	0.0120	0.0038	0.0010	0.0002	0.0000	0.0000							
	4	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000						
	5	0.9998	0.9936	0.9581	0.9347	0.8671	0.7175	0.5344	0.4122	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0009	0.0003	0.0000					
	6	1.0000	0.9988	0.9882	0.9794	0.9487	0.8610	0.7217	0.6085	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0039	0.0014	0.0002	0.0000				
	7		0.9998	0.9973	0.9947	0.9837	0.9431	0.8593	0.7767	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0144	0.0061	0.0012	0.0002	0.0000	0.0000		
	8		1.0000	0.9995	0.9989	0.9957	0.9807	0.9404	0.8924	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0433	0.0210	0.0054	0.0009	0.0002	0.0001		
	9			0.9999	0.9998	0.9991	0.9946	0.9790	0.9567	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.1076	0.0596	0.0193	0.0043	0.0011	0.0005	0.0000	
	10			1.0000	1.0000	0.9998	0.9988	0.9939	0.9856	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.2233	0.1407	0.0569	0.0163	0.0053	0.0027	0.0002	
	11					1.0000	0.9998	0.9986	0.9961	0.9938	0.9797	0.9463	0.8811	0.7742	0.6257	0.4509	0.3915	0.2783	0.1390	0.0513	0.0206	0.0118	0.0012	0.0000
	12						1.0000	0.9997	0.9991	0.9986	0.9942	0.9817	0.9519	0.8923	0.7912	0.6450	0.5878	0.4656	0.2825	0.1329	0.0653	0.0419	0.0064	0.0002
	13							1.0000	0.9999	0.9997	0.9987	0.9951	0.9846	0.9589	0.9058	0.8114	0.7689	0.6673	0.4813	0.2836	0.1682	0.1206	0.0282	0.0015
	14								1.0000	1.0000	0.9998	0.9990	0.9962	0.9880	0.9672	0.9217	0.8983	0.8354	0.6943	0.4990	0.3521	0.2798	0.0982	0.0109
	15										1.0000	0.9999	0.9993	0.9975	0.9918	0.9764	0.9674	0.9400	0.8647	0.7287	0.5973	0.5203	0.2662	0.0581
	16											1.0000	0.9999	0.9997	0.9987	0.9954	0.9932	0.9858	0.9605	0.9009	0.8272	0.7759	0.5497	0.2265
	17												1.0000	1.0000	0.9999	0.9996	0.9993	0.9984	0.9944	0.9820	0.9624	0.9464	0.8499	0.6028
	18														1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	0	0.3774	0.1351	0.0456	0.0313	0.0144	0.0042	0.0011	0.0005	0.0003	0.0001	0.0000												
	1	0.7547	0.4203	0.1985	0.1502	0.0829	0.0310	0.0104	0.0047	0.0031	0.0008	0.0002	0.0000	0.0000										
	2	0.9335	0.7054	0.4413	0.3643	0.2369	0.1113	0.0462	0.0240	0.0170	0.0055	0.0015	0.0004	0.0001	0.0000									
	3	0.9868	0.8850	0.6841	0.6070	0.4551	0.2631	0.1332	0.0787	0.0591	0.0230	0.0077	0.0022	0.0005	0.0001	0.0000	0.0000							
	4	0.9980	0.9648	0.8556	0.8011	0.6733	0.4654	0.2822	0.1879	0.1500	0.0696	0.0280	0.0096	0.0028	0.0006	0.0001	0.0001	0.0000						
	5	0.9998	0.9914	0.9463	0.9176	0.8369	0.6678	0.4739	0.3519	0.2968	0.1629	0.0777	0.0318	0.0109	0.0031	0.0007	0.0004	0.0001	0.0000					
	6	1.0000	0.9983	0.9837	0.9719	0.9324	0.8251	0.6655	0.5431	0.4812	0.3081	0.1727	0.0835	0.0342	0.0116	0.0031	0.0019	0.0006	0.0001					
	7		0.9997	0.9959	0.9921	0.9767	0.9225	0.8180	0.7207	0.6656	0.4878	0.3169	0.1796	0.0871	0.0352	0.0114	0.0074	0.0028	0.0005	0.0000	0.0000			
	8		1.0000	0.9992	0.9982	0.9933	0.9713	0.9161	0.8538	0.8145	0.6675	0.4940	0.3238	0.1841	0.0885	0.0347	0.0241	0.0105	0.0023	0.0003	0.0001	0.0000		
	9			0.9999	0.9996	0.9984	0.9911	0.9674	0.9352	0.9125	0.8139	0.6710	0.5000	0.3290	0.1861	0.0875	0.0648	0.0326	0.0089	0.0016	0.0004	0.0001		
	10			1.0000	0.9999	0.9997	0.9977	0.9895	0.9759	0.9653	0.9115	0.8159	0.6762	0.5060	0.3325	0.1855	0.1462	0.0839	0.0287	0.0067	0.0018	0.0008	0.0000	
	11				1.0000	1.0000	0.9995	0.9972	0.9926	0.9886	0.9648	0.9129	0.8204	0.6831	0.5122	0.3344	0.2793	0.1820	0.0775	0.0233	0.0079	0.0041	0.0003	
	12						0.9999	0.9994	0.9981	0.9969	0.9884	0.9658	0.9165	0.8273	0.6919	0.5188	0.4569	0.3345	0.1749	0.0676	0.0281	0.0163	0.0017	
	13						1.0000	0.9999	0.9996	0.9993	0.9969	0.9891	0.9682	0.9223	0.8371	0.7032	0.6481	0.5261	0.3322	0.1631	0.0824	0.0537	0.0086	0.0002
	14							1.0000	0.9999	0.9999	0.9994	0.9972	0.9904	0.9720	0.9304	0.8500	0.8121	0.7178	0.5346	0.3267	0.1989	0.1444	0.0352	0.0020
	15								1.0000	1.0000	0.9999	0.9995	0.9978	0.9923	0.9770	0.9409	0.9213	0.8668	0.7369	0.5449	0.3930	0.3159	0.1150	0.0132
	16										1.0000	0.9999	0.9996	0.9985	0.9945	0.9830	0.9760	0.9538	0.8887	0.7631	0.6357	0.5587	0.2946	0.0665
	17											1.0000	1.0000	0.9998	0.9992	0.9969	0.9953	0.9896	0.9690	0.9171	0.8498	0.8015	0.5797	0.2453
	18													1.0000	0.9999	0.9997	0.9995	0.9989	0.9958	0.9856	0.9687	0.9544	0.8649	0.6226
	19														1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	x \ p	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950
20	0	0.3585	0.1216	0.0388	0.0261	0.0115	0.0032	0.0008	0.0003	0.0002	0.0000	0.0000												
	1	0.7358	0.3917	0.1756	0.1304	0.0692	0.0243	0.0076	0.0033	0.0021	0.0005	0.0001	0.0000											
	2	0.9245	0.6769	0.4049	0.3287	0.2061	0.0913	0.0355	0.0176	0.0121	0.0036	0.0009	0.0002	0.0000										
	3	0.9841	0.8670	0.6477	0.5665	0.4114	0.2252	0.1071	0.0604	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000									
	4	0.9974	0.9568	0.8298	0.7687	0.6296	0.4148	0.2375	0.1515	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000	0.0000							
	5	0.9997	0.9887	0.9327	0.8982	0.8042	0.6172	0.4164	0.2972	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0002	0.0000						
	6	1.0000	0.9976	0.9781	0.9629	0.9133	0.7858	0.6080	0.4793	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0009	0.0003	0.0000					
	7		0.9996	0.9941	0.9887	0.9679	0.8982	0.7723	0.6615	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0037	0.0013	0.0002	0.0000				
	8		0.9999	0.9987	0.9972	0.9900	0.9591	0.8867	0.8095	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0130	0.0051	0.0009	0.0001	0.0000			
	9		1.0000	0.9998	0.9994	0.9974	0.9861	0.9520	0.9081	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0376	0.0171	0.0039	0.0006	0.0001	0.0000		
	10			1.0000	0.9999	0.9994	0.9961	0.9829	0.9624	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0919	0.0480	0.0139	0.0026	0.0006	0.0002	0.0000	
	11				1.0000	0.9999	0.9991	0.9949	0.9870	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1905	0.1133	0.0409	0.0100	0.0028	0.0013	0.0001	
	12					1.0000	0.9998	0.9987	0.9963	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.3385	0.2277	0.1018	0.0321	0.0113	0.0059	0.0004	
	13						1.0000	0.9997	0.9991	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.5207	0.3920	0.2142	0.0867	0.0371	0.0219	0.0024	0.0000
	14							1.0000	0.9998	0.9998	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.7028	0.5836	0.3828	0.1958	0.1018	0.0673	0.0113	0.0003
	15								1.0000	1.0000	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.8485	0.7625	0.5852	0.3704	0.2313	0.1702	0.0432	0.0026
	16										1.0000	0.9997	0.9987	0.9951	0.9840	0.9556	0.9396	0.8929	0.7748	0.5886	0.4335	0.3523	0.1330	0.0159
	17											1.0000	0.9998	0.9991	0.9964	0.9879	0.9824	0.9645	0.9087	0.7939	0.6713	0.5951	0.3231	0.0755
	18												1.0000	0.9999	0.9995	0.9979	0.9967	0.9924	0.9757	0.9308	0.8696	0.8244	0.6083	0.2642
	19													1.0000	1.0000	0.9998	0.9997	0.9992	0.9968	0.9885	0.9739	0.9612	0.8784	0.6415
	20															1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The Poisson distribution: cumulative probabilities

$$P(X \leqslant x) = \sum_{r=0}^{x} e^{-\lambda} \frac{\lambda^r}{r!}$$

x \ λ	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2	.....	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000

x \ λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4	.....	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5	.....	.....	.....	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000
7	.....	.....	.....	.....	.....	.....	.....	.....	.....

x \ λ	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
0	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496
1	0.7358	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337
2	0.9197	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037
3	0.9810	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747
4	0.9963	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559
5	0.9994	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868
6	0.9999	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966
7	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992
8	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
9	.....	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000

x \ λ	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90
0	0.1353	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550
1	0.4060	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146
2	0.6767	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460
3	0.8571	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696
4	0.9473	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318
5	0.9834	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258
6	0.9955	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713
7	0.9989	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901
8	0.9998	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969
9	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991
10	.....	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998
11	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	0.9999
12	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

x \ λ	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
0	0.0498	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202
1	0.1991	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992
2	0.4232	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531
3	0.6472	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532
4	0.8153	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484
5	0.9161	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006
6	0.9665	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995
7	0.9881	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546
8	0.9962	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815
9	0.9989	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931
10	0.9997	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977
11	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
13	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

x \ λ	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90
0	0.0183	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074
1	0.0916	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439
2	0.2381	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333
3	0.4335	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793
4	0.6288	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582
5	0.7851	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335
6	0.8893	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767
7	0.9489	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769
8	0.9786	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382
9	0.9919	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717
10	0.9972	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880
11	0.9991	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953
12	0.9997	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983
13	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
15	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

CUMULATIVE POISSON PROBABILITY

x \ λ	5.00	5.10	5.20	5.30	5.40	5.50	5.60	5.70	5.80	5.90
0	0.0067	0.0061	0.0055	0.0050	0.0045	0.0041	0.0037	0.0033	0.0030	0.0027
1	0.0404	0.0372	0.0342	0.0314	0.0289	0.0266	0.0244	0.0224	0.0206	0.0189
2	0.1247	0.1165	0.1088	0.1016	0.0948	0.0884	0.0824	0.0768	0.0715	0.0666
3	0.2650	0.2513	0.2381	0.2254	0.2133	0.2017	0.1906	0.1800	0.1700	0.1604
4	0.4405	0.4231	0.4061	0.3895	0.3733	0.3575	0.3422	0.3272	0.3127	0.2987
5	0.6160	0.5984	0.5809	0.5635	0.5461	0.5289	0.5119	0.4950	0.4783	0.4619
6	0.7622	0.7474	0.7324	0.7171	0.7017	0.6860	0.6703	0.6544	0.6384	0.6224
7	0.8666	0.8560	0.8449	0.8335	0.8217	0.8095	0.7970	0.7841	0.7710	0.7576
8	0.9319	0.9252	0.9181	0.9106	0.9027	0.8944	0.8857	0.8766	0.8672	0.8574
9	0.9682	0.9644	0.9603	0.9559	0.9512	0.9462	0.9409	0.9352	0.9292	0.9228
10	0.9863	0.9844	0.9823	0.9800	0.9775	0.9747	0.9718	0.9686	0.9651	0.9614
11	0.9945	0.9937	0.9927	0.9916	0.9904	0.9890	0.9875	0.9859	0.9841	0.9821
12	0.9980	0.9976	0.9972	0.9967	0.9962	0.9955	0.9949	0.9941	0.9932	0.9922
13	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983	0.9980	0.9977	0.9973	0.9969
14	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991	0.9990	0.9988
15	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9996	0.9996
16	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
17	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

x \ λ	6.00	6.10	6.20	6.30	6.40	6.50	6.60	6.70	6.80	6.90
0	0.0025	0.0022	0.0020	0.0018	0.0017	0.0015	0.0014	0.0012	0.0011	0.0010
1	0.0174	0.0159	0.0146	0.0134	0.0123	0.0113	0.0103	0.0095	0.0087	0.0080
2	0.0620	0.0577	0.0536	0.0498	0.0463	0.0430	0.0400	0.0371	0.0344	0.0320
3	0.1512	0.1425	0.1342	0.1264	0.1189	0.1118	0.1052	0.0988	0.0928	0.0871
4	0.2851	0.2719	0.2592	0.2469	0.2351	0.2237	0.2127	0.2022	0.1920	0.1823
5	0.4457	0.4298	0.4141	0.3988	0.3837	0.3690	0.3547	0.3406	0.3270	0.3137
6	0.6063	0.5902	0.5742	0.5582	0.5423	0.5265	0.5108	0.4953	0.4799	0.4647
7	0.7440	0.7301	0.7160	0.7017	0.6873	0.6728	0.6581	0.6433	0.6285	0.6136
8	0.8472	0.8367	0.8259	0.8148	0.8033	0.7916	0.7796	0.7673	0.7548	0.7420
9	0.9161	0.9090	0.9016	0.8939	0.8858	0.8774	0.8686	0.8596	0.8502	0.8405
10	0.9574	0.9531	0.9486	0.9437	0.9386	0.9332	0.9274	0.9214	0.9151	0.9084
11	0.9799	0.9776	0.9750	0.9723	0.9693	0.9661	0.9627	0.9591	0.9552	0.9510
12	0.9912	0.9900	0.9887	0.9873	0.9857	0.9840	0.9821	0.9801	0.9779	0.9755
13	0.9964	0.9958	0.9952	0.9945	0.9937	0.9929	0.9920	0.9909	0.9898	0.9885
14	0.9986	0.9984	0.9981	0.9978	0.9974	0.9970	0.9966	0.9961	0.9956	0.9950
15	0.9995	0.9994	0.9993	0.9992	0.9990	0.9988	0.9986	0.9984	0.9982	0.9979
16	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9993	0.9992
17	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
19	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000

x \ λ	7.00	7.10	7.20	7.30	7.40	7.50	7.60	7.70	7.80	7.90
0	0.0009	0.0008	0.0007	0.0007	00006	0.0006	0.0005	0.0005	0.0004	0.0004
1	0.0073	0.0067	0.0061	0.0056	0.0051	0.0047	0.0043	0.0039	0.0036	0.0033
2	0.0296	0.0275	0.0255	0.0236	0.0219	0.0203	0.0188	0.0174	0.0161	0.0149
3	0.0818	0.0767	0.0719	0.0674	0.0632	0.0591	0.0554	0.0518	0.0485	0.0453
4	0.1730	0.1641	0.1555	0.1473	0.1395	0.1321	0.1249	0.1181	0.1117	0.1055
5	0.3007	0.2881	0.2759	0.2640	0.2526	0.2414	0.2307	0.2203	0.2103	0.2006
6	0.4497	0.4349	0.4204	0.4060	0.3920	0.3782	0.3646	0.3514	0.3384	0.3257
7	0.5987	0.5838	0.5689	0.5541	0.5393	0.5246	0.5100	0.4956	0.4812	0.4670
8	0.7291	0.7160	0.7027	0.6892	0.6757	0.6620	0.6482	0.6343	0.6204	0.6065
9	0.8305	0.8202	0.8096	0.7988	0.7877	0.7764	0.7649	0.7531	0.7411	0.7290
10	0.9015	0.8942	0.8867	0.8788	0.8707	0.8622	0.8535	0.8445	0.8352	0.8257
11	0.9467	0.9420	0.9371	0.9319	0.9265	0.9208	0.9148	0.9085	0.9020	0.8952
12	0.9730	0.9703	0.9673	0.9642	0.9609	0.9573	0.9536	0.9496	0.9454	0.9409
13	0.9872	0.9857	0.9841	0.9824	0.9805	0.9784	0.9762	0.9739	0.9714	0.9687
14	0.9943	0.9935	0.9927	0.9918	0.9908	0.9897	0.9886	0.9873	0.9859	0.9844
15	0.9976	0.9972	0.9969	0.9964	0.9959	0.9954	0.9948	0.9941	0.9934	0.9926
16	0.9990	0.9989	0.9987	0.9985	0.9983	0.9980	09978	0.9974	0.9971	0.9967
17	0.9996	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9989	0.9988	0.9986
18	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994
19	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998
20	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
21	.....	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000

x \ λ	8.00	8.10	8.20	8.30	8.40	8.50	8.60	8.70	8.80	8.90
0	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001
1	0.0030	0.0028	0.0025	0.0023	0.0021	0.0019	0.0018	0.0016	0.0015	0.0014
2	0.0138	0.0127	0.0118	0.0109	0.0100	0.0093	0.0086	0.0079	0.0073	0.0068
3	0.0424	0.0396	0.0370	0.0346	0.0323	0.0301	0.0281	0.0262	0.0244	0.0228
4	0.0996	0.0940	0.0887	0.0837	0.0789	0.0744	0.0701	0.0660	0.0621	0.0584
5	0.1912	0.1822	0.1736	0.1653	0.1573	0.1496	0.1422	0.1352	0.1284	0.1219
6	0.3134	0.3013	0.2896	0.2781	0.2670	0.2562	0.2457	0.2355	0.2256	0.2160
7	0.4530	0.4391	0.4254	0.4119	0.3987	0.3856	0.3728	0.3602	0.3478	0.3357
8	0.5925	0.5786	0.5647	0.5507	0.5369	0.5231	0.5094	0.4958	0.4823	0.4689
9	0.7166	0.7041	0.6915	0.6788	0.6659	0.6530	0.6400	0.6269	0.6137	0.6006
10	0.8159	0.8058	0.7955	0.7850	0.7743	0.7634	0.7522	0.7409	0.7294	0.7178
11	0.8881	0.8807	0.8731	0.8652	0.8571	0.8487	0.8400	0.8311	0.8220	0.8126
12	0.9362	0.9313	0.9261	0.9207	0.9150	0.9091	0.9029	0.8965	0.8898	0.8829
13	0.9658	0.9628	0.9595	0.9561	0.9524	0.9486	0.9445	0.9403	0.9358	0.9311
14	0.9827	0.9810	0.9791	0.9771	0.9749	0.9726	0.9701	0.9675	0.9647	0.9617
15	0.9918	0.9908	0.9898	0.9887	0.9875	0.9862	0.9848	0.9832	0.9816	0.9798
16	0.9963	0.9958	0.9953	0.9947	0.9941	0.9934	0.9926	0.9918	0.9909	0.9899
17	0.9984	0.9982	0.9979	0.9977	0.9973	0.9970	0.9966	0.9962	0.9957	0.9952
18	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9985	0.9983	0.9981	0.9978
19	0.9997	0.9997	0.9997	0.9996	0.9995	0.9995	0.9994	0.9993	0.9992	0.9991
20	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996
21	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
22	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
23	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

x \ λ	9.00	9.10	9.20	9.30	9.40	9.50	9.60	9.70	9.80	9.90
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
1	0.0012	0.0011	0.0010	0.0009	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005
2	0.0062	0.0058	0.0053	0.0049	0.0045	0.0042	0.0038	0.0035	0.0033	0.0030
3	0.0212	0.0198	0.0184	0.0172	0.0160	0.0149	0.0138	0.0129	0.0120	0.0111
4	0.0550	0.0517	0.0486	0.0456	0.0429	0.0403	0.0378	0.0355	0.0333	0.0312
5	0.1157	0.1098	0.1041	0.0986	0.0935	0.0885	0.0838	0.0793	0.0750	0.0710
6	0.2068	0.1978	0.1892	0.1808	0.1727	0.1649	0.1574	0.1502	0.1433	0.1366
7	0.3239	0.3123	0.3010	0.2900	0.2792	0.2687	0.2584	0.2485	0.2388	0.2294
8	0.4557	0.4426	0.4296	0.4168	0.4042	0.3918	0.3796	0.3676	0.3558	0.3442
9	0.5874	0.5742	0.5611	0.5479	0.5349	0.5218	0.5089	0.4960	0.4832	0.4705
10	0.7060	0.6941	0.6820	0.6699	0.6576	0.6453	0.6329	0.6205	0.6080	0.5955
11	0.8030	0.7932	0.7832	0.7730	0.7626	0.7520	0.7412	0.7303	0.7193	0.7081
12	0.8758	0.8684	0.8607	0.8529	0.8448	0.8364	0.8279	0.8191	0.8101	0.8009
13	0.9261	0.9210	0.9156	0.9100	0.9042	0.8981	0.8919	0.8853	0.8786	0.8716
14	0.9585	0.9552	0.9517	0.9480	0.9441	0.9400	0.9357	0.9312	0.9265	0.9216
15	0.9780	0.9760	0.9738	0.9715	0.9691	0.9665	0.9638	0.9609	0.9579	0.9546
16	0.9889	0.9878	0.9865	0.9852	0.9838	0.9823	0.9806	0.9789	0.9770	0.9751
17	0.9947	0.9941	0.9934	0.9927	0.9919	0.9911	0.9902	0.9892	0.9881	0.9870
18	0.9976	0.9973	0.9969	0.9966	0.9962	0.9957	0.9952	0.9947	0.9941	0.9935
19	0.9989	0.9988	0.9986	0.9985	0.9983	0.9980	0.9978	0.9975	0.9972	0.9969
20	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9986
21	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0.9994
22	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997
23	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
24	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000

x \ λ	10.00	10.10	10.20	10.30	10.40	10.50	10.60	10.70	10.80	10.90
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0005	0.0005	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002
2	0.0028	0.0026	0.0023	0.0022	0.0020	0.0018	0.0017	0.0016	0.0014	0.0013
3	0.0103	0.0096	0.0089	0.0083	0.0077	0.0071	0.0066	0.0062	0.0057	0.0053
4	0.0293	0.0274	0.0257	0.0241	0.0225	0.0211	0.0197	0.0185	0.0173	0.0162
5	0.0671	0.0634	0.0599	0.0566	0.0534	0.0504	0.0475	0.0448	0.0423	0.0398
6	0.1301	0.1240	0.1180	0.1123	0.1069	0.1016	0.0966	0.0918	0.0872	0.0828
7	0.2202	0.2113	0.2027	0.1944	0.1863	0.1785	0.1710	0.1636	0.1566	0.1498
8	0.3328	0.3217	0.3108	0.3001	0.2896	0.2794	0.2694	0.2597	0.2502	0.2410
9	0.4579	0.4455	0.4332	0.4210	0.4090	0.3971	0.3854	0.3739	0.3626	0.3515
10	0.5830	0.5705	0.5580	0.5456	0.5331	0.5207	0.5084	0.4961	0.4840	0.4719
11	0.6968	0.6853	0.6738	0.6622	0.6505	0.6387	0.6269	0.6150	0.6031	0.5912
12	0.7916	0.7820	0.7722	0.7623	0.7522	0.7420	0.7316	0.7210	0.7104	0.6996
13	0.8645	0.8571	0.8494	0.8416	0.8336	0.8253	0.8169	0.8083	0.7995	0.7905
14	0.9165	0.9112	0.9057	0.9000	0.8940	0.8879	0.8815	0.8750	0.8682	0.8612
15	0.9513	0.9477	0.9440	0.9400	0.9359	0.9317	0.9272	0.9225	0.9177	0.9126
16	0.9730	0.9707	0.9684	0.9658	0.9632	0.9604	0.9574	0.9543	0.9511	0.9477
17	0.9857	0.9844	0.9830	0.9815	0.9799	0.9781	0.9763	0.9744	0.9723	0.9701
18	0.9928	0.9921	0.9913	0.9904	0.9895	0.9885	0.9874	0.9863	0.9850	0.9837
19	0.9965	0.9962	0.9957	0.9953	0.9948	0.9942	0.9936	0.9930	0.9923	0.9915
20	0.9984	0.9982	0.9980	0.9978	0.9975	0.9972	0.9969	0.9966	0.9962	0.9958
21	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9986	0.9984	0.9982	0.9980
22	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9994	0.9993	0.9992	0.9991
23	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996
24	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
25	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
26	.....	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

Critical values for the product moment correlation coefficient, r

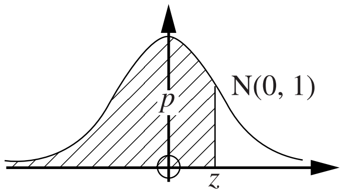
1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%
n				
1	–	–	–	–
2	–	–	–	–
3	0.9877	0.9969	0.9995	0.9999
4	0.9000	0.9500	0.9800	0.9900
5	0.8054	0.8783	0.9343	0.9587
6	0.7293	0.8114	0.8822	0.9172
7	0.6694	0.7545	0.8329	0.8745
8	0.6215	0.7067	0.7887	0.8343
9	0.5822	0.6664	0.7498	0.7977
10	0.5494	0.6319	0.7155	0.7646
11	0.5214	0.6021	0.6851	0.7348
12	0.4973	0.5760	0.6581	0.7079
13	0.4762	0.5529	0.6339	0.6835
14	0.4575	0.5324	0.6120	0.6614
15	0.4409	0.5140	0.5923	0.6411
16	0.4259	0.4973	0.5742	0.6226
17	0.4124	0.4821	0.5577	0.6055
18	0.4000	0.4683	0.5425	0.5897
19	0.3887	0.4555	0.5285	0.5751
20	0.3783	0.4438	0.5155	0.5614
21	0.3687	0.4329	0.5034	0.5487
22	0.3598	0.4227	0.4921	0.5368
23	0.3515	0.4132	0.4815	0.5256
24	0.3438	0.4044	0.4716	0.5151
25	0.3365	0.3961	0.4622	0.5052
26	0.3297	0.3882	0.4534	0.4958
27	0.3233	0.3809	0.4451	0.4869
28	0.3172	0.3739	0.4372	0.4785
29	0.3115	0.3673	0.4297	0.4705
30	0.3061	0.3610	0.4226	0.4629
31	0.3009	0.3550	0.4158	0.4556
32	0.2960	0.3494	0.4093	0.4487
33	0.2913	0.3440	0.4032	0.4421
34	0.2869	0.3388	0.3972	0.4357
35	0.2826	0.3338	0.3916	0.4296
36	0.2785	0.3291	0.3862	0.4238
37	0.2746	0.3246	0.3810	0.4182
38	0.2709	0.3202	0.3760	0.4128
39	0.2673	0.3160	0.3712	0.4076
40	0.2638	0.3120	0.3665	0.4026
41	0.2605	0.3081	0.3621	0.3978
42	0.2573	0.3044	0.3578	0.3932
43	0.2542	0.3008	0.3536	0.3887
44	0.2512	0.2973	0.3496	0.3843
45	0.2483	0.2940	0.3457	0.3801
46	0.2455	0.2907	0.3420	0.3761
47	0.2429	0.2876	0.3384	0.3721
48	0.2403	0.2845	0.3348	0.3683
49	0.2377	0.2816	0.3314	0.3646
50	0.2353	0.2787	0.3281	0.3610
51	0.2329	0.2759	0.3249	0.3575
52	0.2306	0.2732	0.3218	0.3542
53	0.2284	0.2706	0.3188	0.3509
54	0.2262	0.2681	0.3158	0.3477
55	0.2241	0.2656	0.3129	0.3445
56	0.2221	0.2632	0.3102	0.3415
57	0.2201	0.2609	0.3074	0.3385
58	0.2181	0.2586	0.3048	0.3357
59	0.2162	0.2564	0.3022	0.3328
60	0.2144	0.2542	0.2997	0.3301

Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%
n				
1	–	–	–	–
2	–	–	–	–
3	–	–	–	–
4	1.0000	–	–	–
5	0.9000	1.0000	1.0000	–
6	0.8286	0.8857	0.9429	1.0000
7	0.7143	0.7857	0.8929	0.9286
8	0.6429	0.7381	0.8333	0.8810
9	0.6000	0.7000	0.7833	0.8333
10	0.5636	0.6485	0.7455	0.7939
11	0.5364	0.6182	0.7091	0.7545
12	0.5035	0.5874	0.6783	0.7273
13	0.4835	0.5604	0.6484	0.7033
14	0.4637	0.5385	0.6264	0.6791
15	0.4464	0.5214	0.6036	0.6536
16	0.4294	0.5029	0.5824	0.6353
17	0.4142	0.4877	0.5662	0.6176
18	0.4014	0.4716	0.5501	0.5996
19	0.3912	0.4596	0.5351	0.5842
20	0.3805	0.4466	0.5218	0.5699
21	0.3701	0.4364	0.5091	0.5558
22	0.3608	0.4252	0.4975	0.5438
23	0.3528	0.4160	0.4862	0.5316
24	0.3443	0.4070	0.4757	0.5209
25	0.3369	0.3977	0.4662	0.5108
26	0.3306	0.3901	0.4571	0.5009
27	0.3242	0.3828	0.4487	0.4915
28	0.3180	0.3755	0.4401	0.4828
29	0.3118	0.3685	0.4325	0.4749
30	0.3063	0.3624	0.4251	0.4670
31	0.3012	0.3560	0.4185	0.4593
32	0.2962	0.3504	0.4117	0.4523
33	0.2914	0.3449	0.4054	0.4455
34	0.2871	0.3396	0.3995	0.4390
35	0.2829	0.3347	0.3936	0.4328
36	0.2788	0.3300	0.3882	0.4268
37	0.2748	0.3253	0.3829	0.4211
38	0.2710	0.3209	0.3778	0.4155
39	0.2674	0.3168	0.3729	0.4103
40	0.2640	0.3128	0.3681	0.4051
41	0.2606	0.3087	0.3636	0.4002
42	0.2574	0.3051	0.3594	0.3955
43	0.2543	0.3014	0.3550	0.3908
44	0.2513	0.2978	0.3511	0.3865
45	0.2484	0.2945	0.3470	0.3822
46	0.2456	0.2913	0.3433	0.3781
47	0.2429	0.2880	0.3396	0.3741
48	0.2403	0.2850	0.3361	0.3702
49	0.2378	0.2820	0.3326	0.3664
50	0.2353	0.2791	0.3293	0.3628
51	0.2329	0.2764	0.3260	0.3592
52	0.2307	0.2736	0.3228	0.3558
53	0.2284	0.2710	0.3198	0.3524
54	0.2262	0.2685	0.3168	0.3492
55	0.2242	0.2659	0.3139	0.3460
56	0.2221	0.2636	0.3111	0.3429
57	0.2201	0.2612	0.3083	0.3400
58	0.2181	0.2589	0.3057	0.3370
59	0.2162	0.2567	0.3030	0.3342
60	0.2144	0.2545	0.3005	0.3314

The Normal distribution: values of $\Phi(z) = p$

The table gives the probability, p, of a random variable distributed as N(0, 1) *being less than z.*



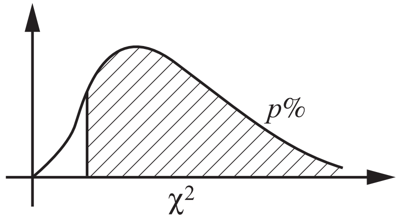
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	1	2	3	4	5	6	7	8	9
											(add)								
0.0	.5000	5040	5080	5120	5160	5199	5239	5279	5319	5359	4	8	12	16	20	24	28	32	36
0.1	.5398	5438	5478	5517	5557	5596	5636	5675	5714	5753	4	8	12	16	20	24	28	32	35
0.2	.5793	5832	5871	5910	5948	5987	6026	6064	6103	6141	4	8	12	15	19	23	27	31	35
0.3	.6179	6217	6255	6293	6331	6368	6406	6443	6480	6517	4	8	11	15	19	23	26	30	34
0.4	.6554	6591	6628	6664	6700	6736	6772	6808	6844	6879	4	7	11	14	18	22	25	29	32
0.5	.6915	6950	6985	7019	7054	7088	7123	7157	7190	7224	3	7	10	14	17	21	24	27	31
0.6	.7257	7291	7324	7357	7389	7422	7454	7486	7517	7549	3	6	10	13	16	19	23	26	29
0.7	.7580	7611	7642	7673	7704	7734	7764	7794	7823	7852	3	6	9	12	15	18	21	24	27
0.8	.7881	7910	7939	7967	7995	8023	8051	8078	8106	8133	3	6	8	11	14	17	19	22	25
0.9	.8159	8186	8212	8238	8264	8289	8315	8340	8365	8389	3	5	8	10	13	15	18	20	23
1.0	.8413	8438	8461	8485	8508	8531	8554	8577	8599	8621	2	5	7	9	12	14	16	18	21
1.1	.8643	8665	8686	8708	8729	8749	8770	8790	8810	8830	2	4	6	8	10	12	14	16	19
1.2	.8849	8869	8888	8907	8925	8944	8962	8980	8997	9015	2	4	6	7	9	11	13	15	16
1.3	.9032	9049	9066	9082	9099	9115	9131	9147	9162	9177	2	3	5	6	8	10	11	13	14
1.4	.9192	9207	9222	9236	9251	9265	9279	9292	9306	9319	1	3	4	6	7	8	10	11	13
1.5	.9332	9345	9357	9370	9382	9394	9406	9418	9429	9441	1	2	4	5	6	7	8	10	11
1.6	.9452	9463	9474	9484	9495	9505	9515	9525	9535	9545	1	2	3	4	5	6	7	8	9
1.7	.9554	9564	9573	9582	9591	9599	9608	9616	9625	9633	1	2	3	3	4	5	6	7	8
1.8	.9641	9649	9656	9664	9671	9678	9686	9693	9699	9706	1	1	2	3	4	4	5	6	6
1.9	.9713	9719	9726	9732	9738	9744	9750	9756	9761	9767	1	1	2	2	3	4	4	5	5
2.0	.9772	9778	9783	9788	9793	9798	9803	9808	9812	9817	0	1	1	2	2	3	3	4	4
2.1	.9821	9826	9830	9834	9838	9842	9846	9850	9854	9857	0	1	1	2	2	2	3	3	4
2.2	.9861	9864	9868	9871	9875	9878	9881	9884	9887	9890	0	1	1	1	2	2	2	3	3
2.3	.9893	9896	9898	9901	9904	9906	9909	9911	9913	9916	0	1	1	1	1	2	2	2	2
2.4	.9918	9920	9922	9925	9927	9929	9931	9932	9934	9936	0	0	1	1	1	1	1	2	2
2.5	.9938	9940	9941	9943	9945	9946	9948	9949	9951	9952									
2.6	.9953	9955	9956	9957	9959	9960	9961	9962	9963	9964									
2.7	.9965	9966	9967	9968	9969	9970	9971	9972	9973	9974									
2.8	.9974	9975	9976	9977	9977	9978	9979	9979	9980	9981									
2.9	.9981	9982	9982	9983	9984	9984	9985	9985	9986	9986	*differences untrustworthy*								
3.0	.9987	9987	9987	9988	9988	9989	9989	9989	9990	9990									
3.1	.9990	9991	9991	9991	9992	9992	9992	9992	9993	9993									
3.2	.9993	9993	9994	9994	9994	9994	9994	9995	9995	9995									
3.3	.9995	9995	9996	9996	9996	9996	9996	9996	9996	9997									
3.4	.9997	9997	9997	9997	9997	9997	9997	9997	9997	9998									

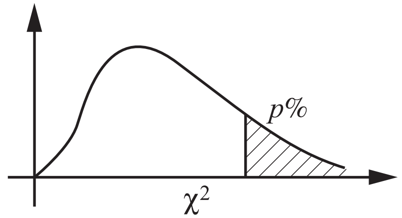
The Inverse Normal function: values of $\Phi^{-1}(p) = z$

p	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.50	.0000	.0025	.0050	.0075	.0100	.0125	.0150	.0175	.0201	.0226
.51	.0251	.0276	.0301	.0326	.0351	.0376	.0401	.0426	.0451	.0476
.52	.0502	.0527	.0552	.0577	.0602	.0627	.0652	.0677	.0702	.0728
.53	.0753	.0778	.0803	.0828	.0853	.0878	.0904	.0929	.0954	.0979
.54	.1004	.1030	.1055	.1080	.1105	.1130	.1156	.1181	.1206	.1231
.55	.1257	.1282	.1307	.1332	.1358	.1383	.1408	.1434	.1459	.1484
.56	.1510	.1535	.1560	.1586	.1611	.1637	.1662	.1687	.1713	.1738
.57	.1764	.1789	.1815	.1840	.1866	.1891	.1917	.1942	.1968	.1993
.58	.2019	.2045	.2070	.2096	.2121	.2147	.2173	.2198	.2224	.2250
.59	.2275	.2301	.2327	.2353	.2378	.2404	.2430	.2456	.2482	.2508
.60	.2533	.2559	.2585	.2611	.2637	.2663	.2689	.2715	.2741	.2767
.61	.2793	.2819	.2845	.2871	.2898	.2924	.2950	.2976	.3002	.3029
.62	.3055	.3081	.3107	.3134	.3160	.3186	.3213	.3239	.3266	.3292
.63	.3319	.3345	.3372	.3398	.3425	.3451	.3478	.3505	.3531	.3558
.64	.3585	.3611	.3638	.3665	.3692	.3719	.3745	.3772	.3799	.3826
.65	.3853	.3880	.3907	.3934	.3961	.3989	.4016	.4043	.4070	.4097
.66	.4125	.4152	.4179	.4207	.4234	.4261	.4289	.4316	.4344	.4372
.67	.4399	.4427	.4454	.4482	.4510	.4538	.4565	.4593	.4621	.4649
.68	.4677	.4705	.4733	.4761	.4789	.4817	.4845	.4874	.4902	.4930
.69	.4959	.4987	.5015	.5044	.5072	.5101	.5129	.5158	.5187	.5215
.70	.5244	.5273	.5302	.5330	.5359	.5388	.5417	.5446	.5476	.5505
.71	.5534	.5563	.5592	.5622	.5651	.5681	.5710	.5740	.5769	.5799
.72	.5828	.5858	.5888	.5918	.5948	.5978	.6008	.6038	.6068	.6098
.73	.6128	.6158	.6189	.6219	.6250	.6280	.6311	.6341	.6372	.6403
.74	.6433	.6464	.6495	.6526	.6557	.6588	.6620	.6651	.6682	.6713
.75	.6745	.6776	.6808	.6840	.6871	.6903	.6935	.6967	.6999	.7031
.76	.7063	.7095	.7128	.7160	.7192	.7225	.7257	.7290	.7323	.7356
.77	.7388	.7421	.7454	.7488	.7521	.7554	.7588	.7621	.7655	.7688
.78	.7722	.7756	.7790	.7824	.7858	.7892	.7926	.7961	.7995	.8030
.79	.8064	.8099	.8134	.8169	.8204	.8239	.8274	.8310	.8345	.8381
.80	.8416	.8452	.8488	.8524	.8560	.8596	.8633	.8669	.8705	.8742
.81	.8779	.8816	.8853	.8890	.8927	.8965	.9002	.9040	.9078	.9116
.82	.9154	.9192	.9230	.9269	.9307	.9346	.9385	.9424	.9463	.9502
.83	.9542	.9581	.9621	.9661	.9701	.9741	.9782	.9822	.9863	.9904
.84	.9945	.9986	1.003	1.007	1.011	1.015	1.019	1.024	1.028	1.032
.85	1.036	1.041	1.045	1.049	1.054	1.058	1.063	1.067	1.071	1.076
.86	1.080	1.085	1.089	1.094	1.099	1.103	1.108	1.112	1.117	1.122
.87	1.126	1.131	1.136	1.141	1.146	1.150	1.155	1.160	1.165	1.170
.88	1.175	1.180	1.185	1.190	1.195	1.200	1.206	1.211	1.216	1.221
.89	1.227	1.232	1.237	1.243	1.248	1.254	1.259	1.265	1.270	1.276
.90	1.282	1.287	1.293	1.299	1.305	1.311	1.317	1.323	1.329	1.335
.91	1.341	1.347	1.353	1.360	1.366	1.372	1.379	1.385	1.392	1.398
.92	1.405	1.412	1.419	1.426	1.433	1.440	1.447	1.454	1.461	1.468
.93	1.476	1.483	1.491	1.499	1.506	1.514	1.522	1.530	1.538	1.546
.94	1.555	1.563	1.572	1.581	1.589	1.598	1.607	1.616	1.626	1.635
.95	1.645	1.655	1.665	1.675	1.685	1.695	1.706	1.717	1.728	1.739
.96	1.751	1.762	1.774	1.787	1.799	1.812	1.825	1.838	1.852	1.866
.97	1.881	1.896	1.911	1.927	1.943	1.960	1.977	1.995	2.014	2.034
.98	2.054	2.075	2.097	2.120	2.144	2.170	2.197	2.226	2.257	2.290
.99	2.326	2.366	2.409	2.457	2.512	2.576	2.652	2.748	2.878	3.090

PERCENTAGE POINTS OF χ^2 AND t – DISTRIBUTIONS

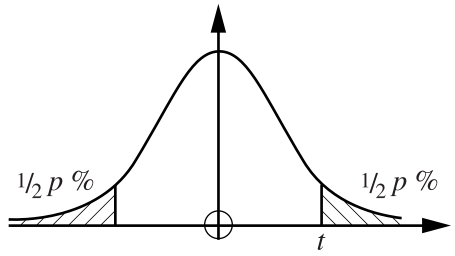
Percentage points of the χ^2 (chi-squared) distribution





p%	99	97.5	95	90	10	5.0	2.5	1.0	0.5
$v = 1$	.0001	.0010	.0039	.0158	2.706	3.841	5.024	6.635	7.879
2	.0201	.0506	0.103	0.211	4.605	5.991	7.378	9.210	10.60
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34	12.84
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28	14.86
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09	16.75
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81	18.55
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	20.28
8	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	21.95
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	23.59
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	25.19
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	26.76
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	28.30
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	29.82
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	31.32
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	32.80
16	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	34.27
17	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	35.72
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	37.16
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
100	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Percentage points of the t-distribution



p%	10	5	2	1	
$v = 1$	6.314	12.71	31.82	63.66	
2	2.920	4.303	6.965	9.925	
3	2.353	3.182	4.541	5.841	
4	2.132	2.776	3.747	4.604	
5	2.015	2.571	3.365	4.032	
6	1.943	2.447	3.143	3.707	
7	1.895	2.365	2.998	3.499	
8	1.860	2.306	2.896	3.355	
9	1.833	2.262	2.821	3.250	
10	1.812	2.228	2.764	3.169	
11	1.796	2.201	2.718	3.106	
12	1.782	2.179	2.681	3.055	
13	1.771	2.160	2.650	3.012	
14	1.761	2.145	2.624	2.977	
15	1.753	2.131	2.602	2.947	
20	1.725	2.086	2.528	2.845	
30	1.697	2.042	2.457	2.750	
50	1.676	2.009	2.403	2.678	
100	1.660	1.984	2.364	2.626	
∞	1.645	1.960	2.326	2.576	= Percentage points of the Normal distribution N(0, 1)

5% points of the *F*-distribution

v_2 \ v_1	1	2	3	4	5	6	7	8	10	12	24	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	241.9	243.9	249.0	254.3
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.5	19.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.64	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.77	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.53	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.14	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.98	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.85	2.79	2.61	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.67	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.60	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.54	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.49	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.45	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.41	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.38	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.35	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.32	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.30	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.27	2.20	2.00	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.25	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.24	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.22	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.20	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.19	2.12	1.91	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.18	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.16	2.09	1.89	1.62
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.14	2.07	1.86	1.59
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.12	2.05	1.84	1.57
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.11	2.03	1.82	1.55
38	4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.09	2.02	1.81	1.53
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.08	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.99	1.92	1.70	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.91	1.83	1.61	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.83	1.75	1.52	1.00

$2^1/_2$% points of the *F*-distribution

v_2 \ v_1	1	2	3	4	5	6	7	8	10	12	24	∞
1	648	800	864	900	922	937	948	957	969	977	997	1018
2	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.5	39.5
3	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.4	14.3	14.1	13.9
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.84	8.75	8.51	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.62	6.52	6.28	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.46	5.37	5.12	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.76	4.67	4.42	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.30	4.20	3.95	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	3.96	3.87	3.61	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.72	3.62	3.37	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.53	3.43	3.17	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.37	3.28	3.02	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.25	3.15	2.89	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.15	3.05	2.79	2.49
15	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.06	2.96	2.70	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	2.99	2.89	2.63	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.92	2.82	2.56	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.87	2.77	2.50	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.82	2.72	2.45	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.77	2.68	2.41	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.73	2.64	2.37	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.70	2.60	2.33	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.67	2.57	2.30	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.64	2.54	2.27	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.61	2.51	2.24	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.59	2.49	2.22	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.57	2.47	2.19	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.55	2.45	2.17	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.53	2.43	2.15	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.51	2.41	2.14	1.79
32	5.53	4.15	3.56	3.22	3.00	2.84	2.72	2.62	2.48	2.38	2.10	1.75
34	5.50	4.12	3.53	3.19	2.97	2.81	2.69	2.59	2.45	2.35	2.08	1.72
36	5.47	4.09	3.51	3.17	2.94	2.79	2.66	2.57	2.43	2.33	2.05	1.69
38	5.45	4.07	3.48	3.15	2.92	2.76	2.64	2.55	2.41	2.31	2.03	1.66
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.39	2.29	2.01	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.27	2.17	1.88	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.16	2.05	1.76	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.05	1.94	1.64	1.00

1% points of the F-distribution

ν_2 \ ν_1	1	2	3	4	5	6	7	8	10	12	24	∞
1	4052	5000	5403	5625	5764	5859	5928	5981	6056	6106	6235	6366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.2	27.1	26.6	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.5	14.4	13.9	13.5
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.05	9.89	9.47	9.02
6	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.87	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.62	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.81	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.26	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.85	4.71	4.33	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.54	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.30	4.16	3.78	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.10	3.96	3.59	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	3.94	3.80	3.43	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.80	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.69	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.59	3.46	3.08	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.51	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.43	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.37	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.31	3.17	2.80	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.26	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.21	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.17	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.13	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.09	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.06	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.03	2.90	2.52	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.00	2.87	2.49	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.98	2.84	2.47	2.01
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	2.93	2.80	2.42	1.96
34	7.45	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.90	2.76	2.38	1.91
36	7.40	5.25	4.38	3.89	3.58	3.35	3.18	3.05	2.86	2.72	2.35	1.87
38	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.83	2.69	2.32	1.84
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.80	2.66	2.29	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.63	2.50	2.12	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.47	2.34	1.95	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.32	2.18	1.79	1.00

0.1% points of the F-distribution

ν_2 \ ν_1	1	2	3	4	5	6	7	8	10	12	24	∞
1	4053	5000	5404	5625	5764	5859	5929	5981	6056	6107	6235	6366
2	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.5	999.5
3	167.0	148.5	141.1	137.1	134.6	132.8	131.5	130.6	129.2	128.3	125.9	123.5
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.05	47.41	45.77	44.05
5	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	26.92	26.42	25.14	23.79
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.41	17.99	16.90	15.75
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.08	13.71	12.73	11.70
8	25.42	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.54	11.19	10.30	9.34
9	22.86	16.39	13.90	12.56	11.71	11.13	10.69	10.37	9.87	9.57	8.72	7.81
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.74	8.44	7.64	6.76
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	7.92	7.63	6.85	6.00
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.29	7.00	6.25	5.42
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.80	6.52	5.78	4.97
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.40	6.13	5.41	4.60
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.08	5.81	5.10	4.31
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.81	5.55	4.85	4.06
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.58	5.32	4.63	3.85
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.39	5.13	4.45	3.67
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.22	4.97	4.29	3.51
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.08	4.82	4.15	3.38
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	4.95	4.70	4.03	3.26
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.83	4.58	3.92	3.15
23	14.19	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.73	4.48	3.82	3.05
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.64	4.39	3.74	2.97
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.56	4.31	3.66	2.89
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.48	4.24	3.59	2.82
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.41	4.17	3.52	2.75
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.35	4.11	3.46	2.69
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.29	4.05	3.41	2.64
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.24	4.00	3.36	2.59
32	13.12	8.64	6.94	6.01	5.43	5.02	4.72	4.48	4.14	3.91	3.27	2.50
34	12.97	8.52	6.83	5.92	5.34	4.93	4.63	4.40	4.06	3.83	3.19	2.42
36	12.83	8.42	6.74	5.84	5.26	4.86	4.56	4.33	3.99	3.76	3.12	2.35
38	12.71	8.33	6.66	5.76	5.19	4.79	4.49	4.26	3.93	3.70	3.06	2.29
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	3.87	3.64	3.01	2.23
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.54	3.32	2.69	1.89
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.24	3.02	2.40	1.54
∞	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	2.96	2.74	2.13	1.00

Critical Values for the Mann-Whitney Test

The critical values in these tables are for the Mann-Whitney test statistic, T. Critical values for the Wilcoxon test statistic, W, may be derived by adding $\frac{1}{2}m(m+1)$ (where m is the size of the sample from which the rank sum has been obtained). These values are tabulated on pages 28 and 29.

1 – tail		5%	2½%	1%	½%
2 – tail		10%	5%	2%	1%
m	*n*				
2	2	–	–	–	–
2	3	–	–	–	–
2	4	–	–	–	–
2	5	0	–	–	–
2	6	0	–	–	–
2	7	0	–	–	–
2	8	1	0	–	–
2	9	1	0	–	–
2	10	1	0	–	–
2	11	1	0	–	–
2	12	2	1	–	–
2	13	2	1	0	–
2	14	3	1	0	–
2	15	3	1	0	–
2	16	3	1	0	–
2	17	3	2	0	–
2	18	4	2	0	–
2	19	4	2	1	0
2	20	4	2	1	0
2	21	5	3	1	0
2	22	5	3	1	0
2	23	5	3	1	0
2	24	6	3	1	0
2	25	6	3	1	0
3	3	0	–	–	–
3	4	0	–	–	–
3	5	1	0	–	–
3	6	2	1	–	–
3	7	2	1	0	–
3	8	3	2	0	–
3	9	4	2	1	0
3	10	4	3	1	0
3	11	5	3	1	0
3	12	5	4	2	1
3	13	6	4	2	1
3	14	7	5	2	1
3	15	7	5	3	2
3	16	8	6	3	2
3	17	9	6	4	2
3	18	9	7	4	2
3	19	10	7	4	3
3	20	11	8	5	3
3	21	11	8	5	3
3	22	12	9	6	4
3	23	13	9	6	4
3	24	13	10	6	4
3	25	14	10	7	5
4	4	1	0	–	–
4	5	2	1	0	–
4	6	3	2	1	0
4	7	4	3	1	0
4	8	5	4	2	1
4	9	6	4	3	1
4	10	7	5	3	2
4	11	8	6	4	2
4	12	9	7	5	3
4	13	10	8	5	3
4	14	11	9	6	4
4	15	12	10	7	5
4	16	14	11	7	5
4	17	15	11	8	6
4	18	16	12	9	6
4	19	17	13	9	7
4	20	18	14	10	8
4	21	19	15	11	8
4	22	20	16	11	9
4	23	21	17	12	9
4	24	22	17	13	10
4	25	23	18	13	10
5	5	4	2	1	0
5	6	5	3	2	1
5	7	6	5	3	1
5	8	8	6	4	2
5	9	9	7	5	3
5	10	11	8	6	4
5	11	12	9	7	5
5	12	13	11	8	6
5	13	15	12	9	7
5	14	16	13	10	7
5	15	18	14	11	8
5	16	19	15	12	9
5	17	20	17	13	10
5	18	22	18	14	11
5	19	23	19	15	12
5	20	25	20	16	13
5	21	26	22	17	14
5	22	28	23	18	14
5	23	29	24	19	15
5	24	30	25	20	16
5	25	32	27	21	17
6	6	7	5	3	2
6	7	8	6	4	3
6	8	10	8	6	4
6	9	12	10	7	5
6	10	14	11	8	6
6	11	16	13	9	7
6	12	17	14	11	9
6	13	19	16	12	10
6	14	21	17	13	11
6	15	23	19	15	12
6	16	25	21	16	13
6	17	26	22	18	15
6	18	28	24	19	16
6	19	30	25	20	17
6	20	32	27	22	18
6	21	34	29	23	19
6	22	36	30	24	21
6	23	37	32	26	22
6	24	39	33	27	23
6	25	41	35	29	24
7	7	11	8	6	4
7	8	13	10	7	6
7	9	15	12	9	7
7	10	17	14	11	9
7	11	19	16	12	10
7	12	21	18	14	12
7	13	24	20	16	13
7	14	26	22	17	15
7	15	28	24	19	16
7	16	30	26	21	18
7	17	33	28	23	19
7	18	35	30	24	21
7	19	37	32	26	22
7	20	39	34	28	24
7	21	41	36	30	25
7	22	44	38	31	27
7	23	46	40	33	29
7	24	48	42	35	30
7	25	50	44	36	32
8	8	15	13	9	7
8	9	18	15	11	9
8	10	20	17	13	11
8	11	23	19	15	13
8	12	26	22	17	15
8	13	28	24	20	17
8	14	31	26	22	18
8	15	33	29	24	20
8	16	36	31	26	22
8	17	39	34	28	24
8	18	41	36	30	26
8	19	44	38	32	28
8	20	47	41	34	30
8	21	49	43	36	32
8	22	52	45	38	34
8	23	54	48	40	35
8	24	57	50	42	37
8	25	60	53	45	39
9	9	21	17	14	11
9	10	24	20	16	13
9	11	27	23	18	16
9	12	30	26	21	18
9	13	33	28	23	20
9	14	36	31	26	22
9	15	39	34	28	24
9	16	42	37	31	27
9	17	45	39	33	29
9	18	48	42	35	31
9	19	51	45	38	33
9	20	54	48	40	36
9	21	57	50	43	38
9	22	60	53	45	40
9	23	63	56	48	43
9	24	66	59	50	45
9	25	69	62	53	47

CRITICAL VALUES FOR THE MANN-WHITNEY TEST

1 – tail		5%	2½%	1%	½%
2 – tail		10%	5%	2%	1%
m	*n*				
10	10	27	23	19	16
10	11	31	26	22	18
10	12	34	29	24	21
10	13	37	33	27	24
10	14	41	36	30	26
10	15	44	39	33	29
10	16	48	42	36	31
10	17	51	45	38	34
10	18	55	48	41	37
10	19	58	52	44	39
10	20	62	55	47	42
10	21	65	58	50	44
10	22	68	61	53	47
10	23	72	64	55	50
10	24	75	67	58	52
10	25	79	71	61	55
11	11	34	30	25	21
11	12	38	33	28	24
11	13	42	37	31	27
11	14	46	40	34	30
11	15	50	44	37	33
11	16	54	47	41	36
11	17	57	51	44	39
11	18	61	55	47	42
11	19	65	58	50	45
11	20	69	62	53	48
11	21	73	65	57	51
11	22	77	69	60	54
11	23	81	73	63	57
11	24	85	76	66	60
11	25	89	80	70	63
12	12	42	37	31	27
12	13	47	41	35	31
12	14	51	45	38	34
12	15	55	49	42	37
12	16	60	53	46	41
12	17	64	57	49	44
12	18	68	61	53	47
12	19	72	65	56	51
12	20	77	69	60	54
12	21	81	73	64	58
12	22	85	77	67	61
12	23	90	81	71	64
12	24	94	85	75	68
12	25	98	89	78	71
13	13	51	45	39	34
13	14	56	50	43	38
13	15	61	54	47	42
13	16	65	59	51	45
13	17	70	63	55	49
13	18	75	67	59	53
13	19	80	72	63	57
13	20	84	76	67	60
13	21	89	80	71	64
13	22	94	85	75	68
13	23	98	89	79	72
13	24	103	94	83	75
13	25	108	98	87	79
14	14	61	55	47	42
14	15	66	59	51	46
14	16	71	64	56	50
14	17	77	69	60	54
14	18	82	74	65	58
14	19	87	78	69	63
14	20	92	83	73	67
14	21	97	88	78	71
14	22	102	93	82	75
14	23	107	98	87	79
14	24	113	102	91	83
14	25	118	107	95	87
15	15	72	64	56	51
15	16	77	70	61	55
15	17	83	75	66	60
15	18	88	80	70	64
15	19	94	85	75	69
15	20	100	90	80	73
15	21	105	96	85	78
15	22	111	101	90	82
15	23	116	106	94	87
15	24	122	111	99	91
15	25	128	117	104	96
16	16	83	75	66	60
16	17	89	81	71	65
16	18	95	86	76	70
16	19	101	92	82	74
16	20	107	98	87	79
16	21	113	103	92	84
16	22	119	109	97	89
16	23	125	115	102	94
16	24	131	120	108	99
16	25	137	126	113	104
17	17	96	87	77	70
17	18	102	93	82	75
17	19	109	99	88	81
17	20	115	105	93	86
17	21	121	111	99	91
17	22	128	117	105	96
17	23	134	123	110	102
17	24	141	129	116	107
17	25	147	135	122	112
18	18	109	99	88	81
18	19	116	106	94	87
18	20	123	112	100	92
18	21	130	119	106	98
18	22	136	125	112	104
18	23	143	132	118	109
18	24	150	138	124	115
18	25	157	145	130	121
19	19	123	113	101	93
19	20	130	119	107	99
19	21	138	126	113	105
19	22	145	133	120	111
19	23	152	140	126	117
19	24	160	147	133	123
19	25	167	154	139	129
20	20	138	127	114	105
20	21	146	134	121	112
20	22	154	141	127	118
20	23	161	149	134	125
20	24	169	156	141	131
20	25	177	163	148	138
21	21	154	142	128	118
21	22	162	150	135	125
21	23	170	157	142	132
21	24	179	165	150	139
21	25	187	173	157	146
22	22	171	158	143	133
22	23	179	166	150	140
22	24	188	174	158	147
22	25	197	182	166	155
23	23	189	175	158	148
23	24	198	183	167	155
23	25	207	192	175	163
24	24	207	192	175	164
24	25	217	201	184	172
25	25	227	211	192	180

For larger values of m, n it is usually adequate to use a Normal approximation with continuity correction, with mean $\frac{1}{2}mn$ and variance $\frac{1}{12}mn(m+n+1)$.

Critical Values for the Wilcoxon Rank Sum 2-Sample Test

1 – tail		5%	2½%	1%	½%
2 – tail		10%	5%	2%	1%
m	*n*				
2	2	–	–	–	–
2	3	–	–	–	–
2	4	–	–	–	–
2	5	3	–	–	–
2	6	3	–	–	–
2	7	3	–	–	–
2	8	4	3	–	–
2	9	4	3	–	–
2	10	4	3	–	–
2	11	4	3	–	–
2	12	5	4	–	–
2	13	5	4	3	–
2	14	6	4	3	–
2	15	6	4	3	–
2	16	6	4	3	–
2	17	6	5	3	–
2	18	7	5	3	–
2	19	7	5	4	3
2	20	7	5	4	3
2	21	8	6	4	3
2	22	8	6	4	3
2	23	8	6	4	3
2	24	9	6	4	3
2	25	9	6	4	3
3	3	6	–	–	–
3	4	6	–	–	–
3	5	7	6	–	–
3	6	8	7	–	–
3	7	8	7	6	–
3	8	9	8	6	–
3	9	10	8	7	6
3	10	10	9	7	6
3	11	11	9	7	6
3	12	11	10	8	7
3	13	12	10	8	7

1 – tail		5%	2½%	1%	½%
2 – tail		10%	5%	2%	1%
m	*n*				
3	14	13	11	8	7
3	15	13	11	9	8
3	16	14	12	9	8
3	17	15	12	10	8
3	18	15	13	10	8
3	19	16	13	10	9
3	20	17	14	11	9
3	21	17	14	11	9
3	22	18	15	12	10
3	23	19	15	12	10
3	24	19	16	12	10
3	25	20	16	13	11
4	4	11	10	–	–
4	5	12	11	10	–
4	6	13	12	11	10
4	7	14	13	11	10
4	8	15	14	12	11
4	9	16	14	13	11
4	10	17	15	13	12
4	11	18	16	14	12
4	12	19	17	15	13
4	13	20	18	15	13
4	14	21	19	16	14
4	15	22	20	17	15
4	16	24	21	17	15
4	17	25	21	18	16
4	18	26	22	19	16
4	19	27	23	19	17
4	20	28	24	20	18
4	21	29	25	21	18
4	22	30	26	21	19
4	23	31	27	22	19
4	24	32	27	23	20
4	25	33	28	23	20

The critical values in these tables are for the Wilcoxon Rank Sum 2-sample test statistic, *W*. Critical values for the Mann-Whitney test statistic, *T*, may be derived by subtracting $\frac{1}{2}m(m+1)$ (where *m* is the size of the sample from which the rank sum has been obtained).

1 – tail		5%	2½%	1%	½%
2 – tail		10%	5%	2%	1%
m	*n*				
5	5	19	17	16	15
5	6	20	18	17	16
5	7	21	20	18	16
5	8	23	21	19	17
5	9	24	22	20	18
5	10	26	23	21	19
5	11	27	24	22	20
5	12	28	26	23	21
5	13	30	27	24	22
5	14	31	28	25	22
5	15	33	29	26	23
5	16	34	30	27	24
5	17	35	32	28	25
5	18	37	33	29	26
5	19	38	34	30	27
5	20	40	35	31	28
5	21	41	37	32	29
5	22	43	38	33	29
5	23	44	39	34	30
5	24	45	40	35	31
5	25	47	42	36	32
6	6	28	26	24	23
6	7	29	27	25	24
6	8	31	29	27	25
6	9	33	31	28	26

1 – tail		5%	2½%	1%	½%
2 – tail		10%	5%	2%	1%
m	*n*				
6	10	35	32	29	27
6	11	37	34	30	28
6	12	38	35	32	30
6	13	40	37	33	31
6	14	42	38	34	32
6	15	44	40	36	33
6	16	46	42	37	34
6	17	47	43	39	36
6	18	49	45	40	37
6	19	51	46	41	38
6	20	53	48	43	39
6	21	55	50	44	40
6	22	57	51	45	42
6	23	58	53	47	43
6	24	60	54	48	44
6	25	62	56	50	45
7	7	39	36	34	32
7	8	41	38	35	34
7	9	43	40	37	35
7	10	45	42	39	37
7	11	47	44	40	38
7	12	49	46	42	40
7	13	52	48	44	41
7	14	54	50	45	43
7	15	56	52	47	44
7	16	58	54	49	46
7	17	61	56	51	47
7	18	63	58	52	49
7	19	65	60	54	50
7	20	67	62	56	52
7	21	69	64	58	53
7	22	72	66	59	55
7	23	74	68	61	57
7	24	76	70	63	58
7	25	78	72	64	60

1 – tail		5%	2½%	1%	½%
2 – tail		10%	5%	2%	1%
m	*n*				
8	8	51	49	45	43
8	9	54	51	47	45
8	10	56	53	49	47
8	11	59	55	51	49
8	12	62	58	53	51
8	13	64	60	56	53
8	14	67	62	58	54
8	15	69	65	60	56
8	16	72	67	62	58
8	17	75	70	64	60
8	18	77	72	66	62
8	19	80	74	68	64
8	20	83	77	70	66
8	21	85	79	72	68
8	22	88	81	74	70
8	23	90	84	76	71
8	24	93	86	78	73
8	25	96	89	81	75
9	9	66	62	59	56
9	10	69	65	61	58
9	11	72	68	63	61
9	12	75	71	66	63
9	13	78	73	68	65
9	14	81	76	71	67
9	15	84	79	73	69
9	16	87	82	76	72
9	17	90	84	78	74
9	18	93	87	80	76
9	19	96	90	83	78
9	20	99	93	85	81
9	21	102	95	88	83
9	22	105	98	90	85
9	23	108	101	93	88
9	24	111	104	95	90
9	25	114	107	98	92

CRITICAL VALUES FOR THE WILCOXON RANK SUM 2-SAMPLE TEST

m	n	1 – tail 5% / 2 – tail 10%	1 – tail 2½% / 2 – tail 5%	1 – tail 1% / 2 – tail 2%	1 – tail ½% / 2 – tail 1%
10	10	82	78	74	71
10	11	86	81	77	73
10	12	89	84	79	76
10	13	92	88	82	79
10	14	96	91	85	81
10	15	99	94	88	84
10	16	103	97	91	86
10	17	106	100	93	89
10	18	110	103	96	92
10	19	113	107	99	94
10	20	117	110	102	97
10	21	120	113	105	99
10	22	123	116	108	102
10	23	127	119	110	105
10	24	130	122	113	107
10	25	134	126	116	110
11	11	100	96	91	87
11	12	104	99	94	90
11	13	108	103	97	93
11	14	112	106	100	96
11	15	116	110	103	99
11	16	120	113	107	102
11	17	123	117	110	105
11	18	127	121	113	108
11	19	131	124	116	111
11	20	135	128	119	114
11	21	139	131	123	117
11	22	143	135	126	120
11	23	147	139	129	123
11	24	151	142	132	126
11	25	155	146	136	129
12	12	120	115	109	105
12	13	125	119	113	109
12	14	129	123	116	112
12	15	133	127	120	115
12	16	138	131	124	119
12	17	142	135	127	122
12	18	146	139	131	125
12	19	150	143	134	129
12	20	155	147	138	132
12	21	159	151	142	136
12	22	163	155	145	139
12	23	168	159	149	142
12	24	172	163	153	146
12	25	176	167	156	149
13	13	142	136	130	125
13	14	147	141	134	129
13	15	152	145	138	133
13	16	156	150	142	136
13	17	161	154	146	140
13	18	166	158	150	144
13	19	171	163	154	148
13	20	175	167	158	151
13	21	180	171	162	155
13	22	185	176	166	159
13	23	189	180	170	163
13	24	194	185	174	166
13	25	199	189	178	170
14	14	166	160	152	147
14	15	171	164	156	151
14	16	176	169	161	155
14	17	182	174	165	159
14	18	187	179	170	163
14	19	192	183	174	168
14	20	197	188	178	172
14	21	202	193	183	176
14	22	207	198	187	180
14	23	212	203	192	184
14	24	218	207	196	188
14	25	223	212	200	192
15	15	192	184	176	171
15	16	197	190	181	175
15	17	203	195	186	180
15	18	208	200	190	184
15	19	214	205	195	189
15	20	220	210	200	193
15	21	225	216	205	198
15	22	231	221	210	202
15	23	236	226	214	207
15	24	242	231	219	211
15	25	248	237	224	216
16	16	219	211	202	196
16	17	225	217	207	201
16	18	231	222	212	206
16	19	237	228	218	210
16	20	243	234	223	215
16	21	249	239	228	220
16	22	255	245	233	225
16	23	261	251	238	230
16	24	267	256	244	235
16	25	273	262	249	240
17	17	249	240	230	223
17	18	255	246	235	228
17	19	262	252	241	234
17	20	268	258	246	239
17	21	274	264	252	244
17	22	281	270	258	249
17	23	287	276	263	255
17	24	294	282	269	260
17	25	300	288	275	265
18	18	280	270	259	252
18	19	287	277	265	258
18	20	294	283	271	263
18	21	301	290	277	269
18	22	307	296	283	275
18	23	314	303	289	280
18	24	321	309	295	286
18	25	328	316	301	292
19	19	313	303	291	283
19	20	320	309	297	289
19	21	328	316	303	295
19	22	335	323	310	301
19	23	342	330	316	307
19	24	350	337	323	313
19	25	357	344	329	319
20	20	348	337	324	315
20	21	356	344	331	322
20	22	364	351	337	328
20	23	371	359	344	335
20	24	379	366	351	341
20	25	387	373	358	348
21	21	385	373	359	349
21	22	393	381	366	356
21	23	401	388	373	363
21	24	410	396	381	370
21	25	418	404	388	377
22	22	424	411	396	386
22	23	432	419	403	393
22	24	441	427	411	400
22	25	450	435	419	408
23	23	465	451	434	424
23	24	474	459	443	431
23	25	483	468	451	439
24	24	507	492	475	464
24	25	517	501	484	472
25	25	552	536	517	505

For larger values of m, n it is usually adequate to use a Normal approximation, with continuity correction,

with mean $\frac{1}{2}mn + \frac{1}{2}m(m+1)$ and variance $\frac{1}{12}mn(m+n+1)$

Critical values for the Wilcoxon Single Sample and Paired Sample tests

1 – tail	5%	2½%	1%	½%
2 – tail	10%	5%	2%	1%
n				
2	–	–	–	–
3	–	–	–	–
4	–	–	–	–
5	0	–	–	–
6	2	0	–	–
7	3	2	0	–
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37
21	67	58	49	42
22	75	65	55	48
23	83	73	62	54
24	91	81	69	61
25	100	89	76	68

1 – tail	5%	2½%	1%	½%
2 – tail	10%	5%	2%	1%
n				
26	110	98	84	75
27	119	107	92	83
28	130	116	101	91
29	140	126	110	100
30	151	137	120	109
31	163	147	130	118
32	175	159	140	128
33	187	170	151	138
34	200	182	162	148
35	213	195	173	159
36	227	208	185	171
37	241	221	198	182
38	256	235	211	194
39	271	249	224	207
40	286	264	238	220
41	302	279	252	233
42	319	294	266	247
43	336	310	281	261
44	353	327	296	276
45	371	343	312	291
46	389	361	328	307
47	407	378	345	322
48	426	396	362	339
49	446	415	379	355
50	466	434	397	373

For larger values of *n*, the Normal approximation with mean $\frac{n(n+1)}{4}$,

variance $\frac{n(n+1)(2n+1)}{24}$ should be used for $T = \min[P,Q]$.

Action and Warning lines for Shewhart Chart for Ranges

Group Size	Action Lines		Warning Lines	
n	D_1	D_2	D_3	D_4
2	0.00	4.12	0.04	2.81
3	0.04	2.99	0.18	2.18
4	0.10	2.58	0.29	1.94
5	0.16	2.36	0.37	1.80
6	0.21	2.22	0.42	1.72
7	0.26	2.12	0.46	1.66
8	0.29	2.05	0.50	1.62
9	0.33	1.99	0.52	1.58
10	0.35	1.94	0.54	1.55

The action and warning lines are obtained by multiplying the values in the table by the mean range of the values obtained from the process.

Random Numbers

68236	35335	71329	96803	24413
62385	36545	59305	59948	17232
64058	80195	30914	16664	50818
64822	68554	90952	64984	92295
17716	22164	05161	04412	59002
03928	22379	92325	79920	99070
11021	08533	83855	37723	77339
01830	68554	86787	90447	54796
36782	73208	93548	77405	58355
58158	45059	83980	40176	40737
91239	10532	27993	11516	61327
27073	98804	60544	12133	01422
81501	00633	62681	84319	03374
64374	26598	54466	94768	19144
29896	26739	30871	29795	13472
38996	72151	65746	16513	62796
73936	81751	00149	99126	23117
18795	93118	84105	18307	49807
76816	99822	92314	45035	43490
12091	60413	90467	42457	50490
41538	19059	69055	94355	84262
12909	04950	14986	08205	53582
49185	94608	87317	37725	66450
37771	48526	14939	32848	77677
22532	13814	69092	78342	37774
60132	24386	10989	54346	41531
23784	56693	45902	33406	53867
03081	20189	77226	89923	67301
51273	64049	19919	45518	43243
03281	40214	60679	68712	71636

Estimation of standard deviation from range

n	a_n	n	a_n	n	a_n	n	a_n
2	0.8862	5	0.4299	8	0.3512	11	0.3152
3	0.5908	6	0.3946	9	0.3367	12	0.3069
4	0.4857	7	0.3698	10	0.3249	13	0.2998

Random permutations (size 4)

3 1 2 4	2 4 3 1	4 3 2 1
2 3 1 4	4 3 1 2	3 1 4 2
4 2 3 1	3 1 2 4	1 4 3 2
1 3 2 4	1 4 2 3	4 3 2 1
2 4 3 1	1 2 4 3	1 3 2 4
4 3 1 2	2 4 3 1	3 4 1 2
2 1 4 3	4 1 2 3	3 4 1 2
4 3 1 2	3 1 2 4	3 2 1 4
2 3 4 1	3 4 2 1	1 4 2 3
3 2 1 4	3 4 2 1	3 4 1 2
1 4 3 2	4 3 2 1	2 1 4 3
1 4 2 3	2 3 1 4	1 4 2 3
2 3 1 4	3 1 4 2	4 2 3 1
4 1 3 2	2 4 1 3	4 2 3 1
2 1 4 3	1 3 4 2	3 2 4 1
2 3 1 4	1 3 4 2	2 3 1 4
2 3 4 1	2 4 3 1	3 2 1 4
3 1 4 2	3 1 2 4	4 1 2 3
4 3 2 1	4 2 1 3	4 2 3 1
2 3 4 1	4 1 2 3	2 3 4 1
1 4 2 3	2 4 3 1	4 1 3 2
2 4 3 1	4 1 2 3	1 3 4 2
1 2 4 3	1 2 3 4	4 3 2 1
2 3 4 1	1 3 4 2	2 4 3 1
4 3 1 2	4 3 2 1	1 4 3 2
1 4 2 3	3 2 1 4	2 1 4 3
3 4 1 2	1 4 3 2	1 4 2 3
2 4 1 3	4 2 3 1	3 4 2 1
1 4 3 2	4 3 2 1	2 3 1 4
1 3 2 4	4 2 1 3	4 2 1 3
1 4 3 2	3 2 1 4	4 1 2 3
2 4 1 3	3 4 1 2	4 3 1 2
3 4 1 2	4 3 1 2	3 4 2 1
4 2 1 3	2 3 4 1	4 3 1 2
3 2 4 1	4 3 2 1	2 3 4 1
3 2 4 1	4 3 1 2	3 2 1 4
1 2 3 4	2 4 1 3	1 3 4 2
4 3 1 2	3 4 1 2	4 1 3 2
1 2 4 3	1 4 3 2	2 1 4 3
4 1 2 3	3 1 2 4	4 3 1 2

Random permutations (size 5)

5 2 3 4 1	4 2 3 5 1	3 1 5 4 2
2 5 1 3 4	3 1 2 4 5	5 3 2 4 1
4 5 3 2 1	2 1 4 3 5	2 1 5 4 3
2 5 3 4 1	1 5 3 4 2	1 4 3 2 5
5 2 3 1 4	5 3 4 1 2	2 5 4 3 1
3 5 1 4 2	5 4 3 2 1	5 1 4 3 2
2 3 4 1 5	4 5 2 3 1	2 5 3 4 1
1 2 5 4 3	2 4 5 3 1	3 4 1 2 5
2 4 1 5 3	1 2 3 5 4	4 1 2 5 3
2 5 1 3 4	3 5 2 1 4	5 4 2 1 3
3 4 1 5 2	5 2 3 1 4	3 2 1 5 4
2 1 5 3 4	3 1 4 2 5	1 4 5 2 3
2 4 1 3 5	3 1 5 2 4	1 2 3 5 4
5 1 3 2 4	4 2 3 5 1	4 5 1 3 2
3 2 4 1 5	1 5 3 4 2	1 3 5 2 4
5 2 4 3 1	1 5 2 4 3	3 4 1 5 2
3 2 4 5 1	4 5 3 1 2	5 3 1 4 2
3 4 1 5 2	1 5 3 4 2	3 5 4 1 2
4 2 1 5 3	1 5 3 4 2	1 2 5 4 3
4 2 1 5 3	2 3 5 1 4	5 1 4 3 2
2 1 4 3 5	1 4 3 5 2	5 2 4 3 1
5 3 2 4 1	1 3 5 4 2	5 1 4 2 3
2 4 3 5 1	3 5 2 1 4	2 5 4 1 3
4 1 5 3 2	1 3 5 2 4	4 1 5 3 2
2 4 5 1 3	3 5 4 1 2	4 1 5 3 2
5 3 4 1 2	1 2 3 4 5	5 4 3 1 2
5 1 2 4 3	4 3 1 2 5	2 1 3 5 4
5 2 4 1 3	5 2 3 1 4	4 3 5 2 1
4 5 2 1 3	2 5 3 4 1	2 4 3 5 1
5 2 4 1 3	2 5 3 1 4	3 1 5 4 2
3 5 4 1 2	5 4 2 3 1	3 5 4 1 2
5 2 1 4 3	1 5 3 2 4	3 4 5 2 1
5 1 3 4 2	1 2 5 3 4	2 5 4 1 3
2 1 5 3 4	3 5 4 2 1	2 1 5 4 3
2 3 4 5 1	3 4 2 5 1	3 5 2 1 4
1 3 5 4 2	4 1 5 3 2	5 3 1 2 4
5 1 4 2 3	5 3 1 4 2	5 1 3 4 2
3 5 1 2 4	1 5 2 3 4	1 5 3 2 4
3 4 1 5 2	4 3 5 1 2	1 5 2 4 3
2 5 1 3 4	1 2 3 5 4	2 3 5 1 4

Random permutations (size 10)

5 8 1 6 7 9 2 10 3 4	8 2 5 7 9 10 1 4 6 3
7 2 4 8 6 1 3 10 5 9	3 5 8 6 1 9 2 10 7 4
10 1 2 4 9 3 7 5 6 8	9 1 3 10 7 4 6 5 8 2
5 2 6 7 1 3 10 9 4 8	6 3 10 9 7 4 5 1 2 8
5 1 8 4 9 6 3 10 7 2	8 4 10 3 9 5 7 6 1 2
3 2 7 6 10 8 5 1 4 9	6 9 2 5 8 3 10 4 7 1
10 6 5 9 7 4 3 1 2 8	9 6 1 4 2 5 10 7 3 8
8 4 7 9 10 6 3 1 2 5	9 1 6 8 2 3 10 5 7 4
5 9 2 4 3 7 1 6 8 10	4 6 10 8 1 9 7 5 3 2
10 8 7 4 5 9 3 6 2 1	5 6 2 7 1 4 8 9 3 10
6 9 5 7 2 10 8 3 4 1	10 6 8 2 9 5 4 1 7 3
10 8 2 6 3 7 4 9 5 1	7 3 6 5 2 8 9 10 4 1
6 3 4 8 5 10 2 9 7 1	9 6 1 3 4 8 10 2 5 7
9 4 7 10 6 2 1 5 8 3	2 6 5 1 9 8 7 3 4 10
9 3 7 8 2 5 4 6 1 10	8 4 7 9 1 10 5 6 2 3
3 2 9 4 1 6 10 7 8 5	8 1 9 10 3 5 4 2 6 7
7 2 10 9 1 4 3 5 8 6	9 10 8 1 4 2 6 7 3 5
1 10 3 4 6 2 9 8 7 5	2 9 8 4 5 6 1 10 3 7
7 9 2 1 6 3 10 4 8 5	10 2 1 7 4 9 8 5 3 6
1 5 6 9 2 8 3 7 4 10	10 6 3 1 8 9 7 2 4 5
10 4 7 5 8 3 1 6 2 9	4 8 7 9 6 10 3 5 2 1
9 10 6 5 7 3 1 8 4 2	2 3 1 8 9 5 6 10 4 7
4 8 2 9 10 1 7 5 3 6	9 2 8 1 10 6 3 5 7 4
3 8 5 2 9 7 4 6 10 1	10 5 8 2 6 9 4 1 7 3
9 3 4 7 1 6 10 2 5 8	3 4 1 2 9 5 8 10 7 6
1 4 7 3 8 10 5 6 9 2	1 3 4 2 6 10 9 5 8 7
3 7 8 9 5 4 2 6 10 1	3 4 5 8 9 7 10 1 2 6
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2 8 5 4 6 10 1 3 7 9	2 4 6 8 7 1 9 3 10 5
9 10 8 6 7 2 3 1 4 5	8 5 6 4 1 7 10 2 3 9
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6 4 3 2 10 5 7 8 9 1	9 8 1 10 5 3 2 7 4 6
4 7 8 1 6 10 2 9 5 3	8 10 7 4 6 3 5 2 9 1
1 7 8 9 3 4 2 6 5 10	2 10 4 1 9 7 6 3 8 5
6 9 1 3 7 2 5 8 4 10	7 9 3 6 4 10 1 2 5 8
5 9 2 7 10 3 4 6 1 8	8 1 9 2 3 7 4 10 5 6