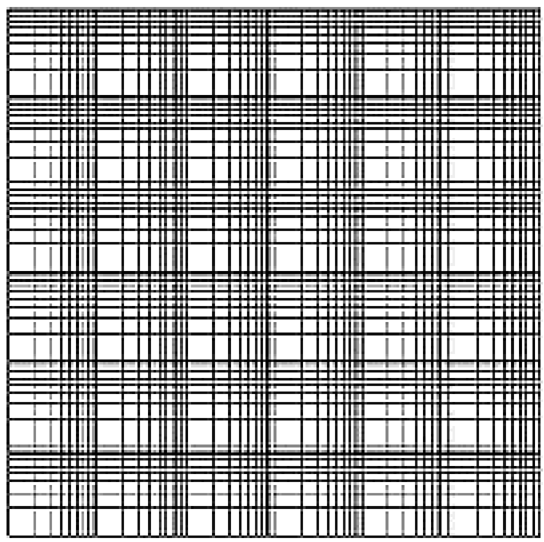
Stars and Slopes

Engagement

First, let us remember what a base 10 logarithm is: when you calculate the logarithm of a number (log), you are calculating the power to which 10 is raised to obtain that number. For example, log1=0 (i.e., $10^0 = 1$), log10=1 (i.e., $10^1 = 10$), log100=2 (i.e., $10^2 = 100$), etc. A sample of logarithmic graph paper is shown below. Notice that is has logarithmic scales on both axes (as opposed to the linear scales you may be used to) and the tick marks, or grids, are made on the axes according to the logarithms of numbers. The divisions from 1-10, 10-100, 100-1000 along the axes are called "cycles".



(A larger version is at the end of this handout)

Let us now turn to a simple physics lab for an example. For many relationships in the real-world, if you plot the data on a rectangular coordinate system, you do not get a straight line. Instead, you get a curve such as a hyperbola, a parabola, or some form of power or exponential curve. In this lesson, we concentrate on power laws, i.e. variables which have a relationship which can be expressed as $Y= k X^n$, where n can have any value.

The following data (see next page) have been gathered from an experiment meant to determine the relationship which exists between the diameter of a ring and its period as a pendulum. Five steel rings with varying diameters were individually suspended from a knife edge mounted on the wall, and caused to swing back and forth about this axis. Each diameter was measured, and each period was determined by measuring the number of cycles per unit of time.

Ring Diameter (cm)	3.51	7.26	13.7	28.5	38.7
Time for Completing 25 Swings (sec)	9.35	13.3	19.2	26.98	32.88

We expect these data to follow a relationship of the form $T = Ad^n$ where T is the period of oscillation, A is the constant of proportionality, d is the diameter of the ring, and n is a constant. Given this, and our desire to end up with a straight line on our graph, we consider the following:

if $T = Ad^n$, then $\log T = \log A + n \log d$.

This equation should have a very familiar form to you - it is the equation for a straight line if you plot log T vs. log d, regardless of the value of n.

Now, plot the data in the following ways: (1) as log T versus log d on Cartesian graph paper; and (2) as T versus d on log-log graph paper. Compare the two plots and answer the following questions:

1. What do you see?

2. What are the values of A and n?

3. How do these values compare with the equation for the period of oscillation of a simple pendulum,

$$2\pi\sqrt{\frac{l}{g}}$$

where g is the acceleration due to gravity, l is the length of the pendulum, and π= 3.14?

4. For a pendulum and a ring to have equal periods of oscillation, what must be true?

5. Given any set of measurements for T and d, what value for the acceleration due to gravity here on Earth do you calculate?

Shown below are the appropriate plots of data from the previous experiment.