

A-level Physics

OCR B (Advancing Physics)

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Contents

Articles

AS	1
Optics	2
Lenses	2
Refraction	5
Communications	7
Digital Storage	7
Digital Processing	8
Digitisation	11
Signal Frequencies	13
Bandwidth	17
Electricity	18
Charge	18
Current	19
Voltage	20
Power	22
Resistance and Conductance	23
Internal Resistance	25
Potential Dividers	26
Sensors	28
Resistivity and Conductivity	30
Semiconductors	31
Material Structure	32
Stress, Strain & the Young Modulus	32
Metals	34
Polymers	35
Waves	38
What is a wave?	38
Phasors	41
Standing Waves	42

Young's Slits	44
Diffraction	47
Finding the Distance of a Remote Object	48
Quantum Physics	49
Light	49
Quantum Behaviour	51
Electron Behaviour	54
Mechanics	56
Vectors	56
Graphs	59
Kinematics	62
Forces and Power	64
A2	66
Decay	67
Exponential Relationships	67
Capacitors	69
Radioactive Decay	72
Half-lives	73
Gravity	75
Force	75
Field	77
Potential Energy	78
Potential	80
Mechanics	81
Simple Harmonic Motion	81
Energy in Simple Harmonic Motion	84
Damping	85
Resonance	86
Conservation of Momentum	88
Forces and Impulse in Collisions	89
Rockets, Hoses and Machine Guns	90
Circular Motion	91
Astrophysics	93

Radar and Triangulation	93
Large Units	95
Orbits	96
Doppler Effect	97
The Big Bang	99
Thermodynamics	101
Heat and Energy	101
Specific Heat Capacity	102
Ideal Gases	103
Kinetic Theory	105
Boltzmann Factor	106
Magnetic Fields	108
Flux	108
Induction	109
Force	110
Transformers	113
Motors	115
Generators	117
Electric Fields	119
Force	119
Field	120
Potential	122
Potential Energy	123
Particle Physics	125
The Standard Model	125
Quarks	127
Bosons	128
Leptons	130
Millikan's Experiment	131
Pair Production and Annihilation	132
Particle Accelerators	133
Cloud Chambers and Mass Spectrometers	134
Nuclear Physics	138
Quantum Principles	138

Radioactive Emissions	139
Energy Levels	140
Fission	141
Fusion	142
Binding Energy	143
Risks, Doses and Dose Equivalents	145
Appendices	147
Trigonometry	147
Logarithms	148
Delta - 'difference in'	148
Sigma - 'sum of'	149
Derivation of Equations for Simple Harmonic Motion	149
Worked Solutions	151
Lenses	151
Refraction	152
Digital Storage	153
Digital Processing	154
Digitisation	155
Signal Frequencies	156
Bandwidth	157
Charge	158
Current	158
Voltage	159
Power	159
Resistance and Conductance	160
Internal Resistance	161
Potential Dividers	162
Sensors	163
Resistivity and Conductivity	165
Semiconductors	166
Stress, Strain & the Young Modulus	166
Metals	168
Polymers	168
What is a wave?	169
Phasors	170
Standing Waves	170

Young's Slits	171
Diffraction	172
Light	172
Electron Behaviour	173
Vectors	174
Graphs	175
Kinematics	178
Forces and Power	179
Exponential Relationships	179
Capacitors	181
Radioactive Decay	182
Half-lives	183
Gravitational Force	184
Gravitational Field	185
Gravitational Potential Energy	186
Gravitational Potential	187
Simple Harmonic Motion	188
Energy in Simple Harmonic Motion	190
Damping	191
Conservation of Momentum	193
Forces and Impulse in Collisions	195
Rockets, Hoses and Machine Guns	196
Circular Motion	197
Radar and Triangulation	198
Large Units	199
Orbits	199
Doppler Effect	200
The Big Bang	201
Heat and Energy	201
Specific Heat Capacity	202
Ideal Gases	202
Kinetic Theory	203
Boltzmann Factor	204
Magnetic Flux	205
Induction	206
Magnetic Force	207
Transformers	208
Motors	209

Generators	210
Electric Force	211
Electric Field	212
Electric Potential	213
Electric Potential Energy	215
The Standard Model	215
Quarks	216
Bosons	216
Leptons	217
Millikan's Experiment	218
Pair Production and Annihilation	219
Particle Accelerators	220
Cloud Chambers and Mass Spectrometers	221
Radioactive Emissions	222
Energy Levels	223
Fission	224
Fusion	225
Binding Energy	225
Risks, Doses and Dose Equivalents	226

References

Article Sources and Contributors	227
Image Sources, Licenses and Contributors	231

Article Licenses

License	234
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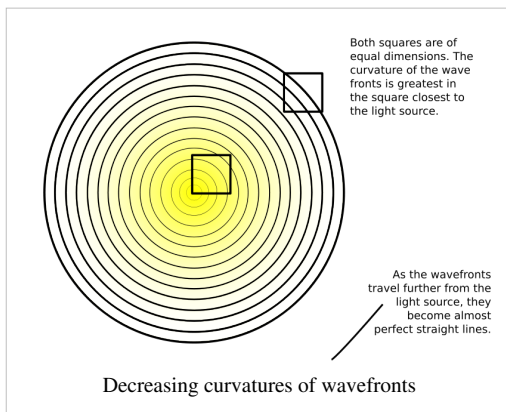
AS

Optics

Lenses

Curvature of Wavefronts

Light can be viewed as beams travelling between points. However, from most light sources, the light radiates outwards as a series of wavefronts. Light from a light source is bent - wavefronts of light have a property known as curvature.



As light travels further away from its source, its curvature decreases. Consider a sphere expanding gradually from a point, which represents a given wavefront of light. As the sphere expands, the curvature of its surface decreases when we look at any part of the surface with a constant area. It should be noted at this point that light from a source infinitely far away has 0 curvature - it is straight. This is useful, as ambient light (light from a source that is far away) can be assumed to have a curvature of 0, as the difference between this and its actual curvature is negligible.

The curvature of a wavefront is given as:

$$C = \frac{1}{f},$$

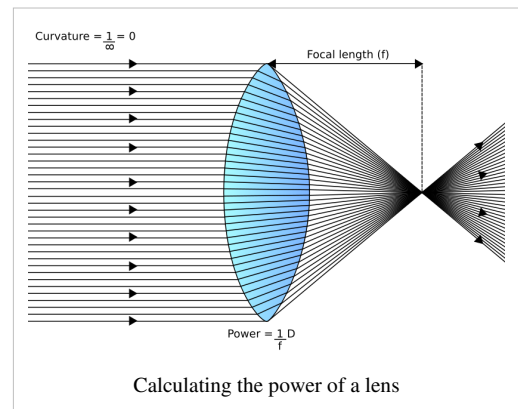
where v is the distance from the wavefront to the in-focus image depicted by the light. Curvature is measured in dioptres (D).

Power of lenses

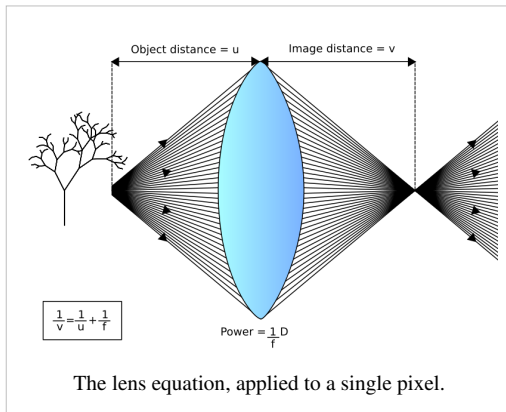
The function of a lens is to increase or decrease the curvature of a wavefront. Lenses have a 'power'. This is the curvature which the lens adds to the wavefront. Power is measured in dioptres, and is given by the formula:

$$P = \frac{1}{f},$$

where f equals the focal length of the lens. This is the distance between the lens and the point where an image will be in focus, if the wavefronts entering the other side of the lens are parallel.



The Lens Equation



Overall, then, the formula relating the curvature of the wavefronts leaving a lens to the curvature of the wavefronts entering it is:

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

where v is the distance between the lens and the in-focus image formed, u is the distance between the object which the in-focus image is of, and f is the focal length of the lens. The power of the lens can be substituted in for the reciprocal of f , as they are the same thing.

The Cartesian Convention

If we were to place a diagram of the lens on a grid, labelled with cartesian co-ordinates, we would discover that measuring the distance of the object distance is negative, in comparison to the image distance. As a result, the value for u must always be negative. This is known as the Cartesian convention.

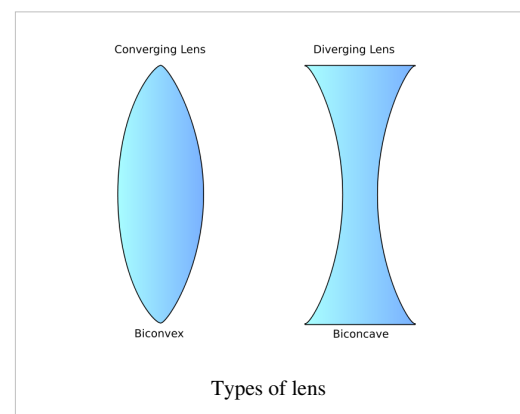
This means that, if light enters the lens with a positive curvature, it will leave with a negative curvature unless the lens is powerful enough to make the light leave with a positive curvature.

Types of Lens

There are two types of lens:

Converging lenses add curvature to the wavefronts, causing them to converge more. These have a positive power, and have a curved surface which is wider in the middle than at the rim.

Diverging lenses remove curvature from the wavefronts, causing them to diverge more. These have a negative power, and have a curved surface with a dip in the middle.



Magnification

Magnification is a measure of how much an image has been enlarged by a lens. It is given by the formula:

$$M = \frac{h_2}{h_1}$$

where h_1 and h_2 are the heights of the image (or object) before and after being magnified, respectively. If an image is shrunk by a lens, the magnification is between 0 and 1.

Magnification can also be given as:

$$M = \frac{v}{u}$$

where v and u are the image and object distances. Therefore:

$$M = \frac{h_2}{h_1} = \frac{v}{u}$$

An easy way to remember this in the middle of an exam is the formula:

$$I = AM$$

where I is image size, A is actual size of the object M is the magnification factor.

Questions

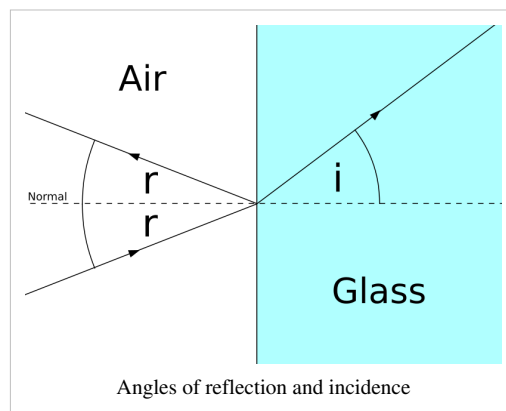
1. A lens has a focal length of 10cm. What is its power, in dioptres?
2. Light reflected off a cactus 1.5m from a 20D lens forms an image. How many metres is it from the other side of the lens?
3. A lens in an RGB projector causes an image to focus on a large screen. What sort of lens is it? Is its power positive or negative?
4. What is the focal length of a 100D lens?
5. The film in a camera is 5mm from a lens when automatically focussed on someone's face, 10m from the camera. What is the power of the lens?
6. The light from a candle is enlarged by a factor of 0.5 by a lens, and produces an image of a candle, 0.05m high, on a wall. What is the height of the candle?

/Worked Solutions/

Refraction

Reflection

Reflection is when light 'bounces' off a material which is different to the one in which it is travelling. You may remember from GCSE (or equivalent) level that we can calculate the direction the light will take if we consider a line known as the 'normal'. The normal is perpendicular to the boundary between the two materials, at the point at which the light is reflected. The angle between the normal and the ray of light is known as the angle of reflection (r). The ray of light will be reflected back at the same angle as it arrived at the normal, on the other side of the normal.



Refraction

Refraction is when light changes velocity when it travels across the boundary between two materials. This causes it to change direction. The angle between the normal and the refracted ray of light is known as the angle of incidence (i).

The Refractive Index

The refractive index is a measure of how much light will be refracted on the boundary between a material and a 'reference material'. This reference material is usually either air or a vacuum. It is given by the following formula:

$$n = \frac{c_0}{c_1}$$

where c_0 is the speed of light in a vacuum (3×10^8 m/s) and c_1 is the speed of light in the material.

Snell's Law

We can relate the refractive index to the angles of incidence and reflection using the following formula, known as Snell's Law:

$$n = \frac{\sin i}{\sin r} = \frac{c_0}{c_1}$$

Total Internal Reflection

Normally, when light passes through a non-opaque material, it is both reflected and refracted. However, sometimes, rays of light are totally internally reflected; in other words, they are not refracted, so no light goes outside the material. This is useful in optic fibres, which allow a signal to be transmitted long distances at the speed of light because the light is totally internally reflected.

Critical Angle

The critical angle is the minimum angle of reflection, for a given material, at which rays of light are totally internally reflected. At the critical angle (C), the angle of incidence must be 90° , as any smaller angle of incidence will result in refraction. Therefore:

$$n = \frac{\sin 90}{\sin r}$$

Since $\sin 90^\circ = 1$:

$$n = \frac{1}{\sin r}$$

$$\sin r = \frac{1}{n} = \sin C$$

In word form, in a material with refractive index n , light will be totally internally reflected at angles greater than the inverse sine of the reciprocal of the refractive index.

Questions

1. A ray of light is reflected from a mirror. Its angle to the normal when it reaches the mirror is 70° . What is its angle of reflection?
2. The speed of light in diamond is 1.24×10^8 m/s. What is its refractive index?
3. The refractive index of ice is 1.31. What is the speed of light in ice?
4. A ray of light passes the boundary between air and a transparent material. The angle of reflection is 20° , and the angle of incidence is 10° . What is the speed of light in this material? Why is it impossible for this material to exist?
5. What is the critical angle of a beam of light leaving a transparent material with a refractive index of 2?

/Worked Solutions/

See also

- Optics/Refraction

Communications

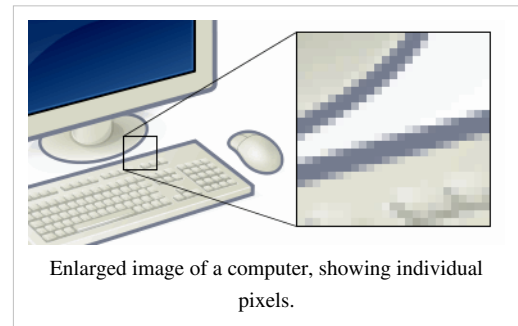
Digital Storage

Digital Data

There are two different types of data: analogue and digital. Analogue data can, potentially, take on any value. Examples include a page of handwritten text, a cassette, or a painting. Digital data can only take on a set range of values. This enables it to be processed by a computer. Examples include all files stored on computers, CDs, DVDs, etc.

Pixels

Digital images are made up of pixels. A pixel represents the value of an individual square of the image, and it has a value assigned to it. The total number of pixels in an image is just like the formula for the area of a rectangle: number of pixels across multiplied by number of pixels down. When representing text, each pixel is one character (for example, a letter, a number, a space, or a new line).



Bits

Each pixel's value is digital: it takes on a definite value. In a higher quality image, each pixel can take on a greater variety of values. Each pixel's value is encoded as a number of bits. A bit is a datum with a value of either 0 or 1. The more values a pixel can take on, the more bits must be used to represent its value. The number of values (N) that a pixel represented by I bits can take on is given by the formula:

$$N = 2^I$$

Hence:

$$I = \frac{\log N}{\log 2} \approx \frac{\log N}{0.3} \text{ (What on earth is this?)}$$

A pixel may be represented by values for red, green and blue, in which case each colour channel will have to be encoded separately. When dealing with text, the number of values is equal to the number of possible characters.

Overall, for an image:

Amount of information in an image (bits) = number of pixels x bits per pixel.

Bytes

A byte is equal to 8 bits. The major difference between bytes and SI units is that when prefixes (such as kilo-, mega-, etc.) are attached, we do not multiply by 10^3 as the prefix increases. Instead, we multiply by 1024. So, 1 kilobyte = 1024 bytes, 1 megabyte = 1024^2 bytes, 1 gigabyte = 1024^3 bytes, and 1 terabyte = 1024^4 bytes.

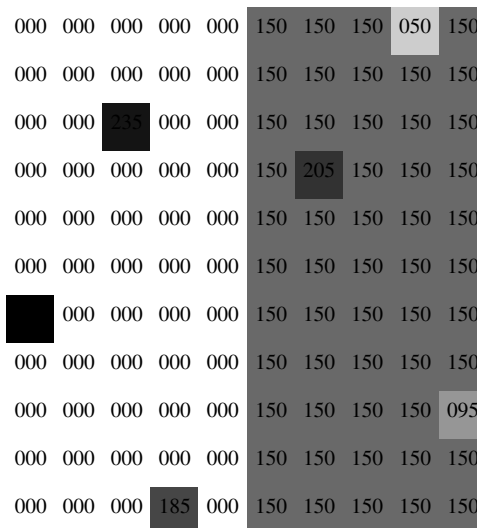
Questions

1. An image transmitted down a SVGA video cable is 800 pixels wide, and 600 pixels high. How many pixels are there in the image?
2. A grayscale image is encoded using 3 bits. How many possible values can each pixel have?
3. The characters in a text document are numbered from 0 - 255. How many bits should each character be encoded with?
4. A page contains 30 lines of text, with an average of 15 characters on each line. Each character is represented by 4 bits. How many megabytes of uncompressed storage will a book consisting of 650 pages like this fill on a computer's hard disk?
5. A 10cm wide square image is scanned into a computer. Each pixel is encoded using 3 channels (red, green and blue), and each channel can take on 256 possible values. One pixel is 0.01 mm wide. How much information does the scanned image contain? Express your answer using an appropriate unit.

/Worked Solutions/

Digital Processing

As we have already seen, a digital image consists of pixels, with each pixel having a value which represents its colour. For the purposes of understanding how digital images are manipulated, we are going to consider an 8-bit grayscale image, with pixel values ranging from 0 to 255, giving us 256 (2^8) levels of grey. 0 represents white, and 255 represents black. This is the image we are going to consider:



The image consists of an edge, and some random noise. There are two methods of smoothing this image (ie. removing noise) that you need to know about:

Mean Smoothing

In order to attempt to remove noise, we can take the mean average of all the pixels surrounding each pixel (and the pixel itself) as the value of the pixel in the smoothed image, as follows:

000	000	000	000	050	100	150	133	133	133
000	026	026	026	050	100	150	139	139	139
000	026	026	026	050	106	173	173	150	150
000	026	026	026	050	106	173	173	150	150
000	000	000	000	050	106	173	173	150	150
043	028	000	000	050	100	150	150	150	150
043	028	000	000	050	100	150	150	150	150
043	028	000	000	050	100	150	150	144	141
000	000	000	000	050	100	150	150	144	141
000	000	021	021	071	100	150	150	144	141
000	000	31	31	081	100	150	150	150	150

This does remove the noise, but it blurs the image.

Median Smoothing

A far better method is, instead of taking the mean, to take the median, as follows:

000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150
000	000	000	000	000	150	150	150	150	150

For this image, this gives a perfect result. In more complicated images, however, data will still be lost, although, in general, less data will be lost by taking the median than by taking the mean.

Edge Detection

We can detect the positioning of edges in an image using the 'Laplace rule', or 'Laplace kernel'. For each pixel in the image, we multiply its value by 4, and then subtract the values of the pixels above and below it, and on either side of it. If the result is negative, we treat it as 0. So, taking the median-smoothed image above, edge detection gives the following result:

```

000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
000 000 000 000 000 150 000 000 000 000
    
```

Questions

1. How could the above methods be applied to a digital sound sample?
2. Which of the above methods would be suitable for smoothing sharp edges? Why?
3. Use median smoothing to remove noise from the following image of a white cat in a snowstorm (the black pixels have a value of 255):

```

000 ■ 000 000
000 000 000 ■
■ 000 000 000
000 000 ■ 000
    
```

4. Why would mean sampling not be appropriate for smoothing the image given in question 3?
5. Use mean smoothing to remove noise from the following image of a black cat in a coal cellar:

```

■ ■ ■ ■ ■
■ ■ ■ ■ ■
■ ■ ■ ■ ■
■ ■ ■ ■ ■
■ ■ ■ ■ ■
    
```

/Worked Solutions/

Digitisation

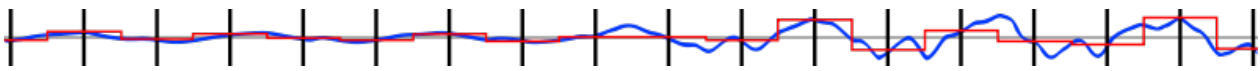
Digitisation of a signal is the process by which an analogue signal is converted to a digital signal.

Digitisation & Reconstruction

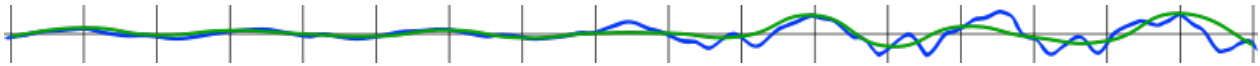
Let us consider the voltage output from a microphone. The signal which enters the microphone (sound) is an analogue signal - it can be any of a potentially infinite range of values, and may look something like this waveform (from an artificial (MIDI) piano):



When the microphone converts this signal to an electrical signal, it samples the signal a number of times, and transmits the level of the signal at that point. The following diagram shows sample times (vertical black lines) and the transmitted signal (the red line):



When we wish to listen to the sound, the digital signal has to be reconstructed. The gaps between the samples are filled in, but, as you can see, the reconstructed signal is not the same as the original sound:



Sampling Rate

The sampling rate when digitising an analogue signal is defined as the number of samples per. second, and is measured in Hertz (Hz), as it is a frequency. You can calculate the sampling rate using the formula:

$$\text{Sampling Rate (Hz)} = \frac{\text{No. of samples}}{\text{No. of seconds}}$$

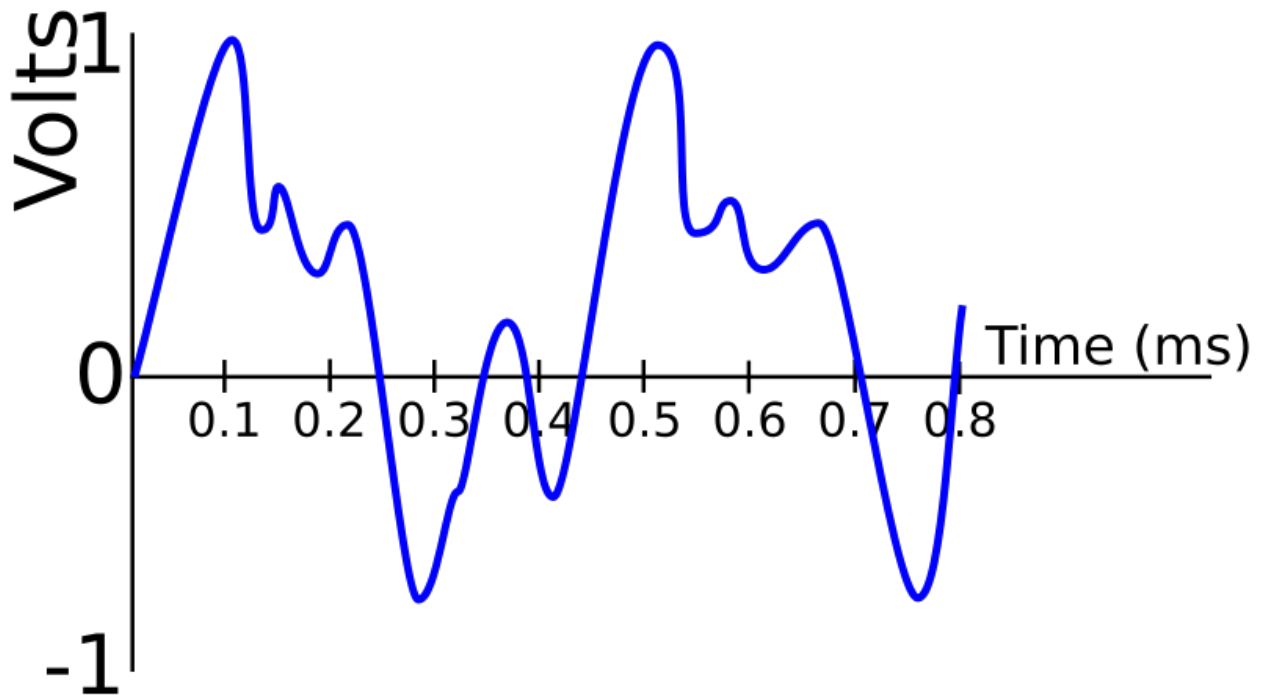
The higher the sampling rate, the closer the reconstructed signal is to the original signal, but, unfortunately, we are limited by the bandwidth available. Theoretically, a sampling rate of twice the highest frequency of the original signal will result in a perfect reconstructed signal. In the example given above, the sampling rate is far too low, hence the loss of information.

Number of Levels

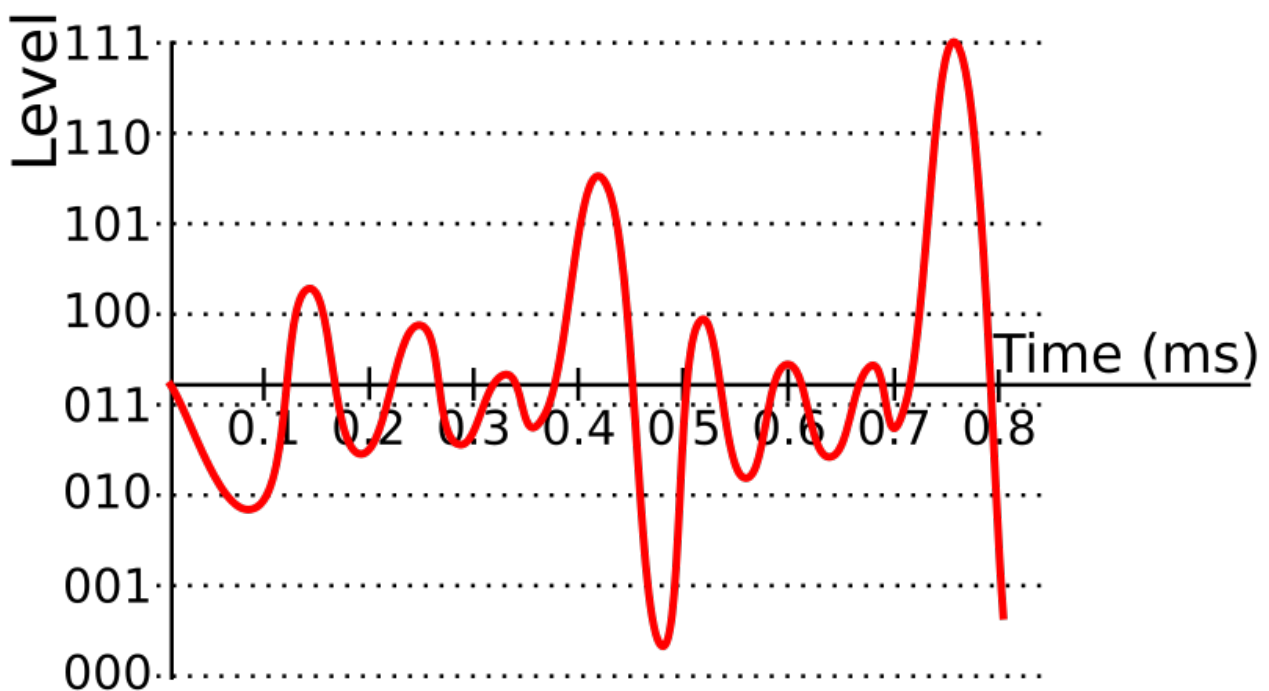
Another factor which may limit the quality of the reconstructed signal is the number of bits with which the signal is encoded. For example, if we use 3 bits per. sample, we only have 8 (2^3) levels, so, when sampling, we must take the nearest value represented by one of these levels. This leads to quantization errors - when a sample does not equal the value of the original signal at a given sample point.

Questions

1. Take samples for the signal below every 0.1ms, and then produce a reconstructed signal. How does it differ from the original?



2. A signal is sampled for 5 seconds at a sampling rate of 20 kHz. How many samples were taken?
3. Most sounds created by human speech except for 'ss' and 'ff' have a maximum frequency of 4 kHz. What is a suitable sampling rate for a low-quality telephone?
4. Using a sampling rate of 20 kHz and 3 bits, sample the following signal, and then produce a reconstructed signal. What is the maximum frequency that can be perfectly reconstructed using this sampling rate?



/Worked Solutions/

Signal Frequencies

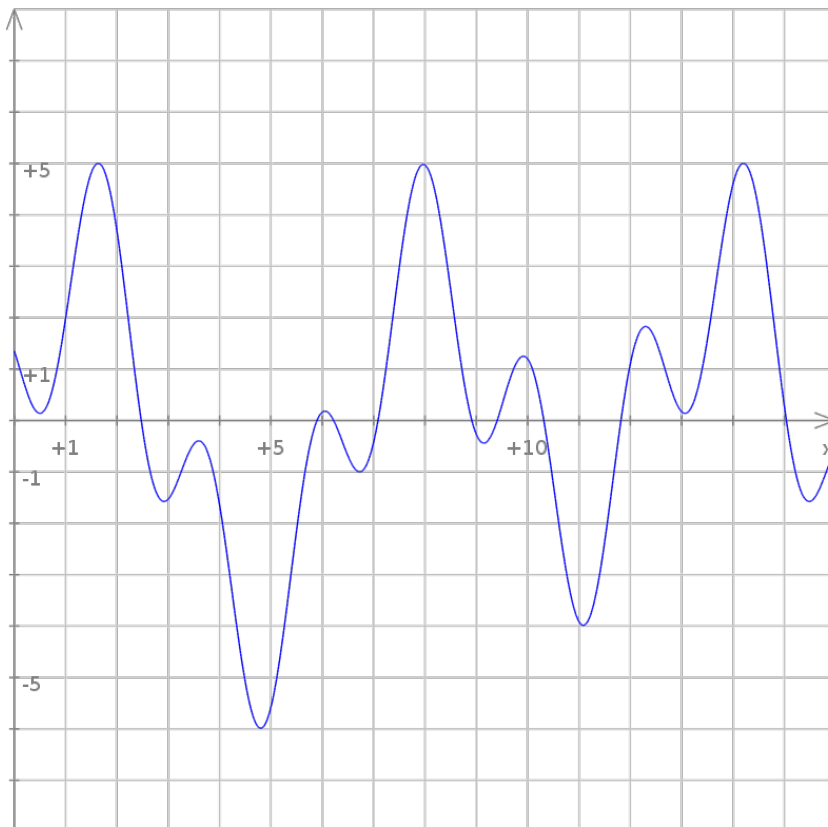
The frequency of a wave describes how many waves go past a certain point in one second. Frequency is measured in Hertz (usually abbreviated Hz), and can be calculated using the formula:

$$V = f\lambda$$

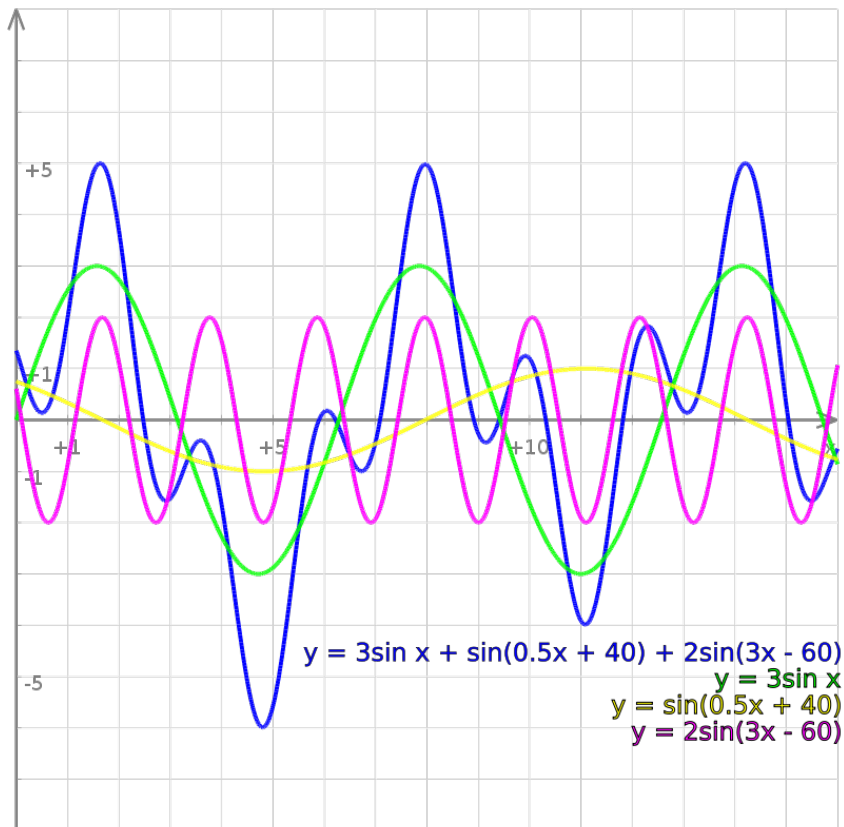
where V is the velocity of the wave (in ms^{-1}), f is the frequency of the wave (in Hz), and λ (the Greek letter lambda) is the wavelength of the wave (distance from one peak / trough to the next, in m).

Multiple Frequencies

Let us consider the following signal (time is in ms, and the y-axis represents volts):



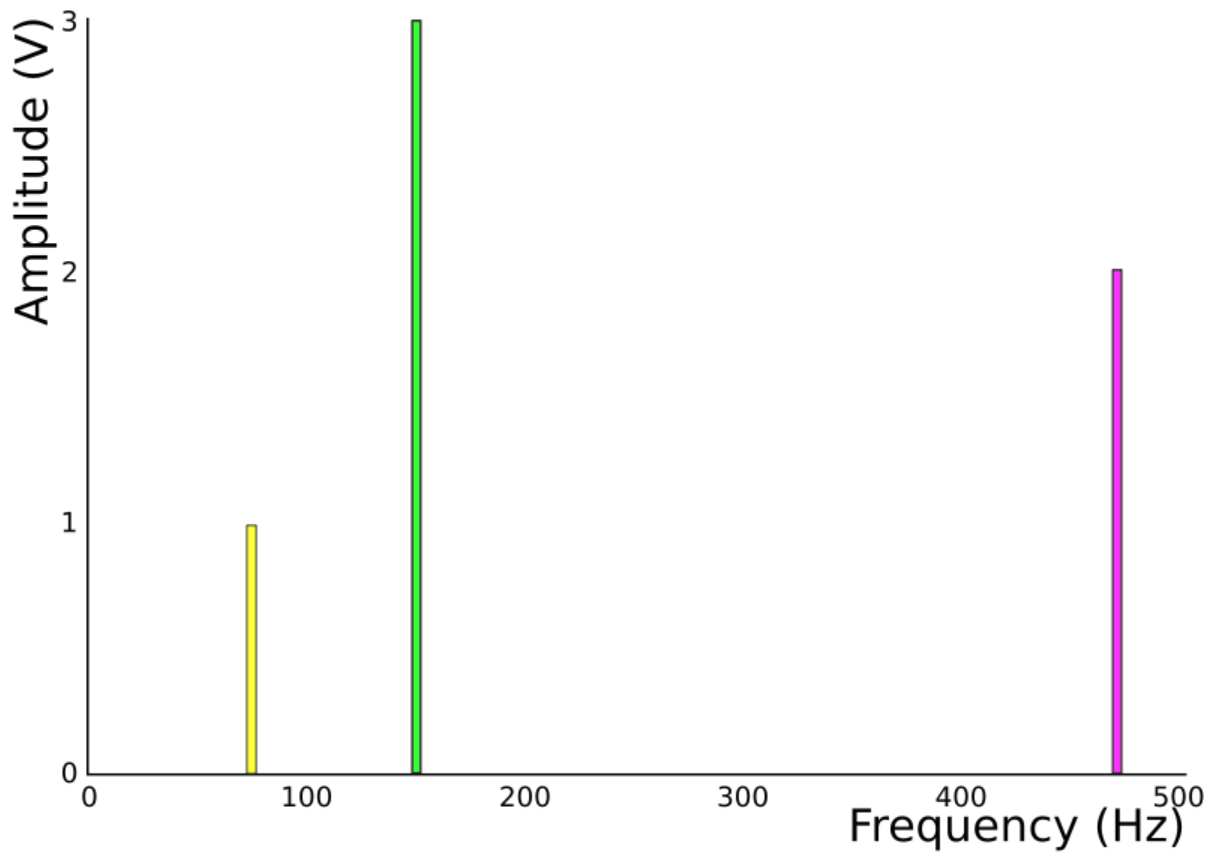
This signal is constructed from a number of different sine waves, with different frequencies, added together. These sine waves are as follows:



Frequency Spectra

Each of these sine waves has a different frequency. You can see this, as they have different distances between their peaks and troughs. These frequencies can be plotted against the amplitude of the wave, as in the table, and chart drawn from it, below:

Wave (y=)	Period (ms)	Amplitude (V)	Frequency (Hz)
$3\sin x$	6.284	3	159
$\sin(0.5x + 40)$	12.566	1	80
$2\sin(3x - 60)$	2.093	2	478



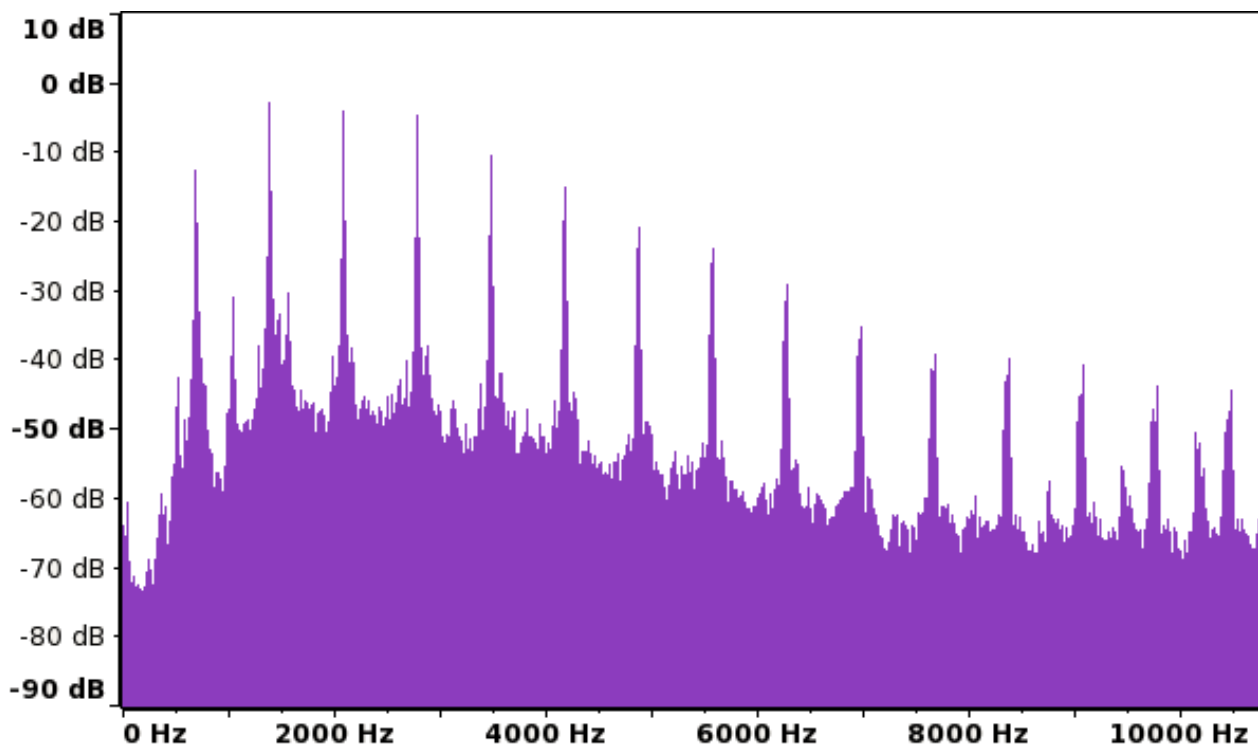
This chart is known as the frequency spectrum of a signal.

Fundamental Frequency

The fundamental frequency is the lowest frequency that makes up a signal. In the above example, the fundamental frequency is 80 Hz. It is always the frequency farthest to the left of a frequency spectrum, ignoring noise. Other frequencies are known as overtones, or harmonics.

Questions

1. What is the frequency of an X-ray (wavelength 0.5nm)?
2. A sound wave, with a frequency of 44 kHz, has a wavelength of 7.7mm. What is the speed of sound?
3. What is the fundamental frequency of the following signal?



4. Approximately how many harmonics does it contain?

5. The three sine waves $\sin x^\circ$, $4\sin(2x-50)^\circ$ and $0.5\sin(3x+120)^\circ$ are added together to form a signal. What are the frequencies of each of the waves? What is the signal's fundamental frequency? Assume that the waves are travelling at the speed of light, and that $60^\circ = 1\text{mm}$.

/Worked Solutions/

Bandwidth

Bandwidth is the frequency of a signal. Although original signals have varying frequencies, when these are transmitted, for example, as FM radio waves, they are modulated so that they only use frequencies within a certain range. FM radio modulates the frequency of a wave, so it needs some variation in the frequencies to allow for transmission of multiple frequencies. Since bandwidth is a frequency, it is the number of bits per. second. The bandwidth required to transmit a signal accurately can be calculated by using 1 as the number of bits, giving the formula:

$$B = \frac{1}{t}$$

where B is bandwidth (in Hz), and t is the time taken to transmit 1 bit of data (in s).

The bandwidth of a signal regulates the bit rate of the signal, as, with a higher frequency, more information can be transmitted. This give us the formula (similar to the formula for lossless digital sampling):

$$b = 2B$$

where b is the bit rate (in bits per. second), and B is the bandwidth (in Hz).

Questions

1. A broadband internet connection has a bit rate of 8Mbit s^{-1} when downloading information. What is the minimum bandwidth required to carry this bit rate?
2. The same connection has a bandwidth of 100 kHz reserved for uploading information. What is the maximum bit rate that can be attained when uploading information using this connection?
3. A lighthouse uses a flashing light and Morse Code to communicate with a nearby shore. A 'dash' consists of the light being on for 2s. The light is left off for 1s between dots and dashes. What is the bandwidth of the connection?
4. The broadband connection in question two is used to upload a 1Mbyte image to a website. How long does it take to do this?

/Worked Solutions/

Electricity

Charge

Electrons, like many other particles, have a charge. While some particles have a positive charge, electrons have a negative charge. The charge on an electron is equal to approximately -1.6×10^{-19} coulombs. Coulombs (commonly abbreviated C) are the unit of charge. One coulomb is defined as the electric charge carried by 1 ampere (amp) of current in 1 second. It is normal to ignore the negative nature of this charge when considering electricity.

If we have n particles with the same charge Q_{particle} , then the total charge Q_{total} is given by:

$$Q_{\text{total}} = n Q_{\text{particle}}$$

By a simple rearrangement:

$$n = \frac{Q_{\text{total}}}{Q_{\text{particle}}}$$

Questions

1. How much charge do 1234 electrons carry?
2. How many electrons does it take to carry 5 C of charge?
3. The total charge on 1 mole of electrons (6×10^{23} particles) is equal to 1 faraday of charge. How many coulombs of charge are equal to 1 faraday?

/Worked Solutions/

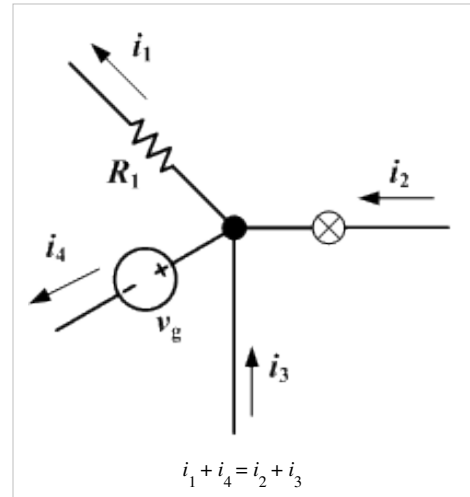
Current

Current is the amount of charge (on particles such as electrons) flowing through part of an electric circuit per second. Current is measured in amperes (usually abbreviated A), where 1 ampere is 1 coulomb of charge per. second. The formula for current is:

$$I = \frac{\Delta Q}{\Delta t} \text{ ([The triangle (greek letter /delta/) means change in the quantity])}$$

where I is current (in A), Q is charge (in C) and t is the time it took for the charge to flow (in seconds).

In a series circuit, the current is the same everywhere in the circuit, as the rate of flow of charged particles is constant throughout the circuit. In a parallel circuit, however, the current is split between the branches of the circuit, as the number of charged particles flowing cannot change. This is Kirchoff's First Law, stating that:



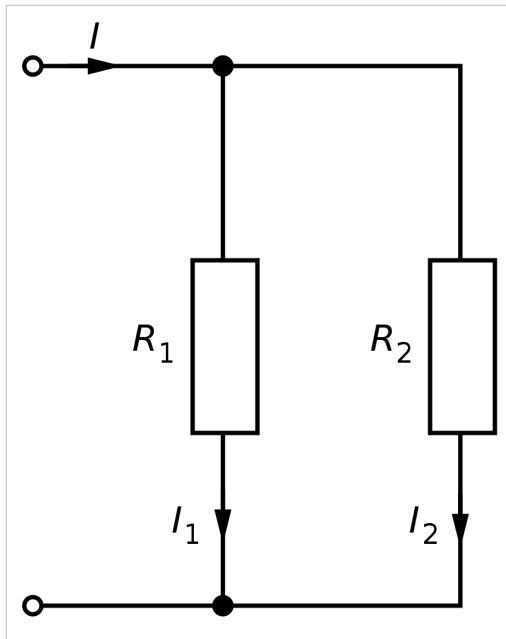
At any point in an electrical circuit where charge density is not changing in time [ie. there is no buildup of charge, as in a capacitor], the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point.



In mathematical form:

$$\sum I_{in} = \sum I_{out} \text{ (The sideways E is the Greek letter /sigma/, meaning 'sum of'.)}$$

Questions



1. 10 coulombs flow past a point in a wire in 1 minute. How much current is flowing through the point?
2. How long does it take for a 2A current to carry 5C?
3. In the diagram on the left, $I = 9\text{A}$, and $I_1 = 4.5\text{A}$. What is the current at I_2 ?
4. What would I equal if $I_1 = 10\text{A}$ and $I_2 = 15\text{A}$?
5. In the diagram on the left, in 5 seconds, 5C of charged particles flow past I_1 , and 6.7C flow past I_2 . How long does it take for 10C to flow past I ?

/Worked Solutions/

Voltage

Charge moves through a circuit, losing potential energy as it goes. This means that the charge travels as an electric current. Voltage is defined as the difference in potential energy per. unit charge, ie.

$$V = \frac{E}{Q}$$

where V is voltage (in V), E is the difference in potential energy (in joules) and Q is charge (in coulombs).

There are two electrical properties which are both measured in volts (commonly abbreviated V), and so both are known under the somewhat vague title of 'voltage'. Both are so called because they change the potential energy of the charge.

Electromotive Force (EMF)

Electrical power sources (such as batteries) 'push' an electric current around a circuit. To do this, they have to exert a force on the electrons. This force is known as electromotive force, or EMF. The current travels around a circuit (from the negative pole of the power source to the positive) because of the difference in EMF between either end of the source. For example, the negative end of a battery may exert 9V of EMF, whereas the positive end exerts no EMF. As a result, the current flows from negative to positive.

Potential Difference

As charge travels around a circuit, each coulomb of charge has less potential energy, so the voltage (relative to the power source) decreases. The difference between the voltage at two points in a circuit is known as potential difference, and can be measured with a voltmeter.

Series Circuits

In a series circuit, the total voltage (EMF) is divided across the components, as each component causes the voltage to decrease, so each one has a potential difference. The sum of the potential differences across all the components is equal to the potential difference (but batteries have their own 'internal resistances', which complicates things slightly, as we will see).

Parallel Circuits

In a parallel circuit, the potential difference across each branch of the circuit is equal to the EMF, as the same 'force' is pushing along each path of the circuit. The number of charge carriers (current) differs, but the 'force' pushing them (voltage) does not.

Questions

1. A battery has an EMF of 5V. What is the total potential difference across all the components in the circuit?
2. The voltages (relative to the voltage of the battery) on either side of a resistor are -6V and -5V. What is the potential difference across the resistor?
3. At a given point in a circuit, 5C of charge have 10 kJ of potential energy. What is the voltage at this point?
4. Why do the electrons move to a point 1cm further along the wire?

/Worked Solutions/

Power

Power is a measure of how much potential energy is dissipated (ie. converted into heat, light and other forms of energy) by a component or circuit in one second. This is due to a drop in the potential energy, and so the voltage, of charge. Power is measured in Watts (commonly abbreviated W), where 1 W is 1 Js^{-1} . It can be calculated by finding the product of the current flowing through a component / circuit and the potential difference across the component / circuit. This gives us the equation:

$$P = \frac{E}{t} = IV$$

where P is the power dissipated (in W), E is the drop in potential energy (in Joules, J), t is the time taken (in s), I is the current (in A) and V is either potential difference or electromotive force (in V), depending on the component being measured.

Since power is the amount of energy changing form per. second, the amount of energy being given out each second will equal the power of the component giving out energy.

You should be able to substitute in values for I and V from other formulae ($V=IR$, $Q=It$) in order to relate power to resistance, conductance, charge and time, giving formulae like these:

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

$$P = \frac{QV}{t}$$

Questions

1. The potential difference across a 9W light bulb is 240V. How much current is flowing through the light bulb?
2. How much energy is dissipated by a 10W component in 1 hour?
3. The potential difference across a top-notch kettle, which can hold up to 1 litre of water, is 240V, and the current is 12.5 A. 4.2 kJ of energy is required to heat a litre of water to 100°C . Assuming 100% efficiency, how long does it take to boil 1 litre of water?
4. How much energy is dissipated by a 100Ω resistor in 10 seconds if 2A of current are flowing?
5. The charge on an electron is $-1.6 \times 10^{-19} \text{ C}$. How long does it take for a mole (6×10^{23} particles) of electrons to flow through a 40W light bulb on a 240V ring main?

/Worked Solutions/

Resistance and Conductance

Conductance is a measure of how well an artefact (such as an electrical component, not a material, such as iron) carries an electric current. Resistance is a measure of how well an artefact resists an electric current.

Resistance is measured in Ohms (usually abbreviated using the Greek letter Omega, Ω) and, in formulae, is represented by the letter R. Conductance is measured in Siemens (usually abbreviated S) and, in formulae, is represented by the letter G.

Resistance and conductance are each other's reciprocals, so:

$$R = \frac{1}{G} \text{ and } G = \frac{1}{R}$$

Ohm's Law

Ohm's Law states that the potential difference across an artefact constructed from Ohmic conductors (ie. conductors that obey Ohm's Law) is equal to the product of the current running through the component and the resistance of the component. As a formula:

$$V = IR$$

where V is potential difference (in V), I is current (in A) and R is resistance (in Ω).

In terms of Resistance

This formula can be rearranged to give a formula which can be used to calculate the resistance of an artefact:

$$R = \frac{V}{I}$$

In terms of Conductance

Since conductance is the reciprocal of resistance, we can deduce a formula for conductance (G):

$$\frac{1}{G} = \frac{V}{I}$$
$$G = \frac{I}{V}$$

The Relationship between Potential Difference and Current

From Ohm's Law, we can see that potential difference is directly proportional to current, provided resistance is constant. This is because two variables (let us call them x and y) are considered directly proportional to one another if:

$$x = ky$$

where k is any positive constant. Since we are assuming that resistance is constant, R can equal k, so $V=RI$ states that potential difference is directly proportional to current. As a result, if potential difference is plotted against current on a graph, it will give a straight line with a positive gradient which passes through the origin. The gradient will equal the resistance.

In Series Circuits

In a series circuit (for example, a row of resistors connected to each other), the resistances of the resistors add up to give the total resistance. Since conductance is the reciprocal of resistance, the reciprocals of the conductances add up to give the reciprocal of the total conductance. So:

$$\Sigma R = R_1 + R_2 + \dots + R_n$$
$$\frac{1}{\Sigma G} = \frac{1}{G_1} + \frac{1}{G_2} + \dots + \frac{1}{G_n}$$

In Parallel Circuits

In a parallel circuit, the conductances of the components on each branch add up to give the total conductance. Similar to series circuits, the reciprocals of the total resistances of each branch add up to give the reciprocal of the total resistance of the circuit. So:

$$\Sigma G = G_1 + G_2 + \dots + G_n$$
$$\frac{1}{\Sigma R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

When considering circuits which are a combination of series and parallel circuits, consider each branch as a separate component, and work out its total resistance or conductance before finishing the process as normal.

Questions

1. The potential difference across a resistor is 4V, and the current is 10A. What is the resistance of the resistor?
2. What is the conductance of this resistor?
3. A conductor has a conductance of 2S, and the potential difference across it is 0.5V. How much current is flowing through it?
4. A graph is drawn of potential difference across an Ohmic conductor, and current. For every 3cm across, the graph rises by 2cm. What is the conductance of the conductor?
5. On another graph of potential difference and current, the graph curves so that the gradient increases as current increases. What can you say about the resistor?
6. 3 resistors, wired in series, have resistances of 1k Ω , 5k Ω and 500 Ω each. What is the total resistance across all three resistors?
7. 2 conductors, wired in parallel, have conductances of 10S and 5S. What is the total resistance of both branches of the parallel circuit?
8. The circuit above is attached in series to 1 10 Ω resistor. What is the total conductance of the circuit now?

/Worked Solutions/

Internal Resistance

Batteries, just like other components in an electric circuit, have a resistance. This resistance is known as internal resistance. This means that applying Ohm's law ($V = IR$) to circuits is more complex than simply feeding the correct values for V , I or R into the formula.

The existence of internal resistance is indicated by measuring the potential difference across a battery. This is always less than the EMF of the battery. This is because of the internal resistance of the battery. This idea gives us the following formula:

PD across battery = EMF of battery - voltage to be accounted for

Let us replace these values with letters to give the simpler formula:

$$V_{\text{external}} = E - V_{\text{internal}}$$

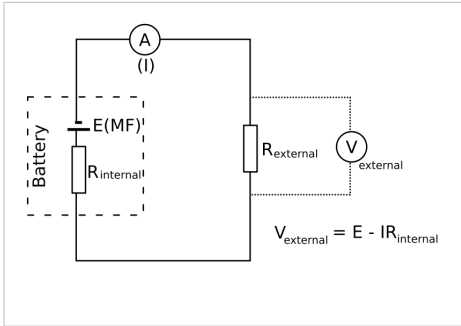
Since $V = IR$:

$$V_{\text{external}} = E - IR_{\text{internal}}$$

You may also need to use the following formula to work out the external potential difference, if you are not given it:

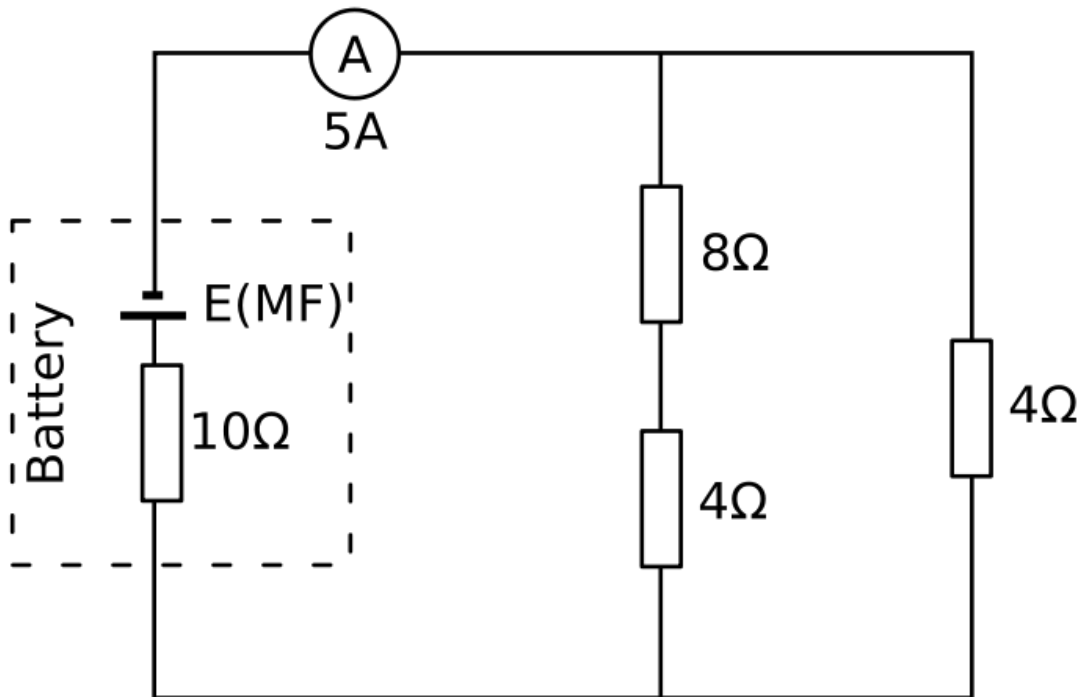
$$V_{\text{external}} = I \Sigma R_{\text{external}}$$

You should also remember the effects of using resistors in both series and parallel circuits.

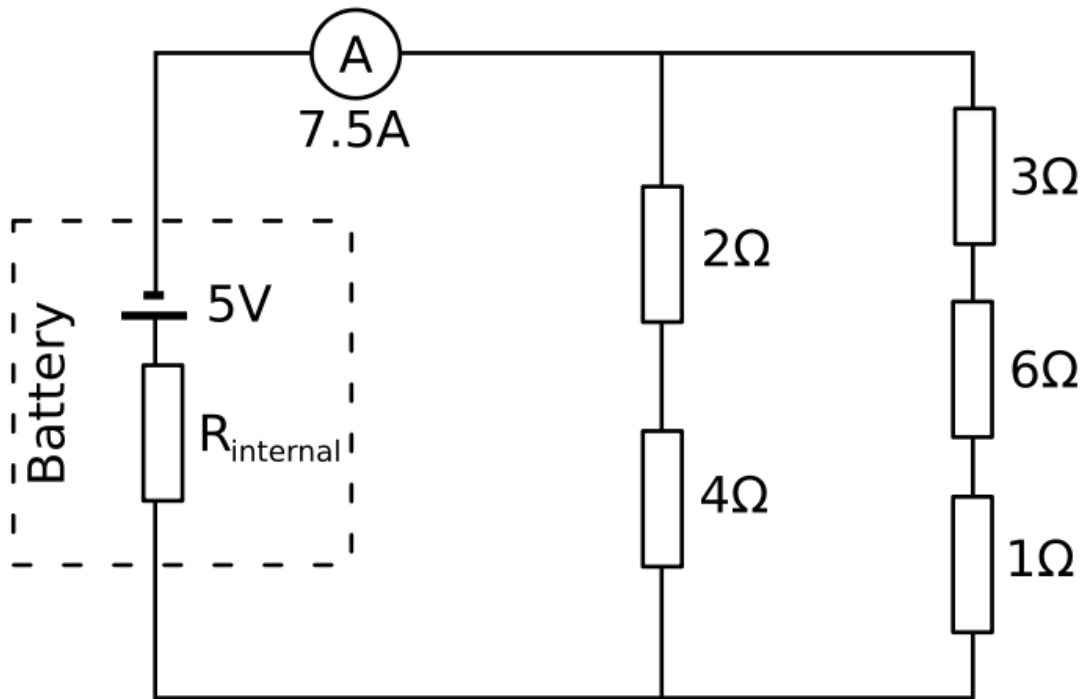


Questions

1. A 9V battery is short-circuited. The potential difference across the battery is found to be 8V, and the current is 5A. What is the internal resistance of the battery?
2. What is the EMF of the battery in the following circuit?



3. What is the internal resistance of the battery in the following circuit?



/Worked Solutions/

Potential Dividers

A potential divider, or potentiometer, consists of a number of resistors, and a voltmeter. The voltage read by the voltmeter is determined by the ratio of the resistances on either side of the point at which one end of the voltmeter is connected.

To understand how a potential divider works, let us consider resistors in series. The resistances add up, so, in a circuit with two resistors:

$$\Sigma R = R_1 + R_2$$

If we apply Ohm's law, remembering that the current is constant throughout a series circuit:

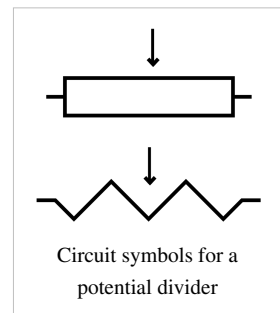
$$\frac{\Sigma V}{I} = \frac{V_1}{I} + \frac{V_2}{I}$$

Multiply by current (I):

$$\Sigma V = V_1 + V_2$$

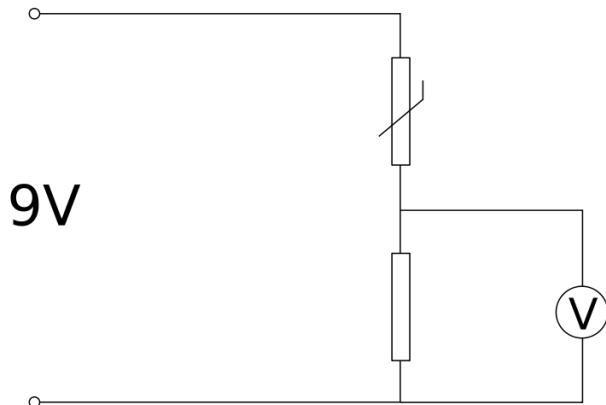
So, just as the resistances in series add up to the total resistance, the potential differences add up to the total potential difference. The ratios between the resistances are equal to the ratios between the potential differences. In other words, we can calculate the potential difference across a resistor using the formula:

$$V_{resistor} = \Sigma V \times \frac{R_{resistor}}{\Sigma R_{external}}$$

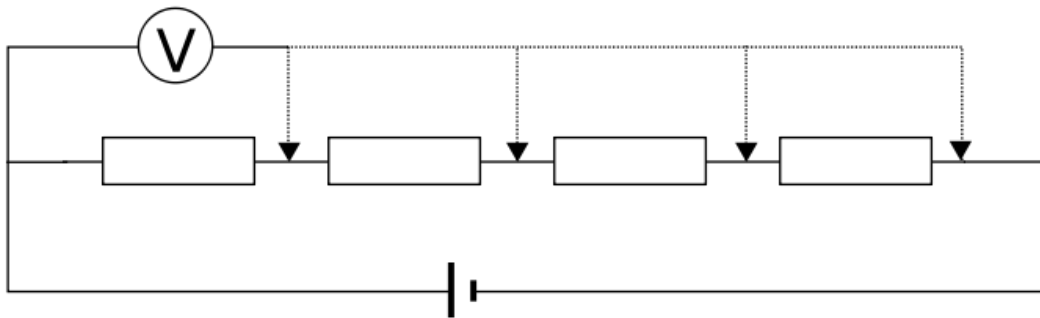


In many cases, you will be told to assume that the internal resistance of the power source is negligible, meaning that you can take the total potential difference as the EMF of the power source.

A potential divider may work by combining a variable resistor such as an LDR or thermistor with a constant resistor, as in the diagram below. As the resistance of the variable resistor changes, the ratio between the resistances changes, so the potential difference across any given resistor changes.



Alternatively, a potential divider may be made of many resistors. A 'wiper' may move across them, varying the number of resistors on either side of the wiper as it moves, as in the following diagram:



Questions

1. A $12\text{ k}\Omega$ resistor and a $20\text{ k}\Omega$ resistor are connected to a 9V battery. A voltmeter is connected across the 12Ω resistor. What is the reading on the voltmeter? (Assume negligible internal resistance.)
2. A potential divider consists of 100 5Ω resistors, with a wiper which moves on one resistor for every 3.6° a handle connected to it turns. The wiper is connected to a voltmeter, and the circuit is powered by a 120V power source with negligible internal resistance. What is the reading on the voltmeter when the handle turns 120° ?
3. A 9V battery with internal resistance 0.8Ω is connected to 3 resistors with conductances of 3, 2 and 1 Siemens. A voltmeter is connected across the 3 and 2 Siemens resistors. An ammeter is placed in the circuit, between the battery and the first terminal of the voltmeter, and reads 2A . What is the reading on the voltmeter?

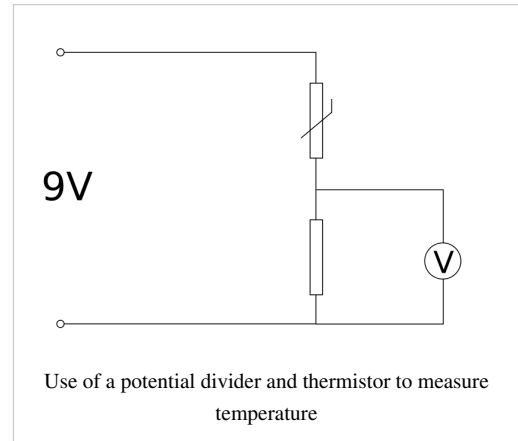
/Worked Solutions/

Sensors

A sensor is a device which converts a physical property into an electrical property (such as resistance). A sensing system is a system (usually a circuit) which allows this electrical property, and so the physical property, to be measured.

Temperature Sensor

A common example of a sensing system is a temperature sensor in a thermostat, which uses a thermistor. In the most common type of thermistor (an NTC), the resistance decreases as the temperature increases. This effect is achieved by making the thermistor out of a semiconductor. The thermistor is then used in a potential divider, as in the diagram on the right. In this diagram, the potential difference is divided between the resistor and the thermistor. As the temperature rises, the resistance of the thermistor decreases, so the potential difference across it decreases. This means that potential difference across the resistor increases as temperature increases. This is why the voltmeter is across the resistor, not the thermistor.



Properties

There are three main properties of sensing systems you need to know about:

Sensitivity

This is the amount of change in voltage output per. unit change in input (the physical property). For example, in the above sensing system, if the voltage on the voltmeter increased by 10V as the temperature increased by 6.3°C:

$$S = \frac{10}{6.3} \approx 1.59 \text{V/}^\circ\text{C}$$

Resolution

This is the smallest change in the physical property detectable by the sensing system. Sometimes, the limiting factor is the number of decimal places the voltmeter can display. So if, for example, the voltmeter can display the voltage to 2 decimal places, the smallest visible change in voltage is 0.01V. We can then use the sensitivity of the sensor to calculate the resolution.

$$S = 1.59 = \frac{0.01}{R} \quad R = \frac{0.01}{1.59} \approx 0.006^\circ\text{C}$$

Response Time

This is the time the sensing system takes to display a change in the physical property it is measuring. It is often difficult to measure.

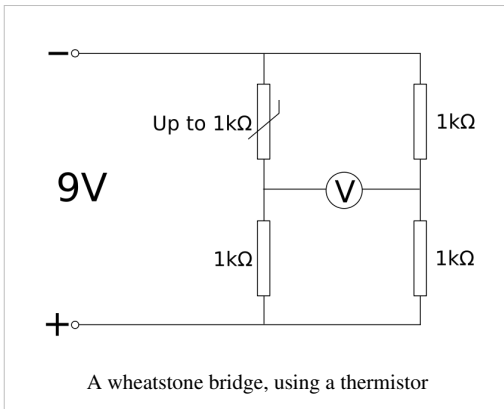
Signal Amplification

Sometimes, a sensing system gives a difference in output voltage, but the sensitivity is far too low to be of any use. There are two solutions to this problem, which can be used together:

Amplification

An amplifier can be placed in the system, increasing the signal. The main problem with this is that the signal cannot exceed the maximum voltage of the system, so values will be chopped off of the top and bottom of the signal because it is so high.

Wheatstone Bridge



This solution is far better, especially when used prior to amplification. Instead of using just one pair of resistors, a second pair is used, and the potential difference between the two pairs (which are connected in parallel) is measured. This means that, if, at the sensing resistor (eg. thermistor / LDR) the resistance is at its maximum, a signal of 0V is produced. This means that the extremes of the signal are not chopped off, making for a much better sensor.

Questions

An LDR's resistance decreases from a maximum resistance of 2kΩ to a minimum resistance of 0Ω as light intensity increases. It is used in a distance sensing system which consists of a 9V power supply, a 1.6 kΩ resistor, the LDR and a multimeter which displays voltage to 2 decimal places measuring the potential difference across one of the two resistors.

1. Across which resistor should the multimeter be connected in order to ensure that, as the distance from the light source to the sensor increases, the potential difference recorded increases?
2. In complete darkness, what voltage is recorded on the multimeter?
3. When a light source moves 0.5m away from the sensor, the voltage on the multimeter increases by 2V. What is the sensitivity of the sensing system when using this light source, in $V m^{-1}$?
4. When the same light source is placed 0m from the sensor, the potential difference is 0V. When the light source is 1m away, what voltage is displayed on the multimeter?
5. What is the resolution of the sensing system?
6. Draw a circuit diagram showing a similar sensing system to this, using a Wheatstone bridge and amplifier to improve the sensitivity of the system.
7. What is the maximum potential difference that can reach the amplifier using this new system (ignore the amplification)?
8. If this signal were to be amplified 3 times, would it exceed the maximum voltage of the system? What would the limits on the signal be?

/Worked Solutions/

Resistivity and Conductivity

Resistivity and conductivity are material properties: they apply to all examples of a certain material anywhere. They are not the same as resistance and conductance, which are properties of individual artefacts. This means that they only apply to a given object. They describe how well a material resists or conducts an electric current.

Symbols and Units

Resistivity is usually represented by the Greek letter rho (ρ), and is measured in $\Omega \text{ m}$. Conductivity is usually represented by the Greek letter sigma (σ), and is measured in S m^{-1} .

Formulae

The formula relating resistivity (ρ) to resistance (R), cross-sectional area (A) and length (L) is:

$$\rho = \frac{RA}{L}$$

Conductivity is the reciprocal of resistivity, just as conductance (G) is the reciprocal of resistance. Hence:

$$\frac{1}{\sigma} = \frac{\frac{1}{G} \times A}{L} = \frac{1}{G} \times \frac{A}{L} = \frac{A}{GL}$$
$$\sigma = \frac{GL}{A}$$

You should be able to rearrange these two formulae to be able to work out resistance, conductance, cross-sectional area and length. For example, it all makes a lot more sense if we write the first formula in terms of ρ , A and L :

$$R = \frac{\rho L}{A}$$

From this, we can see that the resistance of a lump of material is higher if it has a higher resistivity, or if it is longer. Also, if it has a larger cross-sectional area, its resistance is smaller.

Questions

1. A material has a conductivity of 10^6 S m^{-1} . What is its resistivity?
2. A pure copper wire has a radius of 0.5mm, a resistance of 1 $\text{M}\Omega$, and is 4680 km long. What is the resistivity of copper?
3. Gold has a conductivity of 45 MS m^{-1} . What is the resistance of a 0.01m across gold connector, 0.05m long?
4. A strand of metal is stretched to twice its original length. What is its new resistance? State your assumptions.
5. Which has the greater resistivity: a plank or a piece of sawdust, made from the same wood?

/Worked Solutions/

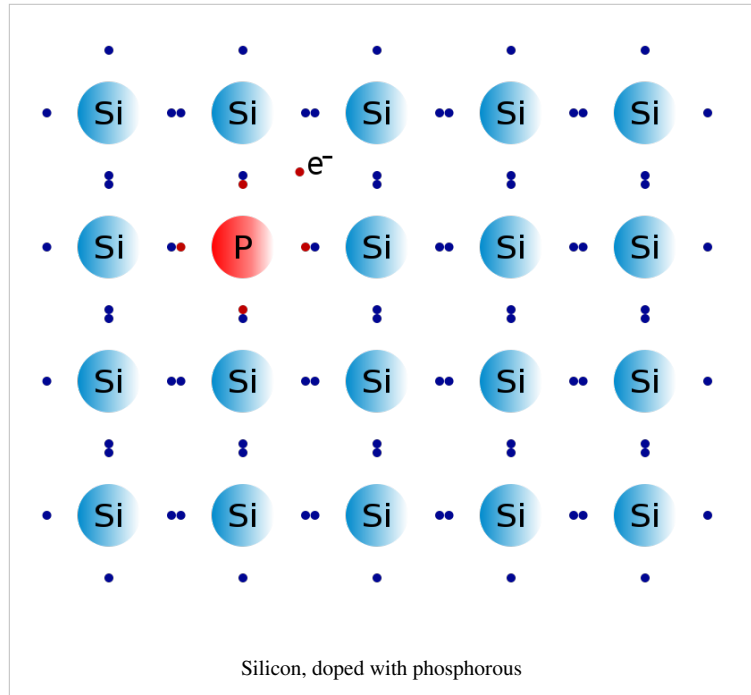
Semiconductors

A semiconductor has a conductivity between that of a conductor and an insulator. They are less conductive than metals, but differ from metals in that, as a semiconductor heats up, its conductivity rises. In metals, the opposite effect occurs.

The reason for this is that, in a semiconductor, very few atoms are ionised, and so very few electrons can move, creating an electric current. However, as the semiconductor heats up, the covalent bonds (atoms sharing electrons, causing the electrons to be relatively immobile) break down, freeing the electrons. As a result, a semiconductor's conductivity rises at an increasing rate as temperature rises.

Examples of semiconductors include silicon and germanium. A full list of semiconductor materials is available at [Wikipedia](#). At room temperature, silicon has a conductivity of about $435 \mu\text{S m}^{-1}$.

Semiconductors are usually 'doped'. This means that ions are added in small quantities, giving the semiconductor a greater or lesser number of free electrons as required. This is controlled by the charge on the ions.



Questions

1. What is the resistivity of silicon, at room temperature?
2. What sort of variable resistor would a semiconductor be useful in?
3. If positive ions are added to silicon (doping it), how does its conductivity change?

/Worked Solutions/

See Also

- The book on Semiconductors.

Material Structure

Stress, Strain & the Young Modulus

Stress

Stress is a measure of how strong a material is. This is defined as how much pressure the material can stand without undergoing some sort of physical change. Hence, the formula for calculating stress is the same as the formula for calculating pressure:

$$\sigma = \frac{F}{A}$$

where σ is stress (in Newtons per square metre but usually Pascals, commonly abbreviated Pa), F is force (in Newtons, commonly abbreviated N) and A is the cross sectional area of the sample.

Tensile Strength

The tensile strength is the level of stress at which a material will fracture. Tensile strength is also known as fracture stress. If a material fractures by 'crack propagation' (i.e., it shatters), the material is brittle.

Yield Stress

The yield stress is the level of stress at which a material will deform permanently. This is also known as yield strength.

Strain

Stress causes strain. Putting pressure on an object causes it to stretch. Strain is a measure of how much an object is being stretched. The formula for strain is:

$$\epsilon = \frac{L - l}{l} = \frac{\Delta l}{l}$$

Young's Modulus

Young's Modulus is a measure of the stiffness of a material. It states how much a material will stretch (i.e., how much strain it will undergo) as a result of a given amount of stress. The formula for calculating it is:

$$E = \frac{\sigma}{\epsilon}$$

The values for stress and strain must be taken at as low a stress level as possible, provided a difference in the length of the sample can be measured. The unit of the Young's Modulus is Pascals, as strain has no units, so the units for stress are retained.

Stress-Strain Graphs

Stress (σ) can be graphed against strain (ϵ). The toughness of a material (i.e., how much it resists stress, in J m^{-3}) is equal to the area under the curve, between the y-axis and the fracture point. Graphs such as the one on the right show how stress affects a material. This image shows the stress-strain graph for low-carbon steel. It has three main features:

Elastic Region

In this region (between the origin and point 2), the ratio between stress and strain (Young's modulus) is constant, meaning that the material is obeying Hooke's law, which states that a material is elastic (it will return to its original shape) if force is directly proportional to extension.

Hooke's Law

Hooke's law of elasticity is an approximation that states that the Force (load) is in direct proportion with the extension of a material as long as this load does not exceed the elastic limit. Materials for which Hooke's law is a useful approximation are known as linear-elastic or "Hookean" materials.

$$F = -kx.$$

The relation is often denoted

$$F \propto -x$$

Plastic Region

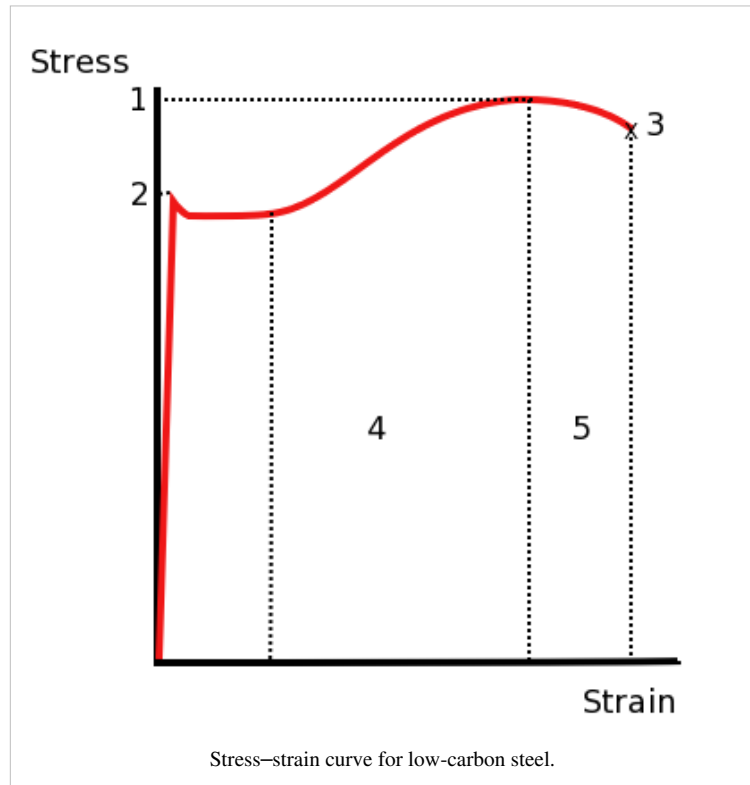
In this region (between points 2 and 3), the rate at which extension is increasing is going up, and the material has passed the elastic limit. It will no longer return to its original shape. After point 1, the amount of stress decreases due to 'necking', so the cross-sectional area is going down. The material will 'give' and extend more under less stress.

Fracture Point

At point 3, the material finally breaks in two completely - it fractures, and the graph stops.

Other Typical Graphs

In a brittle material, such as glass or ceramics, the stress-strain graph will have an extremely short elastic region, and then will fracture. There is no plastic region on the stress-strain graph of a brittle material.



Questions

1. 10N of force are exerted on a wire with cross-sectional area 0.5mm^2 . How much stress is being exerted on the wire?
2. Another wire has a tensile strength of 70MPa, and breaks under 100N of force. What is the cross-sectional area of the wire just before breaking?
3. What is the strain on a Twix bar (original length 10cm) if it is now 12cm long?
4. What is this strain, expressed as a percentage?
5. 50N are applied to a wire with a radius of 1mm. The wire was 0.7m long, but is now 0.75m long. What is the Young's Modulus for the material the wire is made of?
6. Glass, a brittle material, fractures at a strain of 0.004 and a stress of 240 MPa. Sketch the stress-strain graph for glass.
7. (Extra nasty question which you won't ever get in an exam) What is the toughness of glass?

/Worked Solutions/

Metals

There are several physical properties of metals you need to know about:

Electrical Conductivity

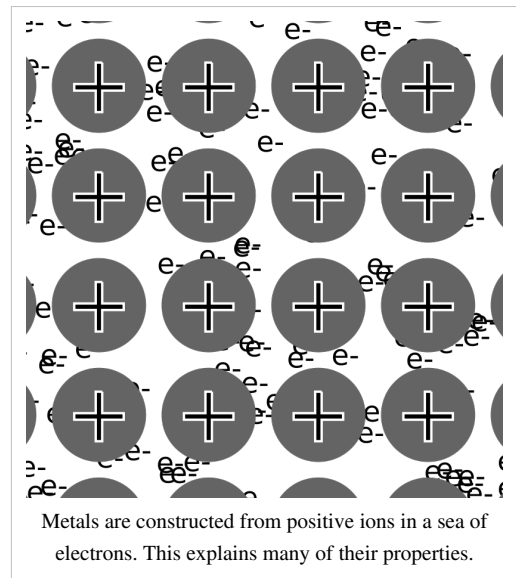
Metals consist of positive metal ions in a 'soup' or 'sea' of free (delocalized) electrons. This means that the electrons are free to move through the metal, conducting an electric current.

Stiffness

The charge between the negatively charged electrons and the positively charged ions holds the ions together, making metals stiff.

Ductility

Since there are no permanent bonds between the ions, they can move about and slide past each other. This makes metals ductile.



Toughness

Metals are tough for the same reason as they are ductile; they can slide past each other while still remaining together. So, instead of breaking apart, they change shape, resulting in increased toughness. This effect is called plasticity.

Elasticity

When a metal is stretched, it can return to its original shape because the sea of electrons which bonds the ions together can be stretched as well.

Brittle

The opposite of Elasticity: a material is likely to crack or shatter upon impact or force.

Questions

1. Would you expect a metal to have more or less conductivity than a semiconductor? Why?
2. How can the stress-strain graph for a metal be explained in terms of ions in a sea of electrons?
3. As a metal heats up, what happens to its conductivity? Why?

/Worked Solutions/

Polymers

A simple polymer consists of a long chain of monomers (components of molecules) joined by covalent bonds. A polymer usually consists of many of these bonds, tangled up. This is known as a bulk polymer.

Types

A bulk polymer may contain two types of regions. In crystalline regions, the chains run parallel to each other, whereas in amorphous regions, they do not. Intermolecular bonds are stronger in crystalline regions. A polycrystalline polymer consists of multiple regions, in which the chains point in a different direction in each region.



Polycrystalline glass

Properties

Transparency

Polymers which are mostly crystalline are transparent. As a polymer becomes more polycrystalline, it becomes more translucent, whilst an amorphous polymer is opaque.

Elasticity

In some polymers, such as polythene, the chains are folded up. When they are stretched, the chains unravel, stretching without breaking. When the stress ceases, they will return to their original shape. If, however, the bonds between the molecules are broken, the material reaches its elastic limit and will not return to its original shape.

Stiffness

Polymer chains may be linked together, causing the polymer to become stiffer. An example is rubber, which, when heated with sulfur, undergoes a process known as vulcanization. The chains in the rubber become joined by sulfur atoms, making the rubber suitable for use in car tyres. A stiffer polymer, however, will usually be more brittle.

Plasticity

When a polymer is stretched, the chains become parallel, and amorphous areas may become crystalline. This causes an apparent change in colour, and a process known as 'necking'. This is when

the chains recede out of an area of the substance, making it thinner, with fatter areas on either side.

Conductivity

Polymers consist of covalent bonds, so the electrons are not free to move according to potential difference. This means that polymers are poor conductors.

Boiling Point

Polymers do not have boiling points. This is because, before they reach a theoretical boiling point, polymers decompose. Polymers do not have melting points for the same reason.

Questions

1. Different crystalline structures have different refractive indexes. Why does this mean that a polycrystalline polymer is translucent?
2. What sort of polymer is a pane of perspex?
3. What sort of polymer does the pane of perspex become when shattered (but still in one piece)?

4. What sort of polymer is a rubber on the end of a pencil?
5. What happens to the translucency of an amorphous polymer when it is put under stress?

/Worked Solutions/

Waves

What is a wave?

At this point in the course, it is easy to get bogged down in the complex theories and equations surrounding 'waves'. However, a better understanding of waves can be gained by going back to basics, and explaining what on Earth a wave is in the first place.

Definitions

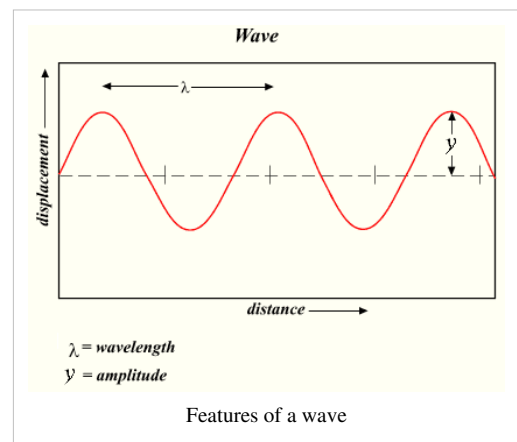
A **wave**, at its most basic level, is a disturbance by which energy is transferred because this disturbance is a store, of sorts, of potential energy. This begs the question "How is this disturbance transferred across space?" In some cases, this is easy to answer, because some waves travel through a medium. The easiest example to think about is a water wave. One area moves up, pulling the next one up with it, and pressure and gravity pull it back to its original position.

However, some waves (electro-magnetic waves) do not appear to travel through a medium. Physicists have puzzled over how light, which behaves like a wave in many situations, travels for a long time. One theory was that there was a mysterious 'ether' which pervaded all of space, and so light was just like water waves, except that the water was invisible. This theory is widely regarded to be incorrect, but, since light is assumed to be a wave, what is it a disturbance in?

Another explanation is that light is not a wave, but instead is a stream of particles. This idea would explain away the need for an 'ether' for light to travel through. This, too, has its problems, as it does not explain why light behaves as a wave.

So, we are left with a paradox. Light behaves as both a wave and a particle, but it can be shown not to be either. Quantum physics is an attempt to explain this paradox. However, since light behaves as both a wave and a particle, we can look at it as both, even if, when doing this, we know that we don't fully understand it yet.

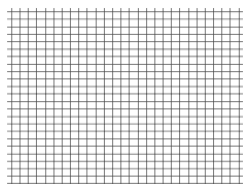
The image on the right shows a waveform. This plots the distance through the medium on the x-axis, and the amount of disturbance on the y-axis. The amount of disturbance is known as the amplitude. Wave amplitudes tend to oscillate between two limits, as shown. The distance in the medium between two 'peaks' or 'troughs' (maxima and minima on the waveform) is known as the wavelength of the wave.



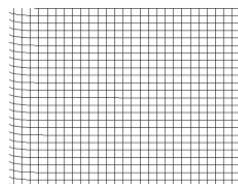
Types of Waves

Waves can be categorised according to the direction of the effect of the disturbance relative to the direction of travel. A wave which causes disturbance in the direction of its travel is known as a longitudinal wave, whereas a wave which causes disturbance perpendicular to the direction of its travel is known as a transverse wave.

Longitudinal wave (eg. sound)



Transverse wave (eg. light)

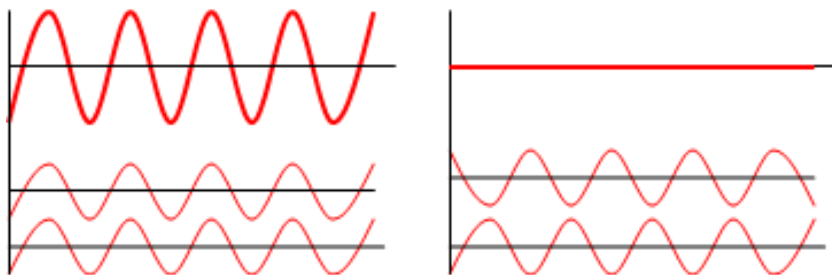


Superposition

One feature of waves is that they superpose. That is to say, when they are travelling in the same place in the medium at the same time, they both affect the medium independently. So, if two waves say "go up" to the same bit of medium at the same time, the medium will rise twice as much. In general, superposition means that the amplitudes of two waves at the same point at the same time at the same polarisation add up.

Interference

Consider two identical waveforms being superposed on each other. The resultant waveform will be like the two other waveforms, except its amplitude at every point will be twice as much. This is known as constructive interference. Alternatively, if one waveform moves on by half a wavelength, but the other does not, the resultant waveform will have no amplitude, as the two waveforms will cancel each other out. This is known as destructive interference. Both these effects are shown in the diagram below:

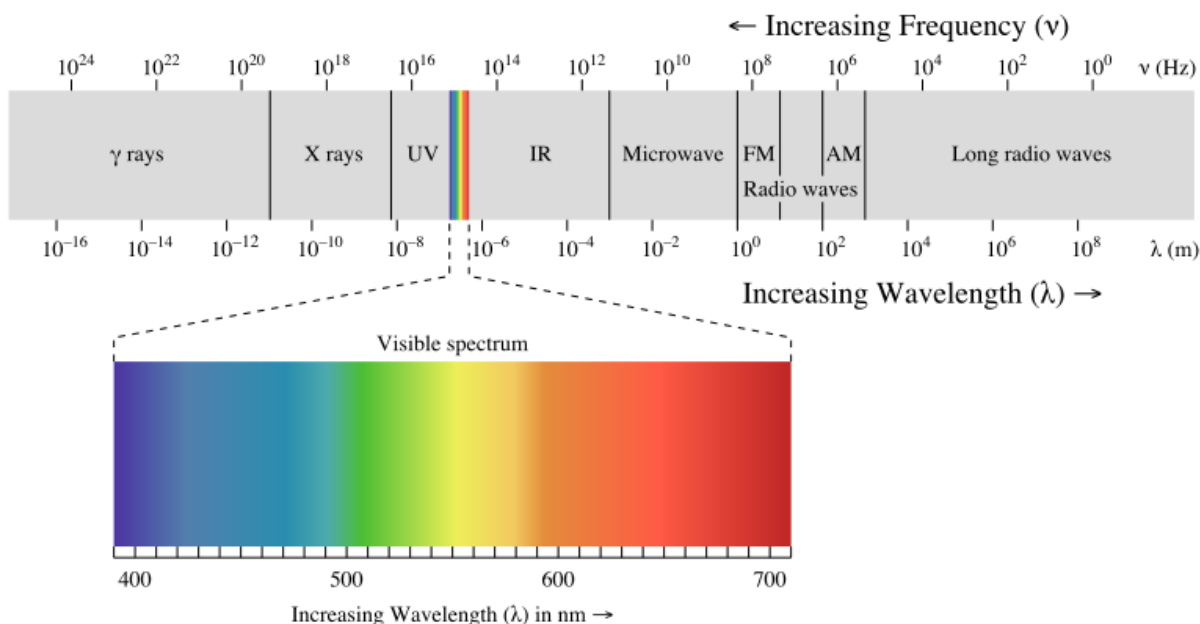


These effects occur because the wavefronts are travelling through a medium, but electromagnetic radiation also behaves like this, even though it does not travel through a medium.

Velocity, frequency and wavelength

You should remember the equation $v = f\lambda$ from earlier in this course, or from GCSE. v = the velocity at which the wave travels through the medium, in ms^{-1} , f = the frequency of the wave, in Hz (no. of wavelengths per. second), and λ = wavelength, in m.

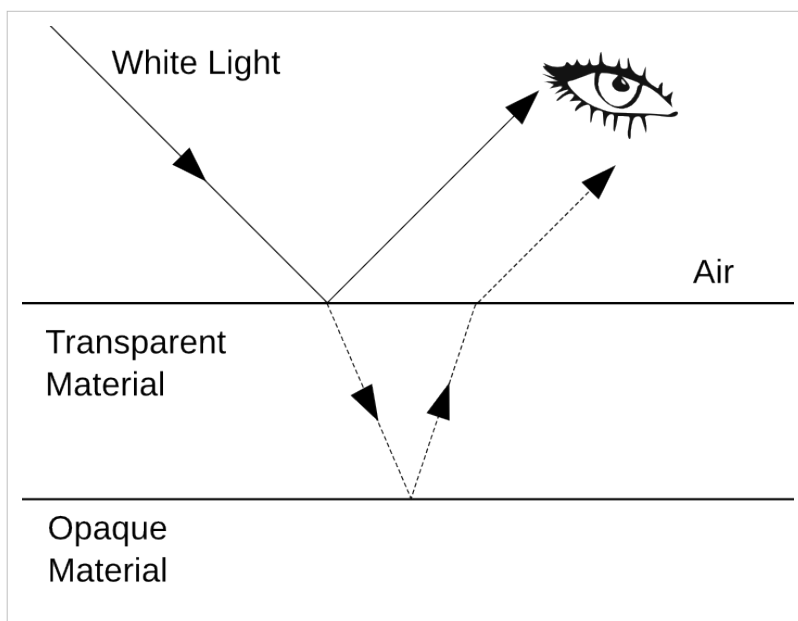
This equation applies to electromagnetic waves, but you should remember that there are different wavelengths of electromagnetic radiation, and that different colours of visible light have different wavelengths. You also need to know the wavelengths of the different types of electromagnetic radiation:



Questions

1. Through what medium are sound waves propagated?
2. What aspects of the behaviour of light make it look like a wave?
3. What aspects of the behaviour of light make it look like a particle?

4. Consider the diagram on the right. White light is partially reflected by the transparent material. Some of the light, however, is refracted into the transparent material and reflected back by the opaque material. The result is two waves travelling in the same place at the same time at the same polarisation (the light is not a single beam). Why does, say, the red light disappear? (Variations on this question are popular with examiners.)

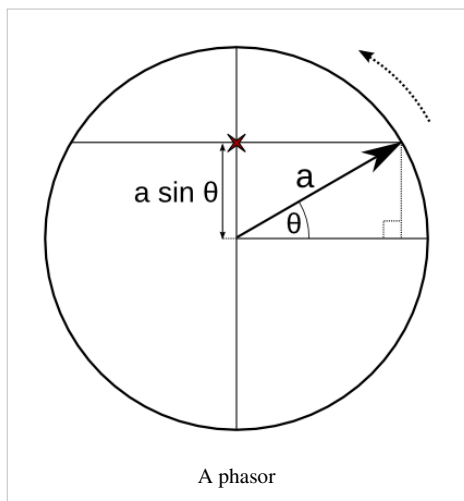
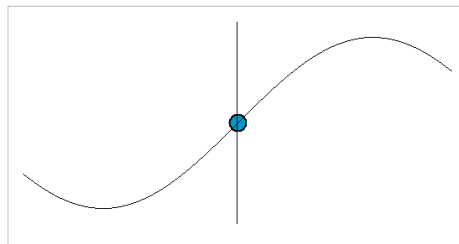


5. What is the wavelength of green light?
6. The lowest frequency sound wave humans can hear has a frequency of approximately 20Hz. Given that the speed of sound in air is 343ms^{-1} , what is the wavelength of the lowest frequency human-audible sound?

/Worked Solutions/

Phasors

Consider the image on the right. It shows a wave travelling through a medium. The moving blue dot represents the displacement caused to the medium by the wave. It can be seen that, if we consider any one point in the medium, it goes through a repeating pattern, moving up and down, moving faster the nearer it is to the centre of the waveform. Its height is determined by the amplitude of the wave at that point in time. This is determined by a sine wave.



Phasors are a method of describing waves which show two things: the displacement caused to the medium, and the point in the repeating waveform which is being represented. They consist of a circle. An arrow moves round the circle anticlockwise as the wave pattern passes. For every wavelength that goes past, the arrow moves 360° , or $2\pi^c$, starting from the right, as in trigonometry. The angle of the arrow from the right is known as the phase angle, and is usually denoted θ , and the radius of the circle is usually denoted a . The height of the point at the end of the arrow represents the displacement caused by the wave to the medium, and so the amplitude of the wave at that point in time. The time taken to rotate 360° is known as the periodic time, and is usually denoted T .

Phase difference is the difference between the angles (θ) of two phasors, which represent two waves. It is never more than 180° , as, since the phasor is moving in a circle, the angle between two lines touching the circumference will always be less than or equal to 180° . It can also be expressed in terms of λ , where λ is the total wavelength (effectively, 360°). You can use trigonometry to calculate the displacement from the angle, and vice-versa, provided you know the radius of the circle. The radius is equal to the maximum amplitude of the wave.

Phasors can be added up, just like vectors: tip-to-tail. So, for example, when two waves are superposed on each other, the phasors at each point in the reference material can be added up to give a new displacement. This explains both constructive and destructive interference as well. In destructive interference, the phasors for each wave are pointing in exactly opposite directions, and so add up to nothing at all. In constructive interference, the phasors are pointing in the same direction, so the total displacement is twice as much.

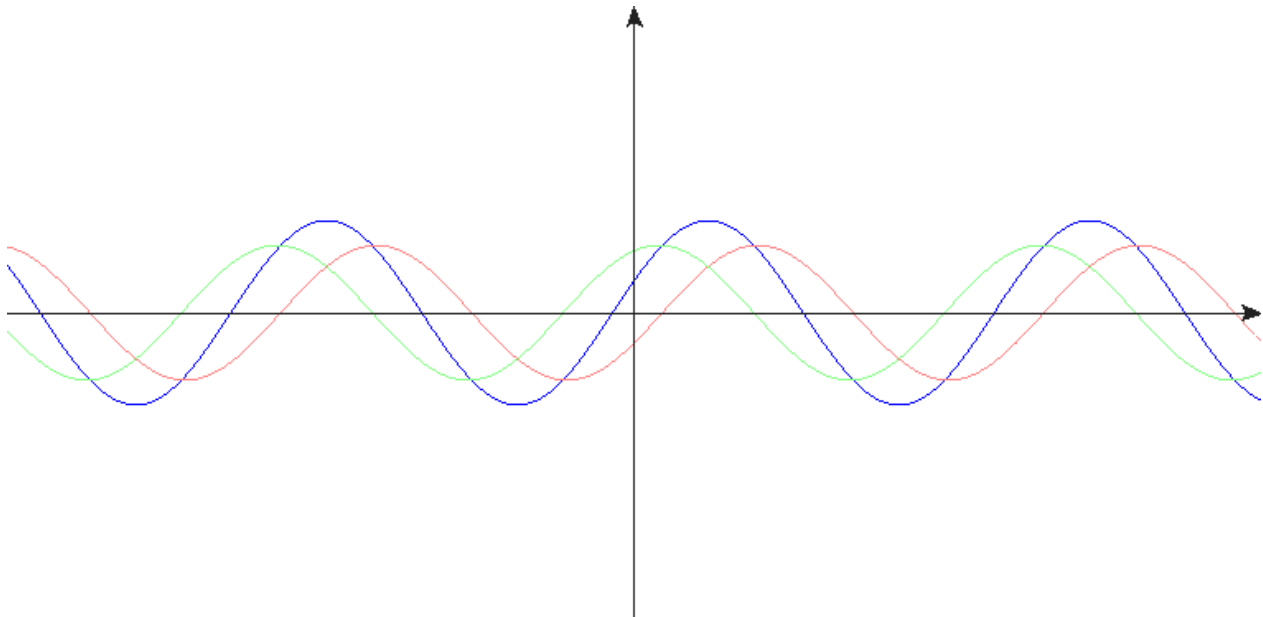
Questions

1. A sine wave with wavelength 0.1m travels through a given point on the surface of the sea. A phasor arrow representing the effect of this wave on this point rotates 1000° . How many wavelengths have gone past in the time taken for the phasor to rotate this much?
2. A sine wave has a maximum amplitude of 500nm. What is its amplitude when the phasor has rotated 60° from its start position?
3. Two waves have a phase difference of 45° . When the first wave is at its minimum amplitude of -0.3m, what is the total amplitude of the superposed waveforms?

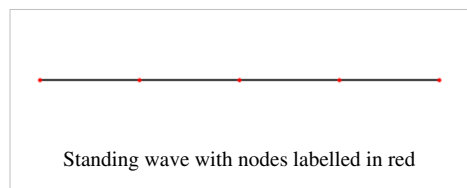
/Worked Solutions/

Standing Waves

When two coherent waves - waves of equal frequency and amplitude - travel in opposite directions through the same area, an interesting superposition effect occurs, as is shown in the following animation:



Some areas of the resultant waveform consistently have an amplitude of 0. These are known as nodes. At other points (half-way between the nodes), the resultant waveform varies from twice the amplitude of its constituent waveforms in both directions. These points are known as antinodes. Everywhere inbetween the nodes and antinodes varies to a lesser degree, depending on its position.



This effect only occurs if the two waveforms have the same amplitude and frequency. If the two waves have different amplitudes, the resultant waveform is similar to a standing wave, except that it has no nodes, and 'moves'.

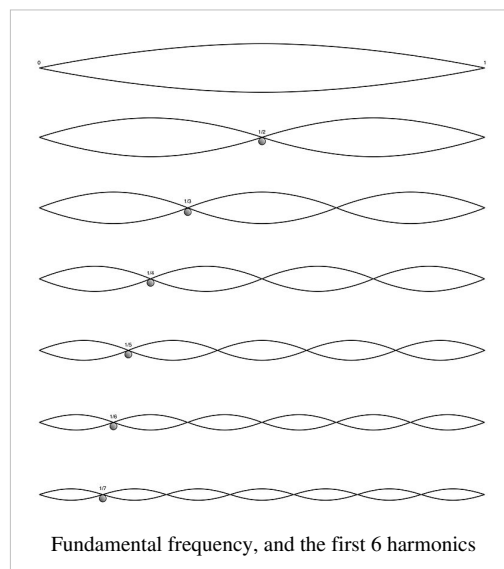
Because of these conditions, standing waves usually only occur when a waveform is reflected back on itself. For example, in a microwave oven, the microwaves are reflected by the metal on the other side of the oven from the transmitter. This creates nodes and antinodes. Since nothing cooks at the nodes, a turntable is necessary to ensure that all of the food passes through the antinodes and gets cooked.

Frequencies

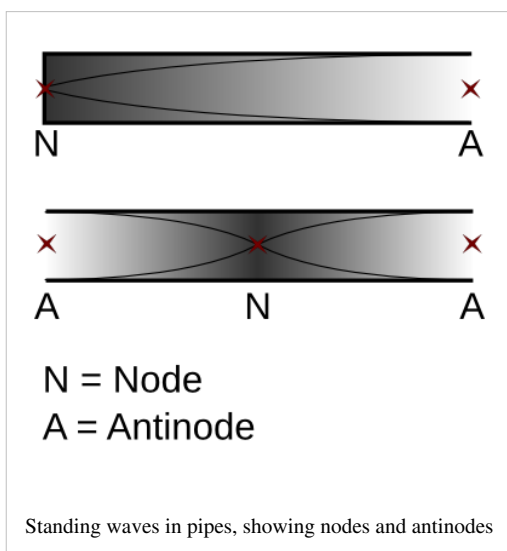
Consider a string, attached at either end, but allowed to move freely in between. If you pluck it, you create a wave which travels along the string in both directions, and is reflected at either end of the string. This process keeps on happening, and so a standing wave pattern is created. The string goes up, and then down, as shown in the first row of the diagram on the right. If you imagine the top arc as the first half of a waveform, you can see that when the string is vibrating at the fundamental frequency, the string is half as long as the wavelength: it is $\frac{1}{2}\lambda$ long.

If you were to pinch the string in the middle, and then pluck it on one side, a different standing wave pattern would be generated. By plucking, you have created an antinode, and by pinching, you have created a node. If you then let go of the string, the standing wave pattern spreads, and the string length now equals the wavelength. This is known as the first harmonic.

As you pinch the string in descending fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc.), you generate successive harmonics, and the total length of the string is equal to additional $\frac{1}{2}\lambda$ wavelengths.



Pipes



Consider a pipe which is open at one end, and closed at the other. In pipes, waves are reflected at the end of the pipe, regardless of whether it is open or not. If you blow across the end of the tube, you create a longitudinal wave, with the air as the medium. This wave travels down the tube, is reflected, travels back, is reflected again, and so on, creating a standing wave pattern.

The closed end of the pipe must be a node; it is the equivalent of pinching a string. Similarly, the open end must be an antinode; blowing across it is the equivalent of plucking the string.

Harmonics can be present in pipes, as well. This is how musical instruments work: an open hole in a wind instrument creates an antinode, changing the frequency of the sound, and so the pitch.

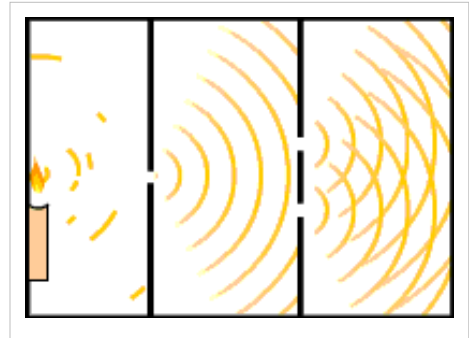
Questions

1. The air in a 3m organ pipe is resonating at the fundamental frequency. Organ pipes are effectively open at both ends. What is the wavelength of the sound?
2. A string is vibrating at the second harmonic frequency. How many wavelengths long is the standing wave created?
3. Express, in terms of λ , the length of a pipe which is closed at one end, where λ is the length of one wave at the fundamental frequency.

/Worked Solutions/

Young's Slits

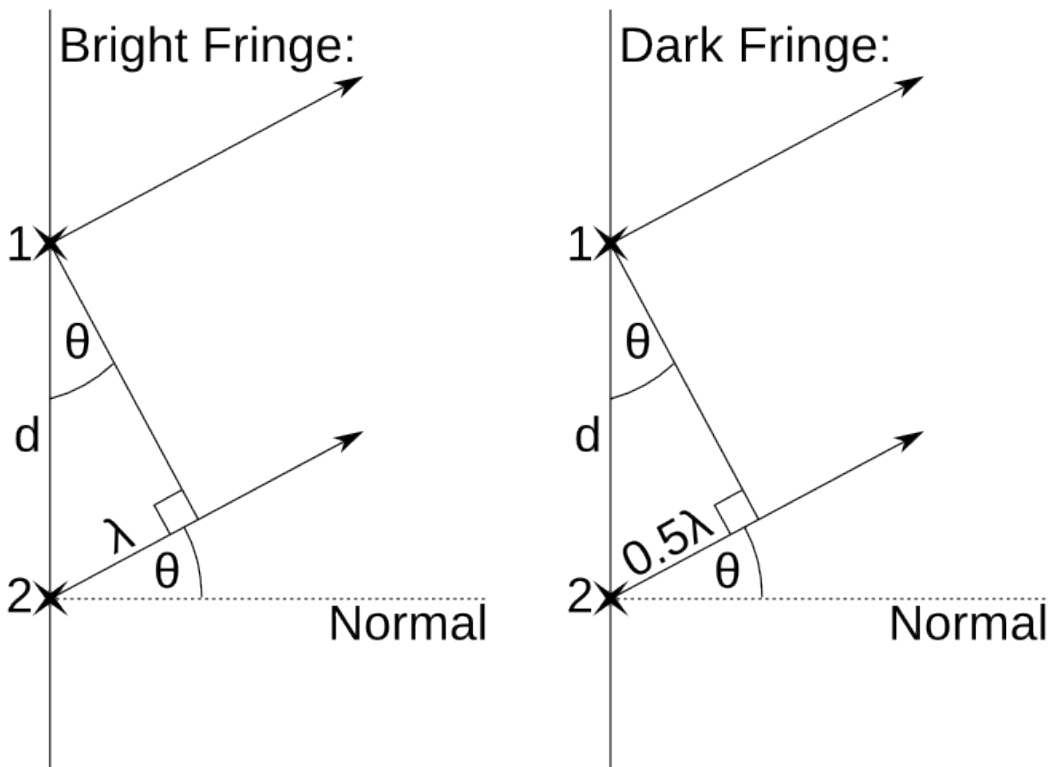
You should be familiar with the idea that, when light passes through a slit, it is diffracted (caused to spread out in arcs from the slit). The amount of diffraction increases the closer the slit width is to the wavelength of the light. Consider the animation on the right. Light from a light source is caused to pass through two slits. It is diffracted at both these slits, and so it spreads out in two sets of arcs.



Now, apply superposition of waves to this situation. At certain points, the peaks (or troughs) of the waves will coincide, creating constructive interference. If this occurs on a screen, then a bright 'fringe' will be visible. On the other hand, if destructive interference occurs (a peak coincides with a trough), then no light will be visible at that point on the screen.

Calculating the angles at which fringes occur

If we wish to calculate the position of a bright fringe, we know that, at this point, the waves must be in phase. Alternatively, at a dark fringe, the waves must be in antiphase. If we let the wavelength equal λ , the angle of the beams from the normal equal θ , and the distance between the slits equal d , we can form two triangles, one for bright fringes, and another for dark fringes (the crosses labelled 1 and 2 are the slits):



The length of the side labelled λ is known as the path difference. For bright fringes, from the geometry above, we know that:

$$\sin \theta = \frac{\lambda}{d}$$

Therefore:

$$\lambda = d \sin \theta$$

However, bright fringes do not only occur when the side labelled λ is equal to 1 wavelength: it can equal multiple wavelengths, so long as it is a whole wavelength. Therefore

$$n\lambda = d \sin \theta,$$

where n is any integer.

Now consider the right-hand triangle, which applies to dark fringes. We know that, in this case:

$$\sin \theta = \frac{0.5\lambda}{d}$$

$$0.5\lambda = d \sin \theta$$

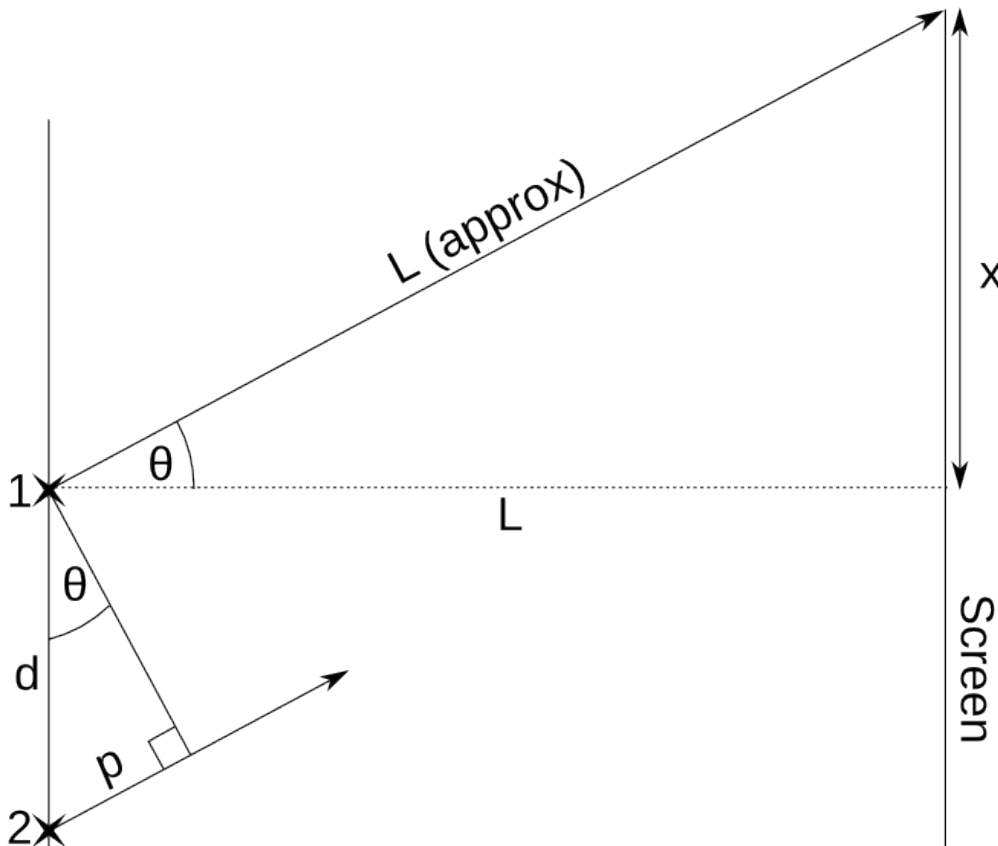
We can generalise this, too, for any dark fringe. However, if 0.5λ is multiplied by an even integer, then we will get a whole wavelength, which would result in a bright, not a dark, fringe. So, n must be an odd integer in the following formula:

$$0.5n\lambda = d \sin \theta$$

$$n\lambda = 2d \sin \theta$$

Calculating the distances angles correspond to on the screen

At this point, we have to engage in some slightly dodgy maths. In the following diagram, p is path difference, L is the distance from the slits to the screen and x is the perpendicular distance from a fringe to the normal:



Here, it is necessary to approximate the distance from the slits to the fringe as the perpendicular distance from the slits to the screen. This is acceptable, provided that θ is small, which it will be, since bright fringes get dimmer as they get further away from the point on the screen opposite the slits. Hence:

$$\sin \theta = \frac{x}{L}$$

If we substitute this into the equation for the path difference p :

$$p = d \sin \theta = \frac{dx}{L}$$

So, at bright fringes:

$$n\lambda = \frac{dx}{L}, \text{ where } n \text{ is an integer.}$$

And at dark fringes:

$$n\lambda = \frac{2dx}{L}, \text{ where } n \text{ is an odd integer.}$$

Diffraction Grating

A diffraction grating consists of a lot of slits with equal values of d . As with 2 slits, when $n\lambda = d \sin \theta$, peaks or troughs from all the slits coincide and you get a bright fringe. Things get a bit more complicated, as all the slits have different positions at which they add up, but you only need to know that diffraction gratings form light and dark fringes, and that the equations are the same as for 2 slits for these fringes.

Questions

1. A 2-slit experiment is set up in which the slits are 0.03m apart. A bright fringe is observed at an angle 10° from the normal. What sort of electromagnetic radiation was being used?
2. Light, with a wavelength of 500nm, is shone through 2 slits, which are 0.05m apart. What are the angles to the normal of the first three dark fringes?
3. Some X-rays, with wavelength 1nm, are shone through a diffraction grating in which the slits are $50\mu\text{m}$ apart. A screen is placed 1.5m from the grating. How far are the first three light fringes from the point at which the normal intercepts the screen?

/Worked Solutions/

Diffraction

We have already seen why fringes are visible when light passes through multiple slits. However, this does not explain why, when light is only passing through 1 slit, a pattern such as the one on the right is visible on the screen.

The answer to this lies in phasors. We already know that the phasor arrows add up to give a resultant phasor. By considering the phasor arrows from many paths which light takes through a slit, we can explain why light and dark fringes occur.

At the normal line, where the brightest fringe is shown, all the phasor arrows are pointing in the same direction, and so add up to create the greatest amplitude: a bright fringe.

At other fringes, we can use the same formulæ as for diffraction gratings, as we are effectively treating the single slit as a row of beams of light, coming from a row of slits.

Now consider the central beam of light. By trigonometry:

$$\sin \theta = \frac{W}{L},$$

where θ = beam angle (radians), W = beam width and L = distance from slit to screen. Since θ is small, we can approximate $\sin \theta$ as θ , so:

$$\theta \approx \frac{W}{L}$$

and since $\lambda = d \sin \theta$:

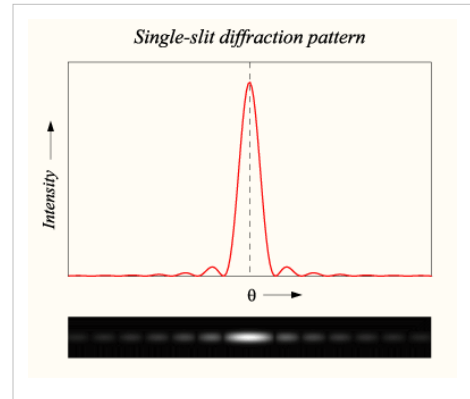
$$\theta \approx \frac{\lambda}{d}$$

Questions

1. What is the width of the central bright fringe on a screen placed 5m from a single slit, where the slit is 0.01m wide and the wavelength is 500nm?

And that's all there is to it ... maybe.

[/Worked Solutions/](#)



Finding the Distance of a Remote Object

In the final section (Section C) of the exam, you have to be able to write about how waves are used to find the distance of a remote object. I would recommend that you pick a method, and then answer the following questions about it:

1. What sort of wave does your system use? What is an approximate wavelength of this wave?
2. What sort of distance is it usually used to measure? What sort of length would you expect the distance to be?
3. Why is measuring this distance useful to society?
4. Draw a labelled diagram of your system.
5. Explain how the system works, and what data are collected.
6. Explain how the distance to the object is calculated using the data collected.
7. What limitations does your system have? (eg. accuracy, consistency)
8. What percentage error would you expect these limitations to cause?
9. How might these problems be solved?

Examples

Some example answers to these questions are given in the following pages:

- /Radar/
- /Sonar/

Quantum Physics

Light

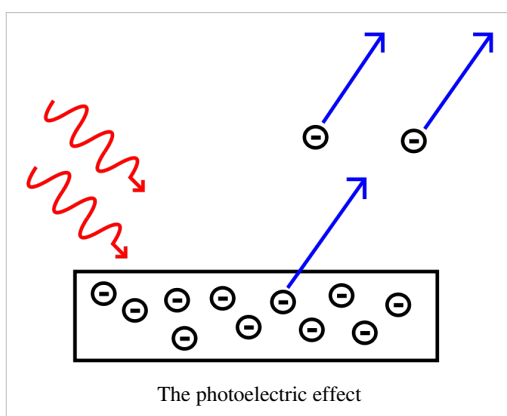
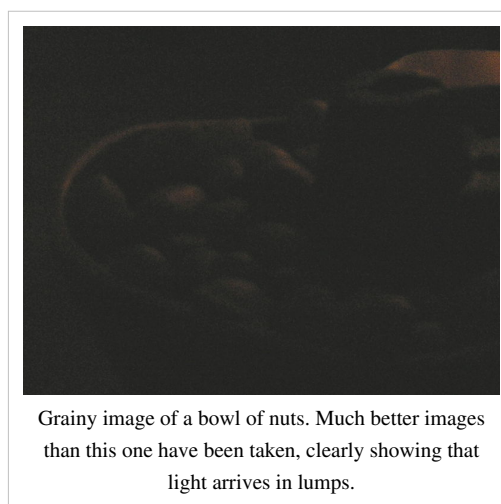
We have already seen how light behaves like both a wave and a particle, yet can be proven not to be either. This idea is not limited to light, but we will start our brief look at quantum physics with light, since it is easiest to understand.

Quantum physics is the study of quanta. A quantum is, to quote Wiktionary, "The smallest possible, and therefore indivisible, unit of a given quantity or quantifiable phenomenon". The quantum of light is the photon. We are not describing it as a particle or a wave, as such, but as a lump of energy which behaves like a particle and a wave in some cases. We are saying that the photon is the smallest part of light which could be measured, given perfect equipment. A photon is, technically, an elementary particle. It is also the carrier of all electromagnetic radiation. However, its behaviour - quantum behaviour - is completely weird, so we call it a quantum.

Evidence for the Quantum Behaviour of Light

Dim Photos

The first, and easiest to understand, piece of evidence is photographic in nature. When you take a photo with very little light, it appears 'grainy', such as the image on the right. This means that the light is arriving at the camera in lumps. If light were a wave, we would expect the photograph to appear dimmer, but uniformly so. In reality, we get clumps of light distributed randomly across the image, although the density of the random lumps is higher on the more reflective materials (the nuts). This idea of randomness, according to rules, is essential to quantum physics.



Photoelectric Effect

The second piece of evidence is more complex, but more useful since a rule can be derived from it. It can be shown experimentally that, when light of an adequate frequency falls on a metallic surface, then the surface absorbs the light and emits electrons. Hence, a current and voltage (between the surface and a positively charged terminal nearby) are produced, which can be measured.

The amount of current produced varies randomly around a certain point. This point changes depending on the frequency of the electromagnetic radiation. Furthermore, if the frequency of the radiation is not high enough, then there is no current at all! If light

were a wave, we would expect energy to build up gradually until an electron was released, but instead, if the photons do not have enough energy, then nothing happens. This is evidence for the existence of photons.

The Relationship between Energy and Frequency

The photoelectric effect allows us to derive an equation linking the frequency of electromagnetic radiation to the energy of each quantum (in this case, photons). This can be achieved experimentally, by exposing the metallic surface to light of different colours, and hence different frequencies. We already know the frequencies of the different colours of light, and we can calculate the energy each photon carries into the surface, as this is the same as the energy required to supply enough potential difference to cause the electron to move. The equation for the energy of the electron is derived as follows:

First, equate two formulae for energy:

$$P = \frac{E}{t} = IV$$

Rearrange to get:

$$E = ItV$$

We also know that:

$$Q = It$$

So, by substituting the previous equation into the equation for energy:

$$E = QV = e\Delta V,$$

where P = power, E = energy, t = time, I = current, V = potential difference, Q = charge, e = charge of 1 electron = -1.602×10^{-19} C, ΔV = potential difference produced between anode and cathode at a given frequency of radiation. This means that, given this potential difference, we can calculate the energy released, and hence the energy of the quanta which caused this energy to be released.

Plotting frequency (on the x-axis) against energy (on the y-axis) gives us an approximate straight line, with a gradient of 6.626×10^{-34} . This number is known as Planck's constant, is measured in Js, and is usually denoted h . Therefore:

$$E = hf$$

In other words, the energy carried by each quantum is proportional to the frequency of the quantum. The constant of proportionality is Planck's constant.

Questions

1. How much energy does a photon with a frequency of 50kHz carry?
2. A photon carries 10^{-30} J of energy. What is its frequency?
3. How many photons of frequency 545 THz does a 20W bulb give out each second?
4. In one minute, a bulb gives out a million photons of frequency 600 THz. What is the power of the bulb?
5. The photons in a beam of electromagnetic radiation carry $2.5\mu\text{J}$ of energy each. How long should the phasors representing this radiation take to rotate?

/Worked Solutions/

Quantum Behaviour

So far, we have identified the fact that light travels in quanta, called photons, and that these photons carry an amount of energy which is proportional to their frequency. We also know that photons aren't waves or particles in the traditional sense of either word. Instead, they are lumps of energy. They don't behave the way we would expect them to.

Many Paths

In fact, what photons do when they are travelling is to take every path possible. If a photon has to travel from point A to point B it will travel in a straight line *and* loop the loop *and* go via Alpha Centauri *and* take every other possible path. This is the photon's so-called 'quantum state'. It is spread out across all space.

However, just because a photon could end up anywhere in space does not mean that it has an equal probability of ending up in any given place. It is far more likely that a photon from a torch I am carrying will end up hitting the ground in front of me than it is that the same photon will hit me on the back of the head. But both are possible. Light can go round corners; just very rarely!

Calculating Probabilities

The probability of a photon ending up at any given point in space relative to another point in space can be calculated by considering a selection of the paths the photon takes to each point. The more paths considered, the greater the accuracy of the calculation. Use the following steps when doing this:

1. Define the light source.
 2. Work out the frequency of the photon.
 3. Define any objects which the light cannot pass through.
 4. Define the first point you wish to consider.
 5. Define a set of paths from the source to the point being considered, the more, the better.
 6. Work out the time taken to traverse of one of the paths.
 7. Work out how many phasor rotations this corresponds to.
 8. Draw an arrow representing the final phasor arrow.
 9. Repeat steps 6-8 for each of the paths.
 10. Add all the phasor arrows together, tip-to-tail.
 11. Square the amplitude of this resultant phasor arrow to gain the intensity of the light at this point. It may help to imagine a square rotating around, instead of an arrow.
 12. Repeat steps 4-11 for every point you wish to consider. The more points you consider, the more accurate your probability distribution will be.
 13. Compare all the resultant intensities to gain a probability distribution which describes the probabilities of a photon arriving at one point to another. For example, if the intensity of light at one point is two times the intensity of light at another, then it is twice as likely that a photon will arrive at the first point than the second.
 14. If all the points being considered were on a screen, the intensities show you the relative brightnesses of light at each of the points.
-

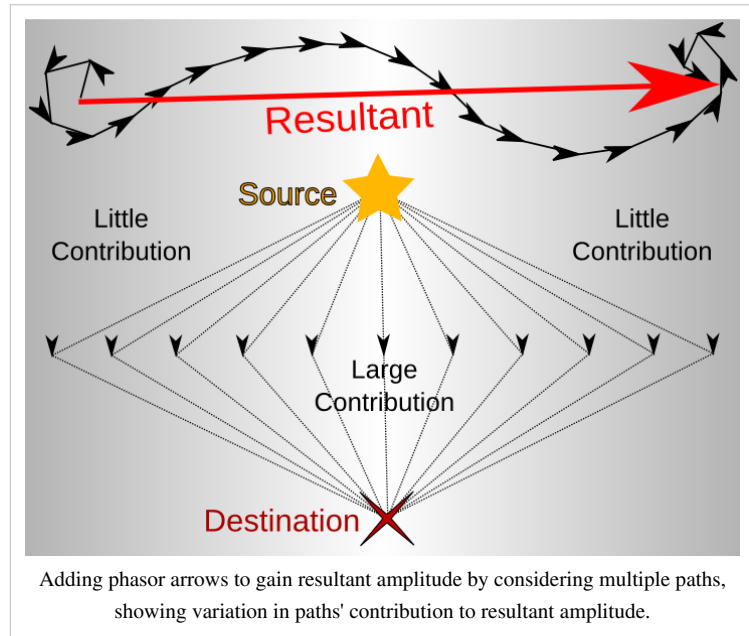
Examples

If we now take this method and apply it to several situations, we find that, in many cases, the results are similar to those obtained when treating light as a wave, with the exception that we can now reconcile this idea with the observable 'lumpiness' of light, and can acknowledge the fact that there is a certain probability that some light will not behave according to some wave laws.

Travelling from A to B

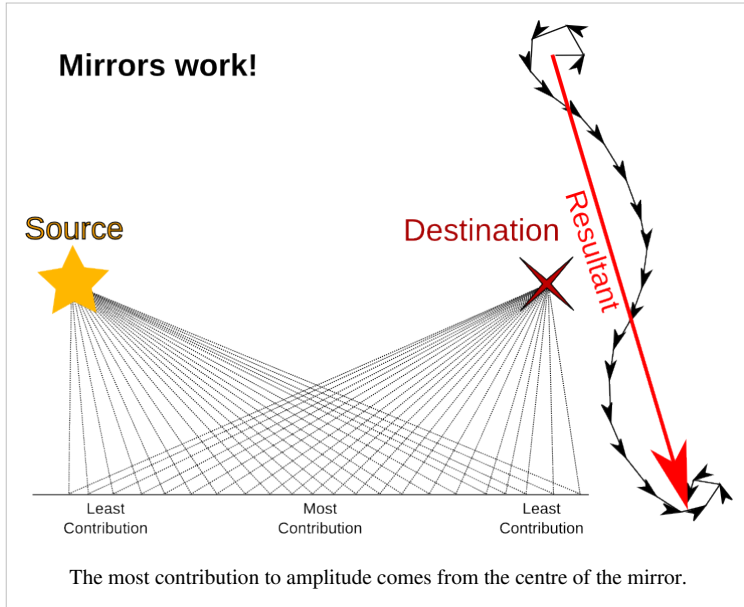
This is the simplest example to consider. If we consider a range of paths going from point A to point B, and calculate the phasor directions at the end of the paths, we get a resultant phasor arrow which gives us some amplitude at point B. Since there are no obstructions, at any point this far away from the source, we will get the same resultant amplitude.

It is important to note that different paths contribute to the resultant amplitude by different amounts. The paths closer to the straight line between the two points are more parallel to the resultant angle, whereas the paths further away vary in direction more, and so tend to cancel each other out. The conclusion: light travelling in straight lines contributes most to the resultant amplitude.



Young's Slits

Here, we just need to consider two paths: one through each slit. We can then calculate two phasor arrows, add them together to gain a resultant phasor arrow, and square its amplitude to gain the intensity of the light at the point the two paths went to. When calculated, these intensities give a pattern of light and dark fringes, just as predicted by the wave theory.

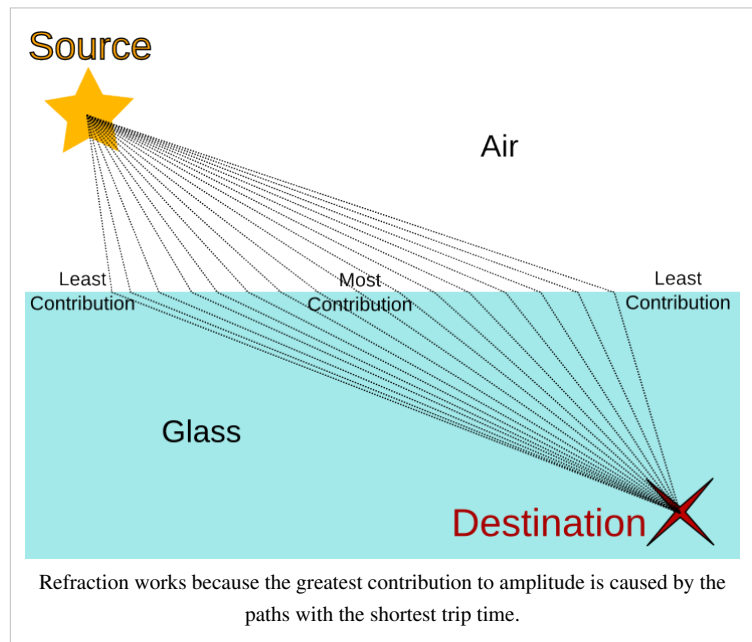


Reflection

This situation is very similar to what happens when light travels in a 'straight line'. The only difference is that we consider the paths which involve rebounding off an obstacle. The results are more or less the same, but the paths from which they were obtained are different. This means that we can assume the same conclusions about these different paths: that most of the resultant amplitude comes from the part of the mirror where the angle of incidence equals the angle of reflection. In other words, the likelihood is that a photon will behave as if mirrors work according to wave theory.

Refraction

Different paths have different lengths, and so photons take different amounts of time to traverse them (these are known as trip times). In the diagram on the right, the photons again traverse all possible paths. However, the paths with the smallest difference between trip times have phasor arrows with the smallest difference in direction, so the paths with the smallest trip times contribute most to the resultant amplitude. This shortest path is given by Snell's law. Yet again, quantum physics provides a more accurate picture of something which has already been explained to some degree.



Diffraction

Diffraction occurs when the photons are blocked from taking every other path. This occurs when light passes through a gap less than 1 wavelength wide. The result is that, where the amplitudes would have roughly cancelled each other out, they do not, and so light spreads out in directions it would not normally spread out in. This explains diffraction, instead of just being able to observe and calculate what happens.

Electron Behaviour

So far, we have considered how quantum physics applies to photons, the quanta of light. In reality, every other particle is also a quantum, but you only need to know about photons and electrons.

The image on the right shows what happens when you fire electrons through a pair of slits: it arrives in lumps, but you get fringes due to superposition as well. The electrons are behaving as both waves and particles. Actually, they are behaving as quanta. The equations describing quantum behaviour in electrons are similar to those describing it in photons.

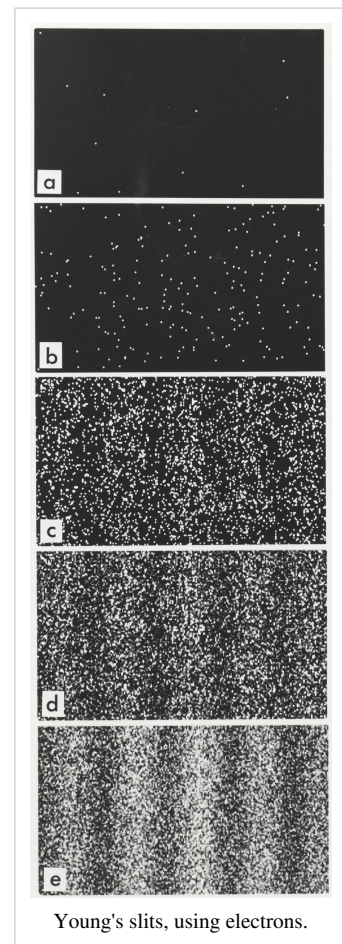
Frequency and Kinetic Energy

We know that, for photons:

$$f = \frac{E}{h}$$

In suggesting that electrons are quanta, we assume that they must have a frequency at which the phasors representing them rotate. We also know that h is a constant; it does not change. So, when adapting the above equation to apply to electrons, all we need to adapt is E . In electrons, this energy is their kinetic energy. If the electron has some form of potential energy, this must first be subtracted from the kinetic energy, as this portion of the kinetic energy does not affect frequency. So:

$$f = \frac{E_{kinetic} - E_{potential}}{h}$$



Young's slits, using electrons.

De Broglie Wavelength

If electrons exhibit some wavelike properties, they must also have a 'wavelength', known as the de Broglie wavelength, after its discoverer. This is necessary in order to work out a probability distribution for the position of an electron, as this is the distance the electron travels for each phasor arrow rotation. The de Broglie wavelength λ is given by the equation:

$$\lambda = \frac{h}{p} = \frac{h}{mv},$$

where h = Planck's constant, p = momentum, m = mass of electron = 9.1×10^{-31} kg and v = velocity of electron.

Potential Difference and Kinetic Energy

Potential difference is what causes electrons to move. You already know how power is related to charge, voltage and time:

$$P = \frac{QV}{t}$$

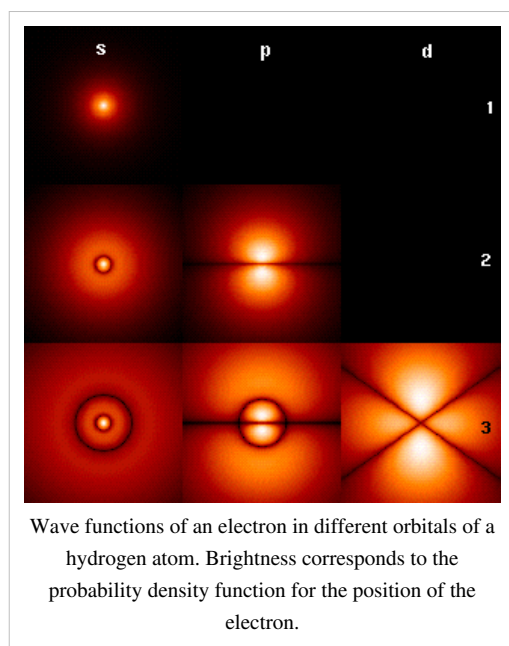
Since power is the rate at which work is done:

$$W = QV$$

We know that the charge on an electron equals -1.6×10^{-19} , and that work done is energy, so:

$$E_{kinetic} = 1.6 \times 10^{-19} \times V$$

Energy, in the SI system of units, is measured in Joules, but, sometimes it is measured in electronvolts, or eV. 1 eV is the kinetic energy of 1 electron accelerated by 1V of potential difference. So, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.



Questions

1. An electron moves at $30,000 \text{ ms}^{-1}$. What is its de Broglie wavelength?
2. What is its frequency?
3. What is its kinetic energy, in eV?
4. Given that it is travelling out of an electron gun, what was the potential difference between the anode and the cathode?
5. An electron is accelerated by a potential difference of 150V. What is its frequency?

/Worked Solutions/

Mechanics

Vectors

What is a vector?

Two types of physical quantity are scalars and vectors. Scalar quantities are simple: they are things like speed, distance, or time. They have a magnitude, but no direction. A vector quantity consists of two parts: both a scalar and a direction. For example, the velocity of an object is made up of both the speed of an object and the direction in which it is moving. Speed is a scalar; add a direction and it becomes velocity, a vector. Similarly, take a distance and give it a direction and it becomes a displacement, such as '2 miles south-east'. Distance is a scalar, whereas displacement is a vector.

Vectors and scalars are both useful. For example, if I run around the room several times and end up back where I started, I may have covered a distance of 50m. My displacement is 0 - the null vector. The null vector is the only vector which has no direction. If I want to calculate how much work I have done, I should use the distance. If I want to know where I am, I should use the displacement.

As we shall see, the directional component of a vector can be expressed in several different ways. '2 miles south-east' is the same as saying '2 miles on a bearing of 135°', or '1.4 miles east, -1.4 miles north'. The scalar component of a vector is known as the **modulus** of a vector.

Vector Notation

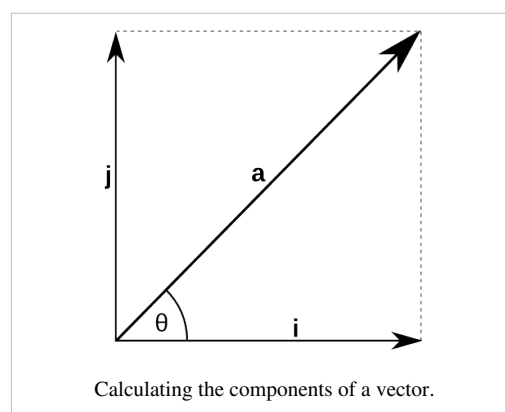
You need to be able to understand the following algebraic representations of vectors:

Symbol	Meaning
\vec{ab}	A vector from point a to point b.
\mathbf{a}	A vector named 'a'. This is used in typed algebra.
\underline{a}	A vector named 'a'. This is used in handwritten algebra.
$ \vec{ab} $ or $ \mathbf{a} $ or $ \underline{a} $	The modulus of a vector.

Vector Components

Sometimes, it is useful to express a vector in terms of two other vectors. These two vectors are usually pointing up and right, and work similarly to the Cartesian co-ordinate system. So, for example, 'an acceleration of 3.4 ms^{-2} west' becomes 'a vertical acceleration of 0 ms^{-2} and an horizontal acceleration of -3.4 ms^{-2} east. However, this is a very simple example.

Consider the diagram on the right. The vector \mathbf{a} consists of a vertical component \mathbf{j} and a horizontal component \mathbf{i} . \mathbf{a} has a modulus $|\mathbf{a}|$. $|\mathbf{i}|$ and $|\mathbf{j}|$ can be calculated using $|\mathbf{a}|$, the angle between \mathbf{i} and \mathbf{a} θ and some basic trigonometry. We know that:



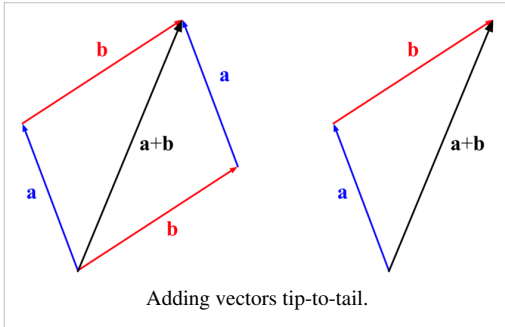
$$\cos \theta = \frac{|\mathbf{i}|}{|\mathbf{a}|} \text{ and } \sin \theta = \frac{|\mathbf{j}|}{|\mathbf{a}|}$$

Hence:

$$|\mathbf{i}| = |\mathbf{a}| \cos \theta \text{ and } |\mathbf{j}| = |\mathbf{a}| \sin \theta.$$

This will be given to you in the formula booklet in the exam.

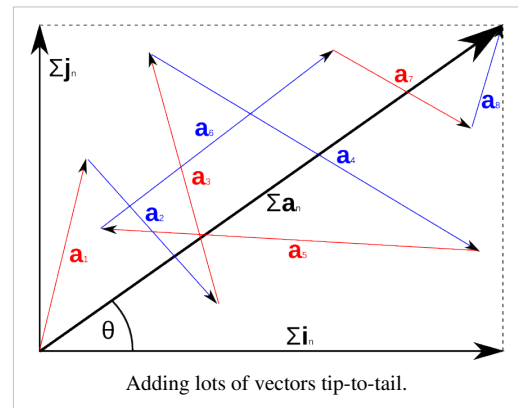
Vector Addition



You also need to know how to add vectors together. This enables us to answer questions such as, "If I travel 5 miles north-west and then 6 miles east, where am I?", or "If I accelerate at 3 ms^{-2} in a northerly direction, and accelerate south-east at 1 ms^{-2} , what is total acceleration?". Vectors can be added 'tip-to-tail'; that is to say, the resultant vector is equal to 'travelling' the first vector, and then travelling the second vector.

This is shown in the diagram on the left. When vectors \mathbf{a} and \mathbf{b} are added together, the resultant vector $\mathbf{a} + \mathbf{b}$ is produced, joining the tail of the first vector to the tip of the last, with the vectors joined together. In practise, the easiest way to add two vectors together is to calculate (if you do not already know this) the vertical and horizontal components, and add them all together to get two total vertical and horizontal components. You can then use Pythagoras' theorem to calculate the modulus of the resultant vector, and some more basic trigonometry to calculate its direction.

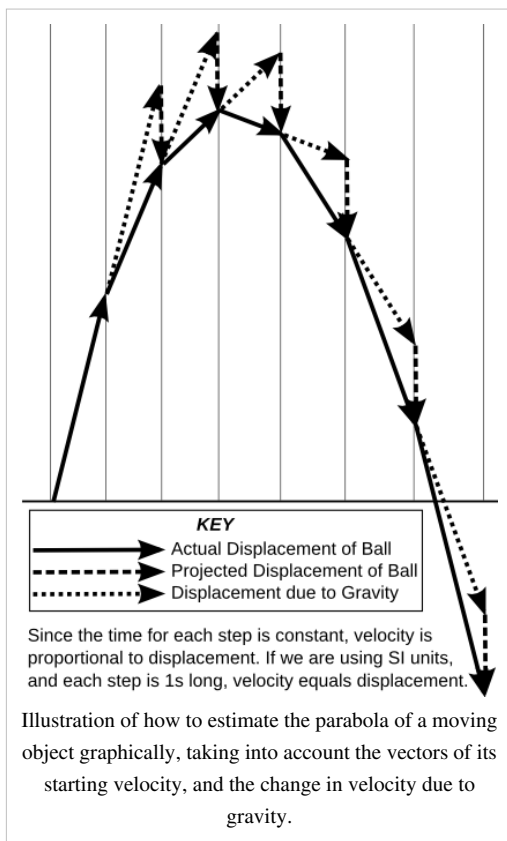
In algebra:



$$\sum |\mathbf{a}|_n = \sqrt{(\sum |\mathbf{i}|_n)^2 + (\sum |\mathbf{j}|_n)^2} \text{ and } \theta = \arctan \frac{\sum |\mathbf{j}|_n}{\sum |\mathbf{i}|_n},$$

where $\mathbf{a}_1 \dots \mathbf{a}_n$ are the vectors to be added together, $\mathbf{i}_1 \dots \mathbf{i}_n$ are their horizontal components, $\mathbf{j}_1 \dots \mathbf{j}_n$ are their vertical components, and θ is the angle between the $\theta=0$ line and the resultant vector $\Sigma \mathbf{a}_n$, as shown in the diagram on the right.

Predicting Parabolas



If you use a diagram to represent vectors (making the lengths of the arrows proportional to the modulus of the vectors they represent, and the directions equal), you can predict graphically the trajectory an object (such as a ball) will take. Use the following steps:

- Draw a vector to represent the velocity of the ball (in ms^{-1}). Since this is the number of metres travelled in 1 second, and each step of the process is 1 second long, this vector represents both the velocity and the displacement of the ball, ie.

$$\Delta \mathbf{v} = \frac{\Delta \mathbf{s}}{\Delta t} = \frac{\Delta \mathbf{s}}{1} = \Delta \mathbf{s}$$

- Copy this vector, and connect its tail to the tip of the first vector. This new vector represents the velocity and displacement that the ball would have had over the next second, if gravity did not exist.
- Draw another vector to represent the change in velocity due to gravity (9.81 ms^{-2}) on Earth. This should be pointing downwards, and be to the same scale as the other vectors. This represents the fact that the velocity of the ball changes due to gravity (velocity is a vector, so both the speed and angle of the ball's travel change).
- Add these vectors together, as shown above, to give a new vector. This new vector represents the velocity and displacement of the ball over the second second.
- Repeat this process until the ball hits something (draw the ground, if in doubt).

Questions

1. Which of the following are vectors?

- 20 cm
- 9.81 ms^{-2} towards the centre of the earth
- 5 km south-east
- 500 ms^{-1} on a bearing of 285.3°

2. A displacement vector **a** is the resultant vector of two other vectors, 5 m north and 10 m south-east. What does **a** equal, as a displacement and a bearing?

3. If I travel at a velocity of 10 ms^{-1} on a bearing of 030° , at what velocity am I travelling north and east?

4. An alternative method of writing vectors is in a column, as follows:

$$\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix},$$

where *x* and *y* are the vertical and horizontal components of the vector respectively. Express *|\mathbf{a}|* and the angle between **a** and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in terms of *x* and *y*.

5. A more accurate method of modelling the trajectory of a ball is to include air resistance as a constant force **F**. How would this be achieved?

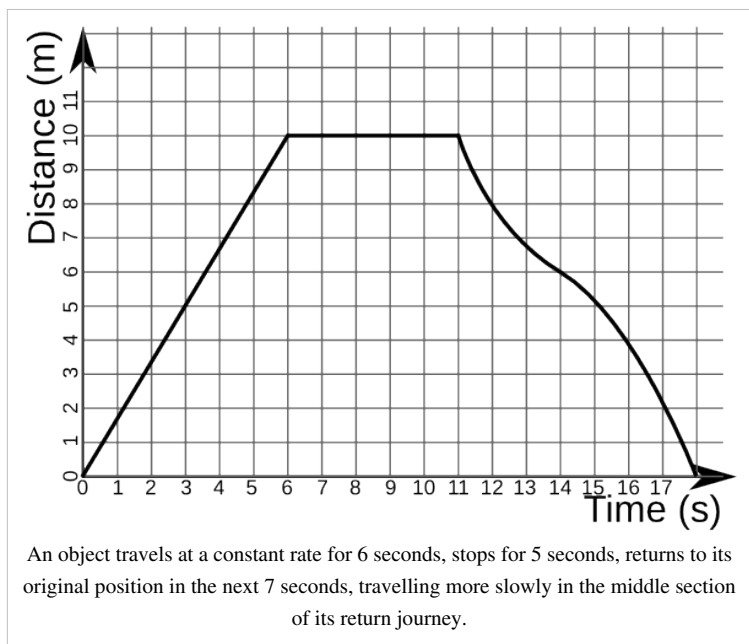
/Worked Solutions/

Graphs

There are two types of graphs of motion you need to be able to use and understand: distance-time graphs and velocity-time graphs.

Distance-time Graphs

A distance-time graph plots the distance of an object away from a certain point, with time on the x-axis and distance on the y-axis. If we were to imagine the line on the graph as a function *f(t)*, giving an equation for $s = f(t)$, we could differentiate this to gain:



$$\frac{ds}{dt} = f'(t),$$

where s is displacement, and t is time. By finding $f'(t)$ at any given time t , we can find the rate at which distance is increasing (or decreasing) with respect to t . This is the gradient of the line. A positive gradient means that distance is increasing, and a negative gradient means that distance is decreasing. A gradient of 0 means that the object is stationary. The velocity of the object is the rate of change of its displacement, which is the same as the gradient of the line on a distance-time graph. This is not necessarily the same as the average velocity of the object v :

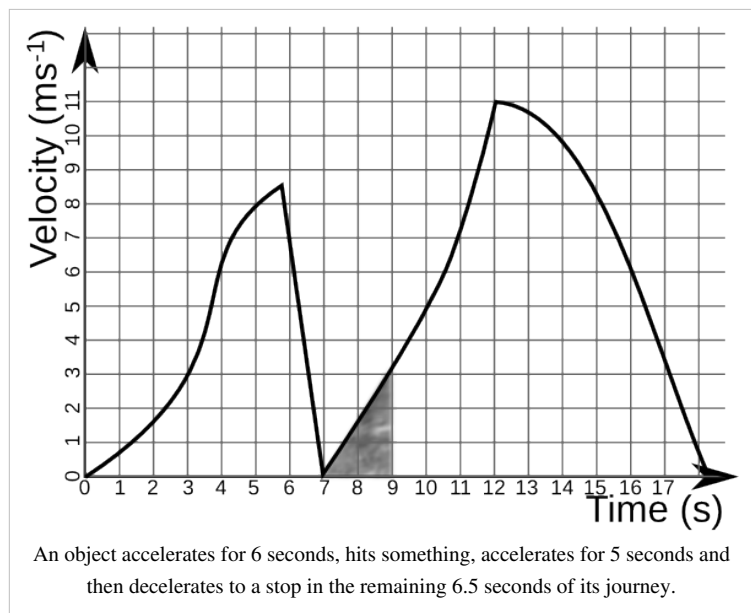
$$v = \frac{s}{t}$$

Here, t and s are the total changes in displacement and time over a certain period - they do not tell you exactly what was happening at any given point in time.

Velocity-time Graphs

A velocity-time graph plots the velocity of an object, relative to a certain point, with time on the x -axis and distance on the y -axis. We already know that velocity is the gradient (derivative) of the distance function. Since integration is the inverse process to differentiation, if we have a velocity-time graph and wish to know the distance travelled between two points in time, we can find the area under the graph between those two points in time. In general:

If $v = f(t)$



$$s = \int_{t_1}^{t_2} f(t) dt$$

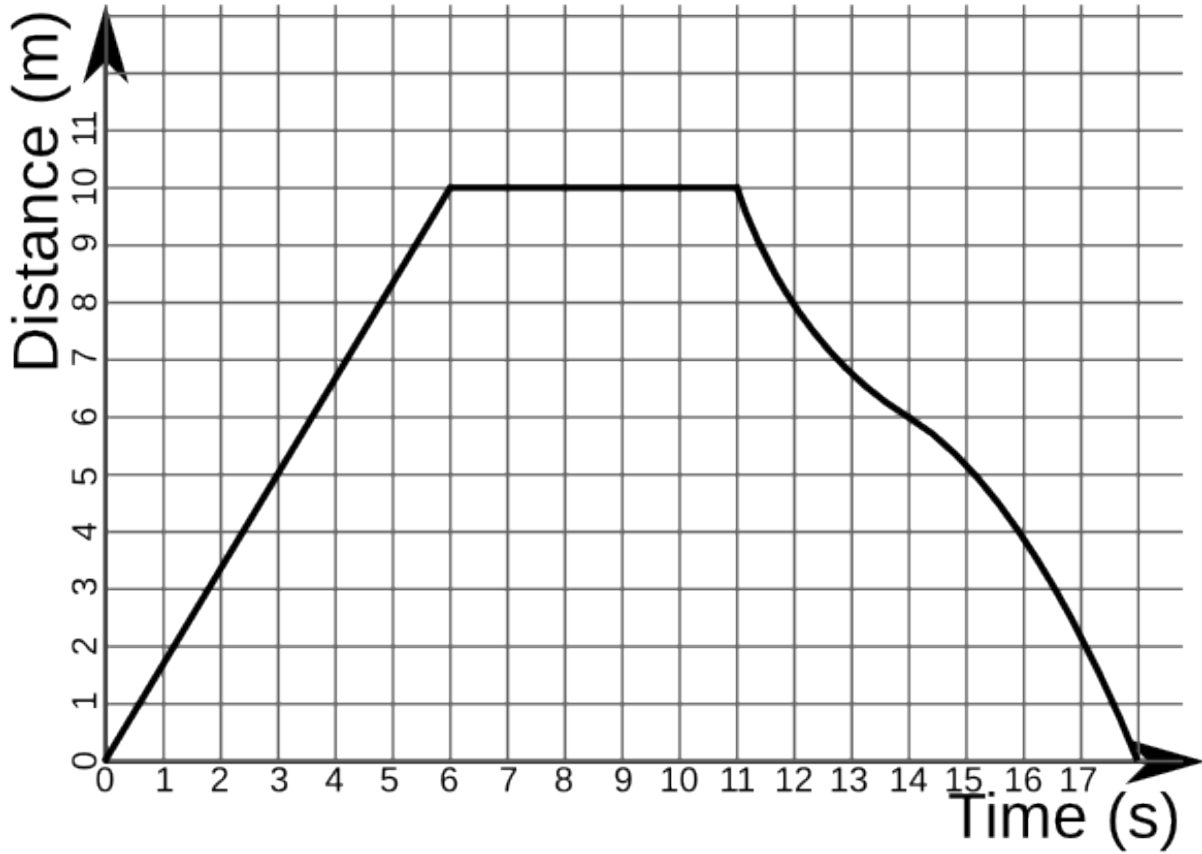
where v is velocity (in ms^{-1}), t is time (in s), and s is the distance travelled (in m) between two points in time t_1 and t_2 .

Also, by differentiation, we know that the gradient (or derivative of $v = f(t)$) is equal to the acceleration of the object at any given point in time (in ms^{-2}) since:

$$a = \frac{dv}{dt}$$

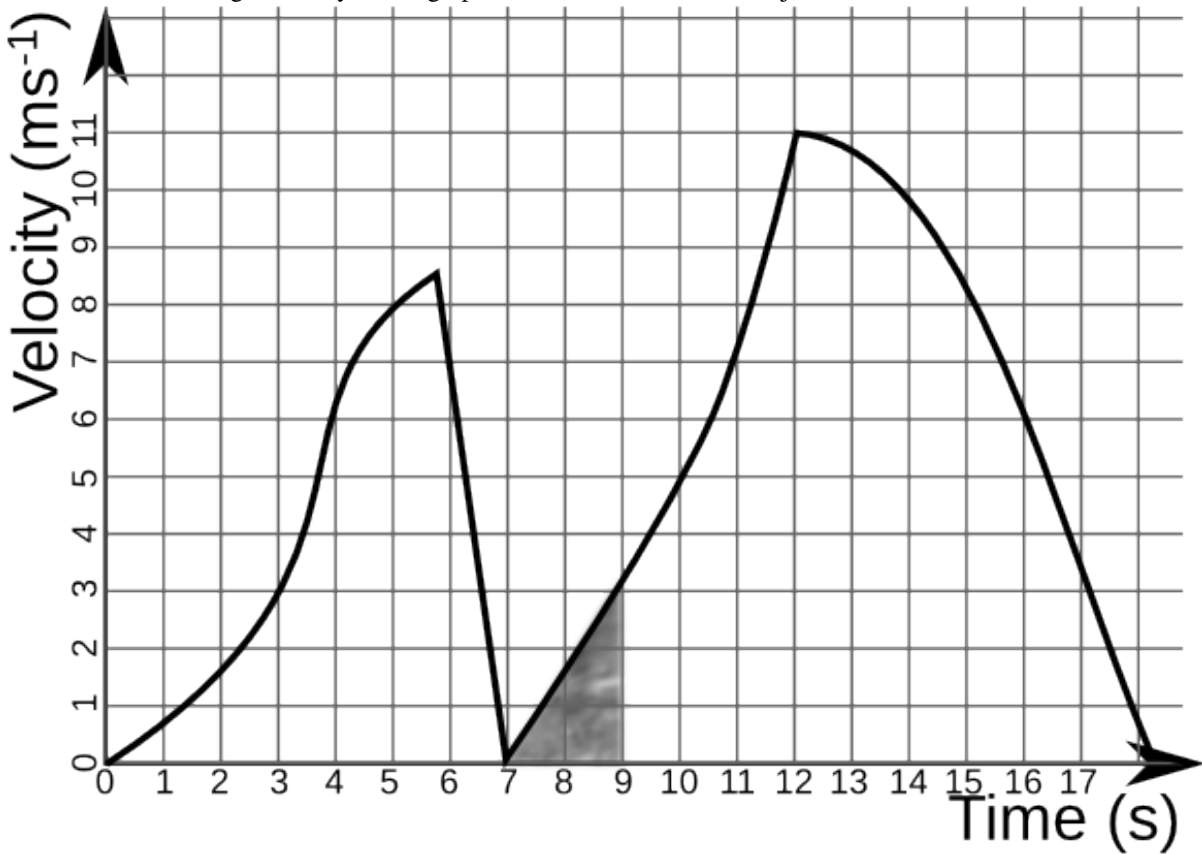
Questions

1. In the following distance-time graph, what is the velocity 4 seconds after the beginning of the object's journey?



2. What is the velocity at 12 seconds?

3. In the following velocity-time graph, how far does the object travel between 7 and 9 seconds?



4. What is the object's acceleration at 8 seconds?
5. A car travels at 10ms^{-1} for 5 minutes in a straight line, and then returns to its original location over the next 4 minutes, travelling at a constant velocity. Draw a distance-time graph showing the distance the car has travelled from its original location.
6. Draw the velocity-time graph for the above situation.

The following question is more difficult than anything you will be given, but have a go anyway:

7. The velocity of a ball is related to the time since it was thrown by the equation $v = 30 - 9.8t$. How far has the ball travelled after 2 seconds?

/Worked Solutions/

Kinematics

Kinematics is the study of how objects move. One needs to understand a situation in which an object changes speed, accelerating or decelerating, and travelling a certain distance. There are four equations you need to be able to use which relate these quantities.

Variables

Before we can understand the kinematic equations, we need to understand the variables involved. They are as follows:

- t is the length of the interval of time being considered, in seconds.
- v is the speed of the object at the beginning of the time interval, in ms^{-1} .
- u is the speed of the object at the end of the time interval, in ms^{-1} .
- a is the acceleration of the object during the time interval, in ms^{-2} .
- s is the displacement (distance travelled) of the object during the time interval, in metres.

Equations

The four equations are as follows:

1. $v = u + at$

2. $s = \frac{u + v}{2}t$

3. $s = ut + \frac{at^2}{2}$

4. $v^2 = u^2 + 2as$

Derivations

It is also useful to know where the above equations come from. We know that acceleration is equal to change in speed per. unit time, so:

$$a = \frac{v - u}{t} (*)$$

$$at = v - u$$

$$v = u + at \quad (1)$$

We also know that the average speed over the time interval is equal to displacement per. unit time, so:

$$\frac{u + v}{2} = \frac{s}{t}$$

$$s = \frac{u + v}{2}t \quad (2)$$

If we substitute the value of v from equation 1 into equation 2, we get:

$$s = \frac{u + (u + at)}{2}t = \frac{2u + at}{2}t = t\left(u + \frac{at}{2}\right) = ut + \frac{at^2}{2} \quad (3)$$

If we take the equation for acceleration (*), we can rearrange it to get:

$$at = v - u$$

$$t = \frac{v - u}{a}$$

If we substitute this equation for t into equation 2, we obtain:

$$s = \frac{u + v}{2} \frac{v - u}{a} = \frac{(v + u)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as \quad (4)$$

Questions

1. A person accelerates from a speed of 1 ms^{-1} to 1.7 ms^{-1} in 25 seconds. How far has he travelled in this time?
2. A car accelerates at a rate of 18 kmh^{-2} to a speed of 60 kmh^{-1} , travelling 1 km in the process. How fast was the car travelling before it travelled this distance?
3. A goose in flight is travelling at 4 ms^{-1} . It accelerates at a rate of 1.5 ms^{-2} for 7 seconds. What is its new speed?
4. How far does an aeroplane travel if it accelerates from 400 kmh^{-1} at a rate of 40 kmh^{-2} for 1 hour?

/Worked Solutions/

Forces and Power

Forces

Forces are vectors. When solving problems involving forces, it is best to think of them as lots of people pulling ropes attached to an object. The forces are all pulling in different directions, with different magnitudes, but the effect is in only one direction, with only one magnitude. So, you have to add the forces up as vectors.

Forces cause things to happen. They cause an object to accelerate in the same direction as the force. In other words, forces cause objects to move in a direction closer to the direction they are pulling in. If the object is already moving, then they will not cause it to move in the direction of the force, as forces do not create velocities: they create accelerations.

If a force is acting on an object, it seems logical that the object will not accelerate as much as a result of the force if it has a lower mass. This gives rise to the equation:

$$F = ma,$$

where F = force applied (in Newtons, denoted N), m = mass (in kg) and a = acceleration (in ms^{-2}). If we rearrange the equation, it makes more sense:

$$a = \frac{F}{m}$$

In other words, the acceleration in a given direction as the result of a force is equal to the force applied per. unit mass in that direction.

Work Done

You should already know how to calculate some types of energy, for example:

$$\text{Kinetic Energy} = \frac{mv^2}{2}$$

$$\text{Gravitational Potential Energy} = mgh$$

The amount of energy converted by a force is equal to the work done, which is equal (as you already know) to the force multiplied by the distance the object it is acting on moves:

$$\text{Work Done} = \Delta E = F\Delta s$$

When answering questions about work done, you may be given a force acting in a direction other than that of the displacement. In this case, you will have to find the displacement in the direction of the force, as shown in the section on Vectors.

Power

Power is the rate of change of energy. It is the amount of energy converted per. unit time, and is measured in Js^{-1} :

$$P = \frac{\Delta E}{t},$$

where E = energy (in J) and t = time (in s). Since ΔE = work done, power is the rate at which work is done. Since:

$$\Delta E = F\Delta s$$

$$\frac{\Delta E}{t} = F\frac{\Delta s}{t}$$

$$P = Fv,$$

where P = power (in Watts, denoted W), F = force and v = velocity.

Gravity

Gravity is something of a special case. The acceleration due to gravity is denoted g , and is equal to 9.81ms^{-2} . It is uniform over small distances from the Earth. The force due to gravity is equal to mg , since $F = ma$. Therefore:

$$a = \frac{F}{m} = \frac{mg}{m} = g$$

Therefore, when things are dropped, they all fall at the same acceleration, regardless of mass. Also, the acceleration due to gravity (in ms^{-2}) is equal to the gravitational field strength (in Nkg^{-1}).

Questions

1. I hit a ball of mass 5g with a cue on a billiards table with a force of 20N. If friction opposes me with a force of 14.2N, what is the resultant acceleration of the ball away from the cue?
2. A 10g ball rolls down a 1.2m high slope, and leaves it with a velocity of 4ms^{-1} . How much work is done by friction?
3. An electric train is powered on a 30kV power supply, where the current is 100A. The train is travelling at 90kmh^{-1} . What is the net force exerted on it in a forwards direction?

/Worked Solutions/

A2

Decay

Exponential Relationships

Many things are governed by exponential relationships. The exponential relationships which we shall be dealing with are of the following form:

$$x = ae^{bt},$$

where t is time, x is a variable, and a and b are constants. e is just a number, albeit a very special number. It is an irrational constant, like π . e is 2.71828182845904523536 to 20 decimal places. However, it is far easier just to find the e (or exp) button on your calculator.

The inverse function of e^t is the natural logarithm, denoted $\ln t$:

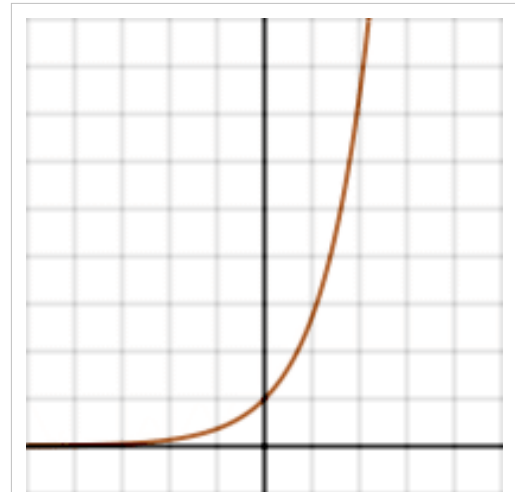
$$\ln t = \log_e t$$

Growth and Decay

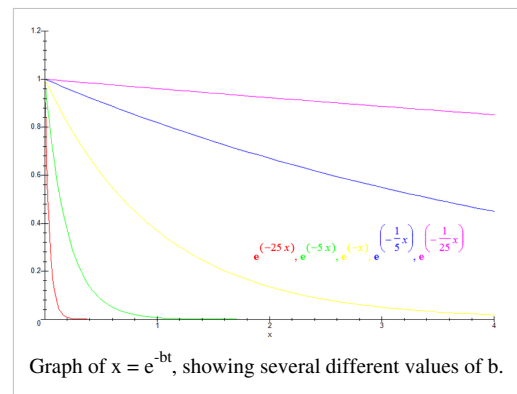
When b is positive, an exponential function increases rapidly. This represents the growth of certain variables very well. When b is negative, an exponential function decreases, flattening out as it approaches the t axis. This represents the decay of certain variables.

Exponential Relationships in the Real World

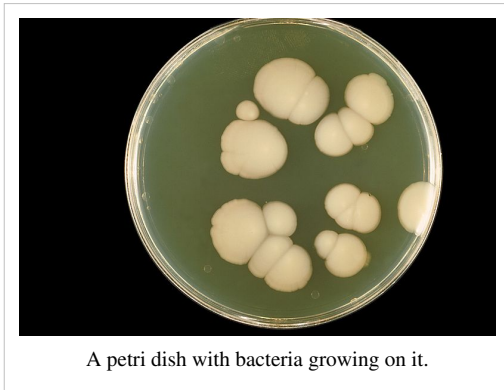
An exponential relationship occurs when the rate of change of a variable depends on the value of the variable itself. You should memorise this definition, as well as understand it. Let us consider some examples:



Graph of $x = ae^{bt}$, where b is positive.



Graph of $x = e^{-bt}$, showing several different values of b .



Population Growth

Let us do some biology for a moment. Consider a Petri dish full of agar jelly (food for bacteria) with a few bacteria on it. These bacteria will reproduce, and so, as time goes by, the number of bacteria on the jelly will increase. However, each bacterium does not care about whether there are other bacteria around or not. It will continue making more bacteria at the same rate. Therefore, as the total number of bacteria increases, their rate of reproduction increases. This is an exponential relationship with a positive value of b .

Of course, this model is flawed since, in reality, the bacteria will eventually have eaten all the agar jelly, and so the relationship will stop being exponential.

Emptying Tank

If you fill a large tank with water, and make a hole in the bottom, at first, the water will flow out very fast. However, as the tank empties, the pressure of the water will decrease, and so the rate of flow will decrease. The rate of change of the amount of water in the tank depends on the amount of water in the tank. This is an exponential relationship with a negative value of b - it is an exponential decay.

Cooling

A hot object cools down faster than a warm object. So, as an object cools, the rate at which temperature 'flows' out of it into its surroundings will decrease. Newton expressed this as an exponential relationship (known as Newton's Law of Cooling):

$$T_t = T_{env} + (T_0 - T_{env})e^{-rt},$$

where T_t is the temperature at a time t , T_0 is the temperature at $t = 0$, T_{env} is the temperature of the environment around the cooling object, and r is a positive constant. Note that a here is equal to $(T_0 - T_{env})$ - but a is still a constant since T_0 and T_{env} are both constants. The '-' sign in front of the r shows us that this is an exponential decay - the temperature of the object is tending towards the temperature of the environment. The reason we add T_{env} is merely a result of the fact that we do not want the temperature to decay to 0 (in whatever unit of temperature we happen to be using). Instead, we want it to decay towards the temperature of the environment.

Mathematical Derivation

We have already said that an exponential relationship occurs when the rate of change of a variable depends on the value of the variable itself. If we translate this into algebra, we get the following:

$$\frac{dx}{dt} = ax, \text{ where } a \text{ is a constant.}$$

By separating the variables:

$$dx = axdt$$

$$\frac{1}{x}dx = adt$$

$$\int \frac{1}{x}dx = \int adt$$

$$\ln x = at + c \text{ (where } c \text{ is the constant of integration)}$$

$$x = e^{at+c} = e^{at}e^c$$

If we let $b = e^c$ (b is a constant, since e^c is a constant):

$$x = be^{at}$$

Questions

1. Simplify Newton's Law of Cooling for the case when I place a warm object in a large tank of water which is on the point of freezing. Measure temperature in °C.
2. What will the temperature of an object at 40 °C be after 30 seconds? (Take $r=10^{-3} \text{ s}^{-1}$.)
3. A body is found in a library (as per Agatha Christie) at 8am. The temperature of the library is kept at a constant temperature of 20 °C for 10 minutes. During these 10 minutes, the body cools from 25 °C to 24 °C. The body temperature of a healthy human being is 36.8 °C. At what time was the person murdered?
4. Suppose for a moment that the number of pages on Wikibooks p can be modelled as an exponential relationship. Let the number of pages required on average to attract an editor be a , and the average number of new pages created by an editor each year be z . Derive an equation expressing p in terms of the time in years since Wikibooks was created t .
5. Wikibooks was created in mid-2003. How many pages should there have been 6 years later? (Take $a = 20$, $z = 10 \text{ yr}^{-1}$.)
6. The actual number of pages in Wikibooks in mid-2009 was 35,148. What are the problems with this model? What problems may develop, say, by 2103?

/Worked Solutions/

Capacitors

If you place two conducting plates near each other, with an insulator (known as a dielectric) in between, and you charge one plate positively and the other negatively, there will be an uniform electric field between them. Since:

$$E = \frac{V}{d},$$

as the distance between the two plates decreases, the voltage between them increases. This system is known as a capacitor - it has a capacitance for storing charge. The capacitance C of a capacitor is:

$$C = \frac{Q}{V},$$

where Q is the charge stored by the capacitor, and V is the potential difference between the plates. C is therefore the amount of charge stored on the capacitor per. unit potential difference. Capacitance is measured in farads (F). Just as 1 coulomb is a massive amount of charge, a 1F capacitor stores a lot of charge per. volt.

Any capacitor, unless it is physically altered, has a constant capacitance. If it is left uncharged, $Q = 0$, and so the potential difference across it is 0. If a DC power source is connected to the capacitor, we create a voltage across the capacitor, causing electrons to move around the circuit. This creates a charge on the capacitor equal to CV . If we then disconnect the power source, the charge remains there since it has nowhere to go. The potential difference



A 10 µF capacitor

across the capacitor causes the charge to 'want' to cross the dielectric, creating a spark. However, until the voltage between the plates reaches a certain level (the breakdown voltage of the capacitor), it cannot do this. So, the charge is stored.

If charge is stored, it can also be released by reconnecting the circuit. If we were to connect a wire of negligible resistance to both ends of the capacitor, all the charge would flow back to where it came from, and so the charge on the capacitor would again, almost instantaneously, be 0. If, however, we put a resistor (or another component with a resistance) in series with the capacitor, the flow of charge (current) is slowed, and so the charge on the capacitor does not become 0 instantly. Instead, we can use the charge to power a component, such as a camera flash.

Exponential Decay

Current is the rate of flow of charge. However, current is given by the formula:

$$I = \frac{V}{R}$$

But, in a capacitor, the voltage depends on the amount of charge left in the capacitor, and so the current is a function of the charge left on the capacitor. The rate of change of charge depends on the value of the charge itself. And so, we should expect to find an exponential relationship:

$$Q = Q_0 e^{-\frac{t}{RC}},$$

where R is the resistance of the resistor in series with the capacitor, Q is the charge on the capacitor at a time t and Q_0 was the charge on the capacitor at $t = 0$. Since $Q = I\Delta t$:

$$I\Delta t = I_0\Delta t e^{-\frac{t}{RC}}$$

$$I = I_0 e^{-\frac{t}{RC}},$$

where I is the current flowing at a time t and I_0 was the initial current flowing at $t = 0$. Since $V = IR$:

$$V = V_0 e^{-\frac{t}{RC}}$$

The power being dissipated across the resistors in the circuit is IV , so:

$$P = I_0 V_0 e^{-\frac{t}{RC}} e^{-\frac{t}{RC}} = P_0 e^{-\frac{2t}{RC}}$$

Energy

The energy stored by a capacitor E is defined as:

$$E = \int Q dV$$

In other words, it is the area under a graph of charge against potential difference. Charge is proportional to potential difference ($Q = CV$), so the area under the graph is that of a triangle with base V and height C . So:

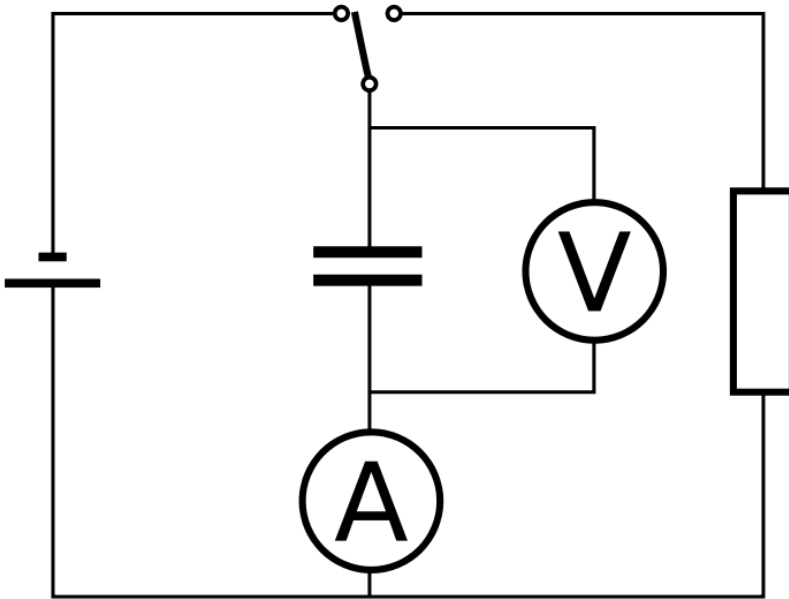
$$E = \frac{1}{2} QV$$

Since $Q = CV$:

$$E = \frac{1}{2} CV^2$$

Circuits

The circuit symbol for a capacitor is $\text{--}||\text{--}$. A simple circuit with a capacitor in series with a resistor, an ideal ammeter (no resistance), and in parallel with an ideal voltmeter (infinite resistance) looks like the following:



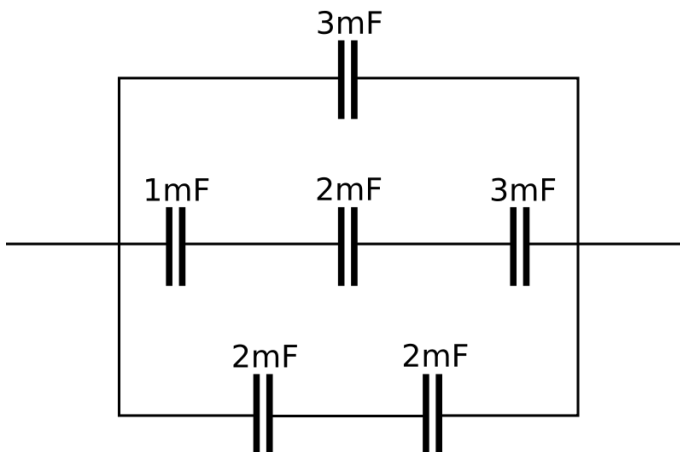
In the position shown, the capacitor is charging. If the switch were put in the other position, the capacitor would be discharging exponentially through the resistor.

If capacitors are placed in parallel, they act as one capacitor with a capacitance equal to the total of all the capacitances of all the individual capacitors. If capacitors are placed in series, the distances between the plates in each of them result in the capacitance of the imaginary resultant capacitor ΣC being given by:

$$\frac{1}{\Sigma C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

Questions

1. A 2 mF capacitor is connected to a 10V DC power supply. How much charge can be stored by the capacitor?
2. What is the highest possible energy stored by this capacitor?
3. The capacitor is placed in series with a 5Ω resistor and charged to capacity. How long would it take for the charge in the capacitor to be reduced to 1 mC?
4. After this time has elapsed, how much energy is stored in the capacitor?
5. What is the capacitance of the equivalent capacitor to the following network of capacitors?



/Worked Solutions/

Radioactive Decay

Decay Constant

We can model radioactive decay by assuming that the probability that any one nucleus out of N nuclei decays in any one second is a constant λ . λ is known as the decay constant, and is measured in s^{-1} (technically the same as Hz, but it is a probability, not a frequency, so we use s^{-1}).

Activity

As our N nuclei decay, the number of nuclei decreases. The activity of the N nuclei we have left is, on average, the probability that any one nucleus will decay per. unit time multiplied by the number of nuclei. If we have 200 nuclei, and the decay constant is 0.5, we would expect, on average, 100 nuclei to decay in one second. This rate would decrease as time goes by. This gives us the following formula for the activity A of a radioactive sample:

$$A = -\frac{dN}{dt} = \lambda N$$

Activity is always positive, and is measured in becquerels (Bq). It is easy to see that the rate of change of the number of nuclei is $-A = -\lambda N$.

Decay

The solution of the differential equation for activity given above is an exponential relationship:

$$N = N_0 e^{-\lambda t},$$

where N is the number of nuclei present at a time t , and N_0 is the number of nuclei present at time $t=0$. You can define $t=0$ to be any point in time you like, provided you are consistent. Since $A = \lambda N$ and therefore $A_0 = \lambda N_0$:

$$A = A_0 e^{-\lambda t},$$

where A is the activity of the sample at a time t , and A_0 is the activity at time $t=0$.

Questions

$$1 \text{ mole} = 6.02 \times 10^{23} \text{ atoms}$$

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

1. Americium-241 has a decay constant of $5.07 \times 10^{-11} \text{ s}^{-1}$. What is the activity of 1 mole of americium-241?
2. How many g of lead-212 ($\lambda = 18.2 \mu\text{s}^{-1}$) are required to create an activity of $0.8 \times 10^{18} \text{ Bq}$?
3. How long does it take for 2kg of lead-212 to decay to 1.5kg of lead-212?
4. Where does the missing 0.5kg go?
5. Some americium-241 has an activity of 3kBq. What is its activity after 10 years?
6. This model of radioactive decay is similar to taking some dice, rolling them once per. second, and removing the dice which roll a one or a two. What is the decay constant of the dice?
7. If you started out with 10 dice, how many dice would you have left after 10s? What is the problem with this model of radioactive decay?

/Worked Solutions/

Half-lives

The half life of something that is decaying exponentially is the time taken for the value of a decaying variable to halve.

Half Life of a Radioisotope

The most common use of half-lives is in radioactive decay. The activity is given by the equation:

$$A_t = A_0 e^{-\lambda t}$$

At $t=t_{1/2}$, $A_t = \frac{1}{2}A_0$, so:

$$\frac{A_0}{2} = A_0 e^{-\lambda t_{1/2}} = \frac{A_0}{e^{\lambda t_{1/2}}}$$

$$2 = e^{\lambda t_{1/2}}$$

$$\ln 2 = \lambda t_{1/2}$$

Therefore:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

It is important to note that the half-life is completely unrelated to the variable which is decaying. At the end of the half-life, all decaying variables will have halved. This also means that you can start at any point in the decay, with any value of any decaying variable, and the time taken for the value of that variable to halve from that time will be the half-life.

Half-Life of a Capacitor

You can also use this formula for other forms of decay simply by replacing the decay constant λ with the constant that was in front of the t in the exponential relationship. So, for the charge on a capacitor, given by the relationship:

$$Q_t = Q_0 e^{\frac{-t}{RC}}$$

So, substitute:

$$\lambda = \frac{1}{RC}$$

Therefore, the half-life of a capacitor is given by:

$$t_{\frac{1}{2}} = RC \ln 2$$

Time Constant of a Capacitor

However, when dealing with capacitors, it is more common to use the time constant, commonly denoted τ , where:

$$\tau = RC = \frac{t_{\frac{1}{2}}}{\ln 2}$$

At $t = \tau$:

$$Q_t = Q_0 e^{\frac{-RC}{RC}} = \frac{Q_0}{e}$$

So, the time constant of a capacitor can be defined as the time taken for the charge, current or voltage from the capacitor to decay to the reciprocal of e (36.8%) of the original charge, current or voltage.

Questions

1. Radon-222 has a decay constant of $2.1\mu\text{s}^{-1}$. What is its half-life?
2. Uranium-238 has a half-life of 4.5 billion years. How long will it take for a 5 gram sample of U-238 to decay to contain 1.25 grams of U-238?
3. How long will it be until it contains 0.5 grams of U-238?
4. Tritium, a radioisotope of Hydrogen, decays into Helium-3. After 1 year, 94.5% is left. What is the half-life of tritium (H-3)?
5. A large capacitor has capacitance 0.5F. It is placed in series with a 5Ω resistor and contains 5C of charge. What is its time constant?
6. How long will it take for the charge in the capacitor to reach $0.677C$? ($0.677 = \frac{5}{e^2}$)

/Worked Solutions/

Gravity

Force

Gravity is a force. Any object with mass exerts a gravitational force on any other object with mass. The force exerted by an object is proportional to its mass. However, this force is also inversely proportional to the distance between the objects squared. This means that, if two objects are twice as far away, the forces they exert on each other are four times smaller.

The gravitational force exerted by a sphere of mass M on another sphere of mass m is given by the following formula:

$$F_{grav} = \frac{-GMm}{r^2},$$

where r is the distance between the spheres, and G is a constant. Experiments have shown that $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$. The minus sign indicates that the force acts in the opposite direction to the radius. Radius is measured outwards from the centre, whereas gravity is an attractive force.

Gravitational Force Inside an Object

Inside a roughly spherical object (such as the Earth), it can be proved geometrically that the effects of the gravitational force resulting from all the mass outside a radius at which an object is located can be ignored, since it all cancels itself out. So, the only mass we need to consider is the mass inside the radius at which the object is located. The density of an object ρ is given by the following equation:

$$\rho = \frac{M}{V},$$

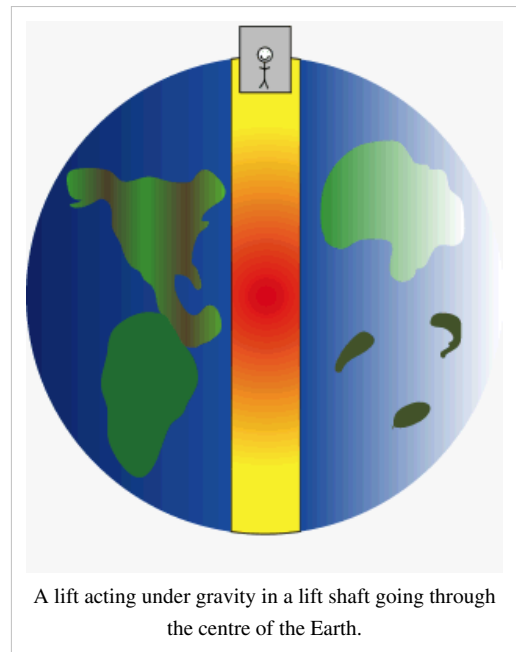
where M is mass, and V is volume. Therefore:

$$M = \rho V$$

If we substitute the volume of a sphere for V :

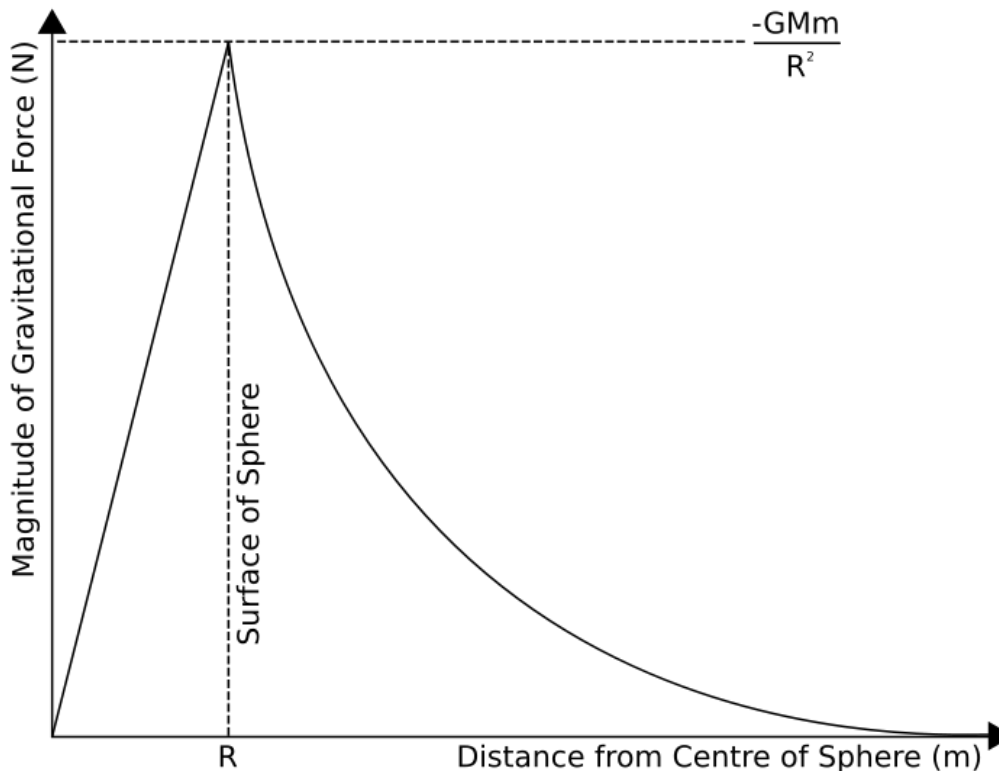
$$M = \frac{4}{3}\pi\rho r^3$$

And if we substitute this mass into the formula for gravitational force given above:



$$F_{grav} = \frac{-Gm\frac{4}{3}\pi\rho r^3}{r^2} = -\frac{4}{3}\pi G\rho mr$$

In other words, inside a sphere of uniform mass, the gravitational force is directly proportional to the distance of an object from the centre of the sphere. Incidentally, this results in a simple harmonic oscillator such as the one on the right. This means that a graph of gravitational force against distance from the centre of a sphere looks like this:



Questions

1. Jupiter orbits the Sun at a radius of around 7.8×10^{11} m. The mass of Jupiter is 1.9×10^{27} kg, and the mass of the Sun is 2.0×10^{30} kg. What is the gravitational force acting on Jupiter? What is the gravitational force acting on the Sun?
2. The force exerted by the Sun on an object at a certain distance is 10^6 N. The object travels half the distance to the Sun. What is the force exerted by the Sun on the object now?
3. How much gravitational force do two 1 kg weights 5 cm apart exert on each other?
4. The radius of the Earth is 6360 km, and its mass is 5.97×10^{24} kg. What is the difference between the gravitational force on 1 kg at the top of your body, and on 1 kg at your head, and 1 kg at your feet? (Assume that you are 2 m tall.)

/Worked Solutions/

Field

The gravitational field, or gravitational field strength is the force exerted by gravity on an object per. unit mass of the object:

$$g = \frac{F_{grav}}{m}$$

The unit of gravitational field strength is Nkg^{-1} . If we consider a planet, the gravitational field strength outside the planet is given by:

$$g = \frac{-GMm}{r^2 m} = \frac{-GM}{r^2}$$

Inside the planet, force is proportional to the distance from the centre, so the field is also proportional to distance.

Acceleration

Since $F = ma$, the gravitational field strength is also the acceleration due to gravity on any object. This acceleration is the same for any object, regardless of mass. Over small distances, such as those in our day-to-day experiences, g remains roughly constant.

Field Lines

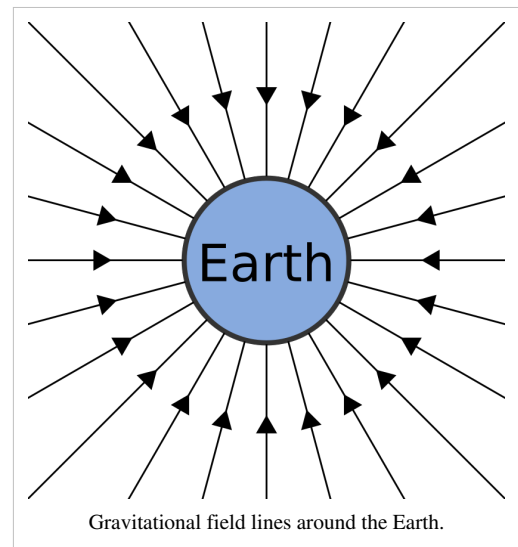
The gravitational field can be represented using field lines. These run in the direction that a mass would be accelerated in initially. The object will not necessarily fall along the field lines, but the acceleration will always be in the direction of the field lines. The closer the field lines are together, the denser the gravitational field.

Questions

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

1. A 15kg object has a weight of 8000N. What is the gravitational field strength at this point?
2. Draw a graph of gravitational field strength against distance.
3. What is the gravitational field strength of the Sun (mass $2 \times 10^{30} \text{ kg}$) on the Earth (mass $6 \times 10^{24} \text{ kg}$, mean orbital radius $15 \times 10^{10} \text{ m}$)?
4. What is the difference in the acceleration due to gravity over a vertical distance d ?
5. How far would one have to travel upwards from the Earth's surface to notice a 1 Nkg^{-1} difference in gravitational field? (The Earth has a radius of 6400 km.)

/Worked Solutions/



Potential Energy

If you throw a ball into the air, you give it kinetic energy. The ball then slows down because of the effect of the Earth's gravitational field on it. However, we know that energy cannot be created or destroyed. The kinetic energy you gave the ball is transformed into gravitational potential energy. The further away from the Earth you manage to throw the ball, the greater the potential there is for kinetic energy to be created. However, for kinetic energy to be created, there must be an acceleration. If there is an acceleration, there must be a force.

You should already know that energy is the same as the work I have to do to move something a distance Δx :

$$\Delta E_{grav} = F \Delta x$$

When I am doing work against gravity, the force I have to exert on an object must equal the force gravity is exerting on that object - this is the object's weight, mg . So, if I move an object against gravity a distance Δx , the work I have to do is given by:

$$\Delta E_{grav} = mg \Delta x$$

It is usual to call this x the height, so you will often see $E_{grav} = mgh$. The deltas are important. They mean that it doesn't matter which distance x I move the object across - I can decide the point at which gravitational potential energy is 0 in a way which makes the maths easy.

The difficulty with this simple formula is that g does not remain the same over large distances:

$$g = \frac{-GM}{r^2}$$

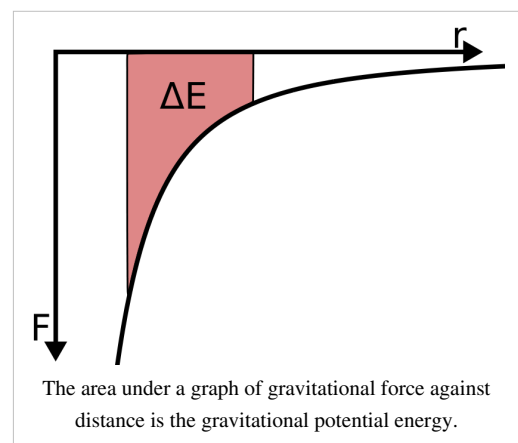
So, over a distance Δr , x becomes r and so:

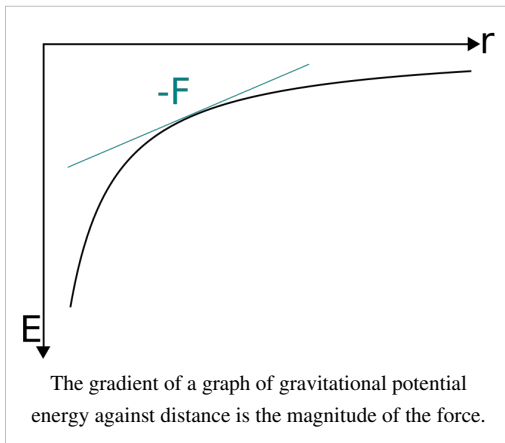
$$E_{grav} = \frac{-GMmr}{r^2} = \frac{-GMm}{r}$$

So, if you're dealing with gravitational potential energy over large distances, use this formula. If you're dealing with gravitational potential energy over short distances, such as with ramps on the Earth's surface, where $g=9.81\text{ms}^{-2}$, use $E_{grav} = mgh$.

Graphs

We have just done something sneaky. You probably didn't notice. Let's see what happens when we integrate the gravitational force F with respect to r between r and ∞ :





$$\int_r^\infty \frac{-GMm}{r^2} dr = \left[\frac{GMm}{r} \right]_r^\infty = \frac{GMm}{\infty} - \frac{GMm}{r}$$

Since dividing anything by infinity gets you practically 0:

$$\int_r^\infty F dr = -\frac{GMm}{r} = E_{grav}$$

And therefore:

$$\frac{dE_{grav}}{dr} = F$$

So, if you have a graph of gravitational potential energy against radius, the gradient of the graph is the gravitational force. If you have a graph of gravitational force against radius, the area under the graph between any point and the F-axis is the gravitational potential energy at this point. The area under the graph between any two points is the difference in gravitational potential energy between them.

Questions

1. A ball rolls down a 3m-high smooth ramp. What speed does it have at the bottom?
2. In an otherwise empty universe, two planets of mass 10^{25} kg are 10^{12} m apart. What are their speeds when they collide?
3. What is the least work a 2000kg car must do to drive up a 100m hill?
4. How does the speed of a planet in an elliptical orbit change as it nears its star?

/Worked Solutions/

Potential

Gravitational potential is the gravitational potential energy given to objects per. unit mass:

$$V_{grav} = \frac{E_{grav}}{m} = \frac{-\frac{GMm}{r}}{m} = \frac{-GM}{r}$$

Over short distances, gravitational potential energy is given by:

$$E_{grav} \approx mg\Delta h$$

So, over short distances, gravitational potential is equal to $g\Delta h$. Gravitational potential is measured in Jkg^{-1} .

Equipotentials

On a field diagram, lines can be drawn which, like contours on a map, show all the points which have the same gravitational potential. These lines are known as equipotentials. Equipotentials are always perpendicular to field lines, and get closer together as the field strength increases, and the density of field lines increases. Over short distances, equipotentials are evenly spaced.

Summary of Gravity

You should now know (if you did the gravity section in the right order) about four attributes of gravitational fields: force, field strength, potential energy and potential. These can be summarised by the following table:

Force $F_{grav} = \frac{-GMm}{r^2}$	\rightarrow integrate \rightarrow with respect to r	Potential Energy $E_{grav} = \frac{-GMm}{r}$
\downarrow per. unit mass \downarrow		
Field Strength $g = \frac{-GM}{r^2}$	\rightarrow integrate \rightarrow with respect to r	Potential $V_{grav} = \frac{-GM}{r}$

Questions

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$g = 9.81 \text{ ms}^{-2}$$

1. What is the gravitational potential at the Earth's surface? (mass of Earth = 5.97×10^{24} kg, radius of Earth = 6371 km)
2. Taking the Earth's surface as $V_{grav} = 0$, what is the gravitational potential 2m above the ground?
3. A 0.2kg firework reaches a gravitational potential relative to the ground of 500 Jkg^{-1} . If the firework is 30% efficient, how much energy was expended to get there?
4. Express gravitational potential in terms of gravitational force.
5. Draw the equipotentials and field lines surrounding the Earth.

/Worked Solutions/

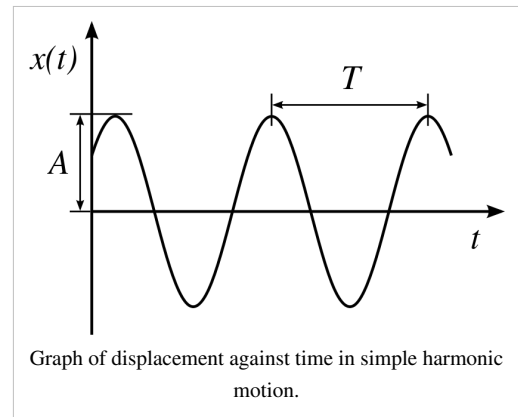
Mechanics

Simple Harmonic Motion

Simple harmonic motion occurs when the force on an object is proportional and in the opposite direction to the displacement of the object. Examples include masses on springs and pendula, which 'bounce' back and forth repeatedly. Mathematically, this can be written:

$$F = -kx,$$

where F is force, x is displacement, and k is a positive constant. This is exactly the same as Hooke's Law, which states that the force F on an object at the end of a spring equals $-kx$, where k is the spring constant. Since $F = ma$, and acceleration is the second derivative of displacement with respect to time t :



$$m \frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = \frac{-kx}{m}$$

The solution of this second order differential equation is:

$$x = A \cos \omega t,$$

where A is the maximum displacement, and ω is the 'angular velocity' of the object. The derivation is given here, since it will seem very scary to those who haven't met complex numbers before. It should be noted that this solution, if given different starting conditions, becomes:

$$x = A \sin \omega t,$$

Angular Velocity

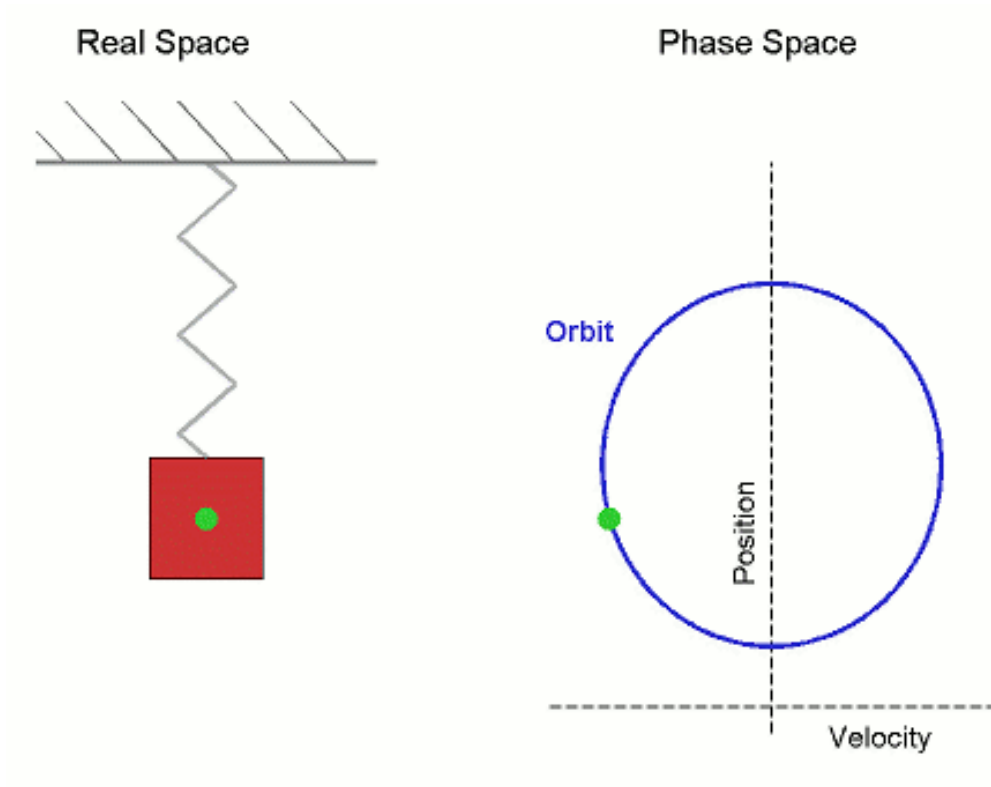
Angular velocity in circular motion is the rate of change of angle. It is measured in radians per. second. Since 2π radians is equivalent to one complete rotation in time period T :

$$\omega = \frac{2\pi}{T} = 2\pi f$$

If we substitute this into the equation for displacement in simple harmonic motion:

$$x = A \cos 2\pi f t$$

The reason the equation includes angular velocity is that simple harmonic motion is very similar to circular motion. If you look at an object going round in a circle side-on, it looks exactly like simple harmonic motion. We have already noted that a mass on a spring undergoes simple harmonic motion. The following diagram shows the similarity between circular motion and simple harmonic motion:



Time Period

The time period of an oscillation is the time taken to repeat the pattern of motion once. In general:

$$T = \frac{2\pi}{\omega}$$

However, depending on the type of oscillation, the value of ω changes. For a mass on a spring:

$$\omega = \sqrt{\frac{k}{m}}$$

For a pendulum:

$$\omega = \sqrt{\frac{g}{l}},$$

where g is the gravitational field strength, and l is the length of the string. By substitution, we may gain the following table:

Type of Oscillation	Spring	Pendulum
Angular Velocity	$\sqrt{\frac{k}{m}}$	$\sqrt{\frac{g}{l}}$
Time Period	$2\pi\sqrt{\frac{m}{k}}$	$2\pi\sqrt{\frac{l}{g}}$

Velocity and Acceleration

The displacement of a simple harmonic oscillator is:

$$x = A \cos \omega t$$

Velocity is the rate of change of displacement, so:

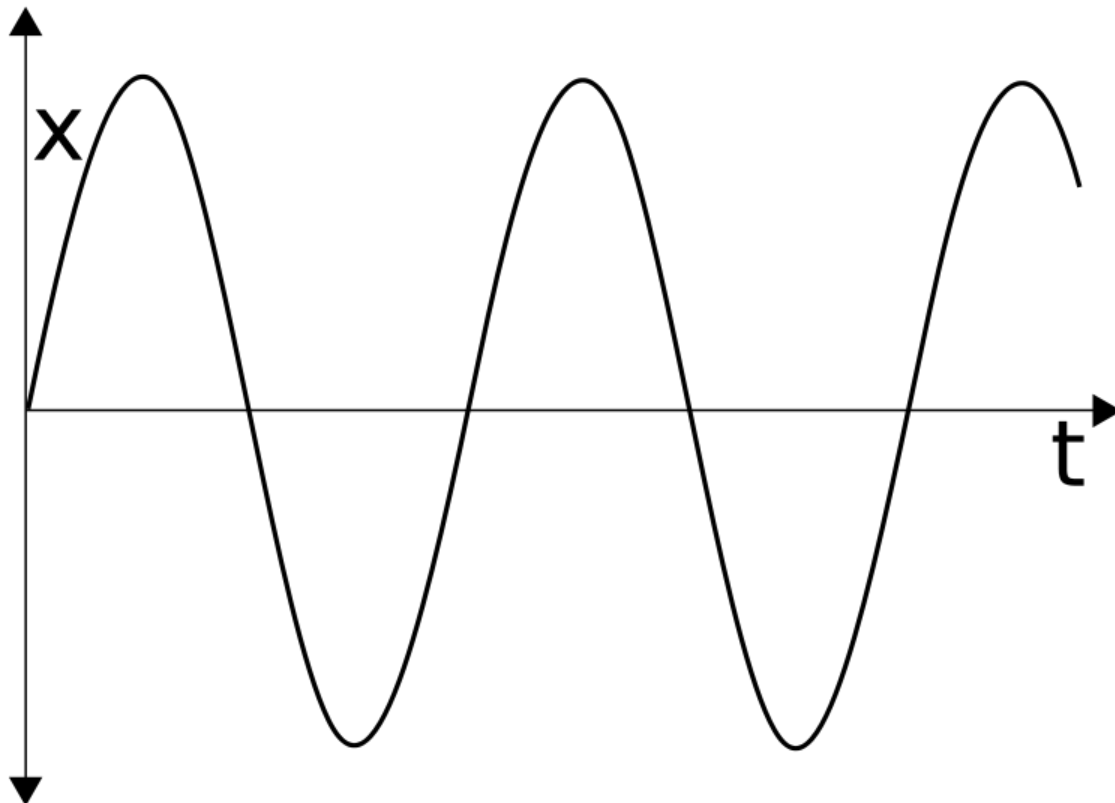
$$v = \frac{dx}{dt} = -A\omega \sin \omega t$$

Acceleration is the rate of change of velocity, so:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2 \cos \omega t = -\omega^2 x$$

Questions

1. A 10N weight extends a spring by 5cm. Another 10N weight is added, and the spring extends another 5cm. What is the spring constant of the spring?
2. The spring is taken into outer space, and is stretched 10cm with the two weights attached. What is the time period of its oscillation?
3. What force is acting on the spring after 1 second? In what direction?
4. A pendulum oscillates with a frequency of 0.5 Hz. What is the length of the pendulum?
5. The following graph shows the displacement of a simple harmonic oscillator. Draw graphs of its velocity, momentum, acceleration and the force acting on it.



6. A pendulum can only be modelled as a simple harmonic oscillator if the angle over which it oscillates is small. Why is this?

/Worked Solutions/

Energy in Simple Harmonic Motion

A mass oscillating on a spring in a gravity-free vacuum has two sorts of energy: kinetic energy and elastic (potential) energy. Kinetic energy is given by:

$$E_k = \frac{1}{2}mv^2$$

Elastic energy, or elastic potential energy, is given by:

$$E_e = \frac{1}{2}kx^2$$

So, the total energy stored by the oscillator is:

$$\Sigma E = \frac{1}{2}(mv^2 + kx^2)$$

This total energy is constant. However, the proportions of this energy which are kinetic and elastic change over time, since v and x change with time. If we give a spring a displacement, it has no kinetic energy, and a certain amount of elastic energy. If we let it go, that elastic energy is all converted into kinetic energy, and so, when the mass reaches its initial position, it has no elastic energy, and all the elastic energy it did have has been converted into kinetic energy. As the mass continues to travel, it is slowed by the spring, and so the kinetic energy is converted back into elastic energy - the same amount of elastic energy as it started out. The nature of the energy oscillates back and forth, but the total energy is constant.

If the mass is oscillating vertically in a gravitational field, the situation gets more complicated since the spring now has gravitational potential energy, elastic potential energy and kinetic energy. However, it turns out (if you do the maths) that the total energy is still constant, although the equilibrium position will be lower.

Questions

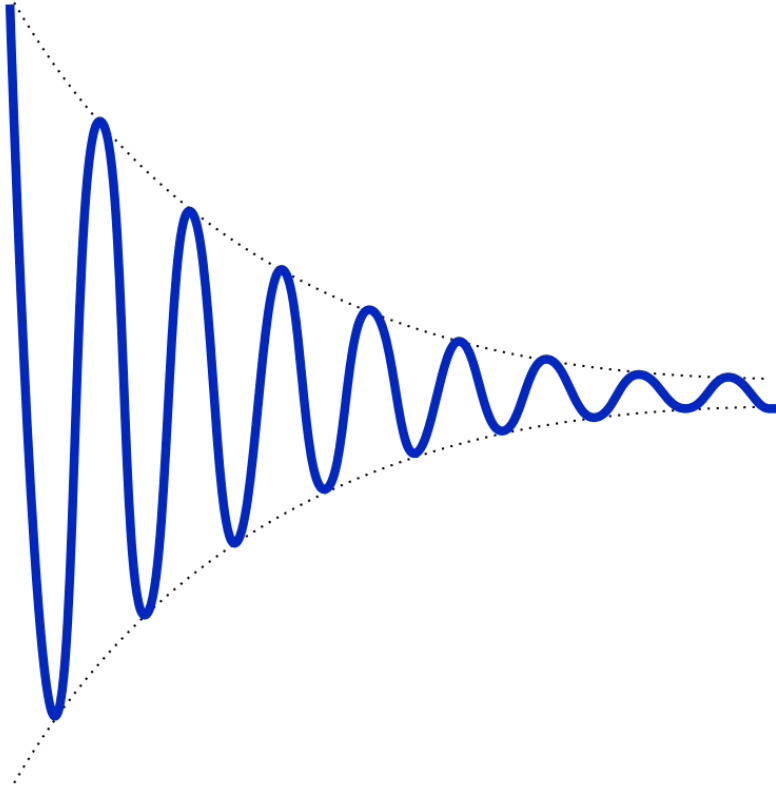
1. A 10g mass causes a spring to extend 5cm. How much energy is stored by the spring?
2. A 500g mass on a spring ($k=100$) is extended by 0.2m, and begins to oscillate in an otherwise empty universe. What is the maximum velocity which it reaches?
3. Another 500g mass on another spring in another otherwise empty universe is extended by 0.5m, and begins to oscillate. If it reaches a maximum velocity of 15ms^{-1} , what is the spring constant of the spring?
4. Draw graphs of the kinetic and elastic energies of a mass on a spring (ignoring gravity).
5. Use the trigonometric formulae for x and v to derive an equation for the total energy stored by an oscillating mass on a spring, ignoring gravity and air resistance, which is constant with respect to time.

/Worked Solutions/

Damping

Previously, our mathematical models of simple harmonic motion have assumed that the energy stored by a simple harmonic oscillator is constant. In reality, of course, resistive forces slow an oscillator down, transferring its energy to its surroundings. A pendulum will lose energy by moving the air. In addition to this, the motion of a mass on a spring will cause the spring to heat up, 'losing' the energy. This process is known as damping.

The principle effect of damping is to reduce the amplitude of an oscillation, not to change its frequency. So, the graph of the amplitude of a normal damped oscillation might look like the following:



Critical Damping

Critical damping occurs when a system is designed to return an oscillator to its equilibrium position in the least time possible. A critically damped oscillator, when damped, ceases to oscillate, and returns to its equilibrium position, where it stops moving. An example is the door closer. Normally, the door would swing back and forth, being damped by friction in the hinges, and air resistance. The door closer forces the door to stop swinging, and shut immediately. When closed, the door is in its equilibrium position.

Questions

1. Draw a graph of displacement for a critically damped oscillation.
2. How would you critically damp an oscillating pendulum?
3. How would you damp an oscillating pendulum using only a weighted polystyrene block?
4. What would the displacement graph look like for this oscillation, before and after damping began?
5. The graph above is an exponentially damped oscillation. If the displacement of the undamped oscillation is given by $\sin \omega t$, what is an approximate equation for the damped oscillation, in terms of a constant k which describes the degree to which the oscillation is damped?

/Worked Solutions/

Resonance

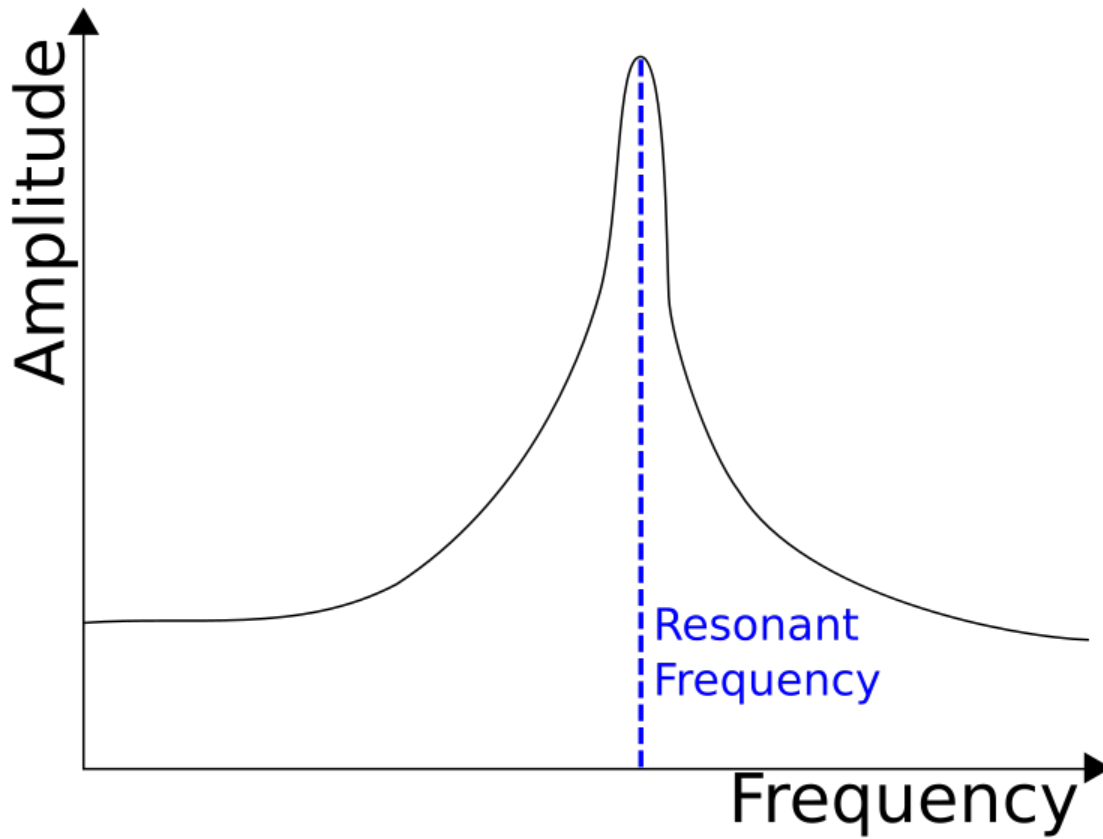
All oscillators have a natural frequency. If you have a mass on a spring, and give it an amplitude, it will resonate at a frequency:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

This frequency is independent of the amplitude you give the oscillator to start with. It is the natural frequency of the oscillator. If you keep giving the oscillator amplitude at this frequency, it will not change the frequency of the oscillation. But, you are still doing work. This energy must go somewhere. The only place it can go is into additional kinetic and gravitational potential energy in the oscillation. If you force an oscillation at its resonant frequency, you add significantly to its amplitude.

Put simply, resonance occurs when the driving frequency of an oscillation matches the natural frequency, giving rise to large amplitudes.

If you were to force an oscillation at a range of frequencies, and measure the amplitude at each, the graph would look something like the following:



There are many types of oscillators, and so practically everything has a resonant frequency. This can be used, or can result in damage if the resonant frequency is not known.

Reading

Instead of doing questions this time, read the following articles on Wikipedia about these different types:

Tacoma Narrows Bridge

Resonance in Water Molecules (Microwave Ovens)

"No Highway" - a novel with a plot that uses things suspiciously similar to resonance.

Conservation of Momentum

Momentum is the product of the mass of an object and its velocity. It is usually denoted p :

$$p = mv,$$

where m is mass, and v is velocity. The total momentum in a closed system is always conserved. This fact is useful, since it allows us to calculate velocities and masses in collisions.

Collisions

Let us consider a basic example: a ball of mass M collides with velocity u with a stationary ball of mass m . The stationary ball has no momentum before the collision, and the moving ball has momentum Mu . This must equal the momentum of both balls after the collision. If we let their velocities be v_1 and v_2 :

$$Mu = Mv_1 + mv_2$$

At this point, we would need to know one of the velocities afterwards in order to calculate the other.

Alternatively, we could have one ball of mass M colliding with another ball of mass m , with both balls moving in opposite directions with velocities u_1 and u_2 respectively. If we define the direction of motion of the ball with mass M as the positive direction:

$$Mu_1 - mu_2 = Mv_1 + mv_2$$

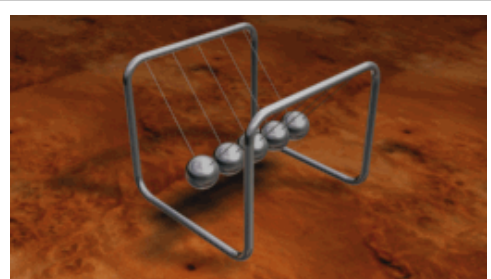
We do not need to worry about the signs on the right-hand side: they will take care of themselves. If one of our velocities turns out to be negative, we know that it is in the opposite direction to u_1 .

Elasticity

Although momentum within a closed system is always conserved, kinetic energy does not have to be. If kinetic energy is conserved in a collision, then it is known as a perfectly (or totally) elastic collision. If it is not conserved, then the collision is inelastic. If the colliding particles stick together, then a totally inelastic collision has occurred. This does not necessarily mean that the particles have stopped. In a totally inelastic collision, the two particles become one, giving the equation:

$$Mu_1 + mu_2 = (M + m)v$$

$$v = \frac{Mu_1 + mu_2}{M + m}$$



The collisions in a Newton's cradle are almost perfectly elastic.

Explosions

In an explosion, two particles which are stuck together are no longer stuck together, and so gain separate velocities:

$$(M + m)u = Mv_1 + mv_2$$

Questions

1. A ball of mass 0.5kg collides with a stationary ball of 0.6kg at a velocity of 3ms^{-1} . If the stationary ball moves off at a speed of 2ms^{-1} , what is the new velocity of the first ball?
2. Two balls are moving in opposite directions with velocities 5ms^{-1} and 10ms^{-1} . They collide, and move off in opposite directions with new velocities of 7.5ms^{-1} each. If the mass of the first ball was 1.25kg, what is the mass of the second ball?
3. A totally elastic collision occurs between two balls of equal mass. One of the balls is stationary. What happens?
4. A particle explodes to become two particles with masses 1kg and 2kg. The 1kg particle moves with velocity 45ms^{-1} . With what velocity does the other particle move?
5. A 3kg ball moving at 3ms^{-1} collides with a 5kg ball moving at -5ms^{-1} . The collision is perfectly elastic. What are the new velocities of the balls?
6. A ball collides with a wall, and rebounds at the same velocity. Why doesn't the wall move?

/Worked Solutions/

Forces and Impulse in Collisions

You should already know that the force exerted on an object is proportional to its acceleration. The constant of proportionality is known as the mass of the object:

$$F = ma$$

In the case of a collision, for one of the particles in the collision, the acceleration is simply the difference between its velocity before the collision (u) and its velocity after the collision (v) per unit. time:

$$F = \frac{m(v - u)}{\Delta t} = \frac{mv - mu}{\Delta t}$$

So, force is the rate of change of momentum. The quantity on top is known as the impulse of the collision, measured in Ns; Δt is the length of time it took for the collision to take place. So, the impulse I is given by:

$$I = \Delta p = mv - mu = F\Delta t$$

In a collision where a certain change in momentum (impulse) occurs, a force is exerted. If the collision time is small, a larger force is exerted. If the collision time is long, a smaller force is exerted. If you have a graph of force against time, impulse is the area under the graph, since:

$$I = \int F dt$$

The impulse on one particle in a simple collision is the negative impulse on the other particle.

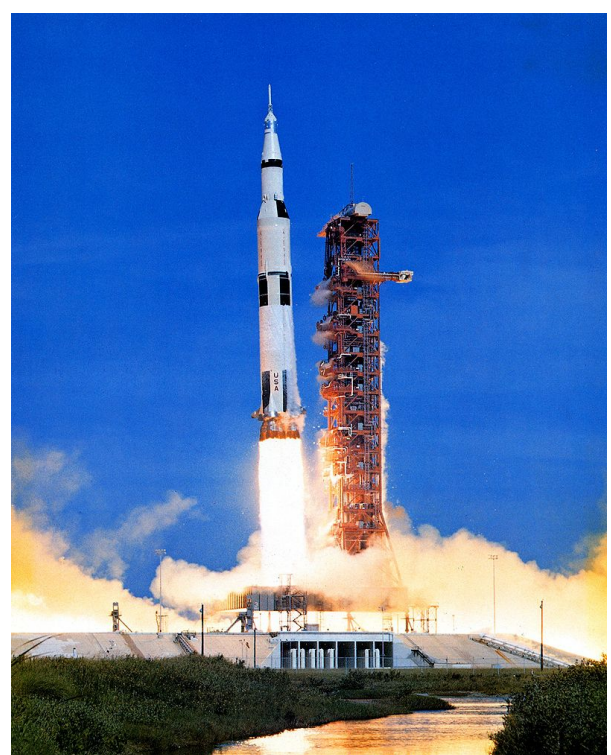
Questions

1. Escape velocity from the Earth is 11.2kms^{-1} . How much impulse must be exerted on a 47000kg payload to get it to travel away from the Earth?
2. Two billiard balls, of mass 10g , collide. One is moving at 5ms^{-1} , and the other at 2ms^{-1} . After the collision, the first billiard ball is moving backwards at 4ms^{-1} . The collision takes 1ms . What force was exerted on this ball?
3. What impulse and force were exerted on the second ball?
4. A 60kg spacewalker uses a jet of gas to exert an impulse of 10Ns . How many times would he have to do this to reach a speed of 1ms^{-1} from stationary?
5. A 5kg bowling ball collides with a stationary tennis ball of mass 0.1kg at 3ms^{-1} , slowing to 2.5ms^{-1} . It exerts a force of 100N on the ball. How long did the collision take?

/Worked Solutions/

Rockets, Hoses and Machine Guns

We have already seen that force is the rate of change of momentum. This applies to continuous flows of momentum as well as to collisions:



Apollo 15 launches itself to the moon by means of the change in momentum of its fuel.

$$F = \frac{dp}{dt} = v \frac{dm}{dt}$$

If I have a machine gun, explosions give the bullets of mass m momentum, causing them to move at a velocity v . This occurs several times each second - the momentum of the bullets is changing, and so there is a roughly continuous force acting on them. Momentum, of course, must be conserved. This results in a change in the momentum of the gun each time it fires a bullet. Overall, this results in a roughly continuous force on the gun which is equal and opposite to the force acting on the bullets.

If I have a tank of water and a hose, with a pump, and I pump the water out of the tank, a similar thing occurs - a force pushes me away from the direction of flow of the water. This force is equal to the flow rate (in kg s^{-1}) of the water multiplied by its velocity. Bear in mind that 1 litre of water has a mass of about 1kg.

Rockets work on this principle - they pump out fuel, causing it to gain momentum. This results in a thrust on the rocket. When designing propulsion systems for rockets, the aim is to give the fuel as high a velocity per. unit mass as possible in order to make the system fuel-efficient, and to get a high enough change in momentum.

Questions

1. A machine gun fires 300 5g bullets per. minute at 800ms^{-1} . What force is exerted on the gun?
2. 1 litre of water is pumped out of a tank in 5 seconds through a hose. If a 2N force is exerted on the tank, at what speed does the water leave the hose?
3. If the hose were connected to the mains, what problems would there be with the above formula?
4. The thrust of the first stage of a Saturn V rocket is 34 MN, using 131000kg of solid fuel in 168 seconds. At what velocity does the fuel leave the tank?
5. Escape velocity from the Earth is 11kms^{-1} . What is the velocity of the rocket after the first stage is used up, if the total mass of the rocket is $3 \times 10^6 \text{ kg}$? How does this compare to escape velocity?

/Worked Solutions/

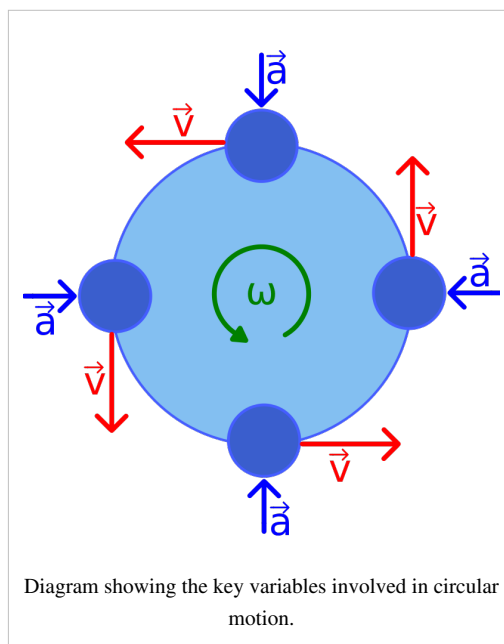
Circular Motion

Very rarely, things move in circles. Some planets move in roughly circular orbits. A conker on a string might move around my head in a circle. A car turning a corner might, briefly, move along the arc of a circle. The key thing to note about circular motion is that there is no force pulling outwards from the circle, and there is no force pulling the moving object tangential to the circle. Centrifugal force does not exist. There is only one force acting in circular motion, which is known as centripetal force. It always acts towards the centre of the circle. The object does not follow a circular path because two forces are balanced. Instead, the centripetal force accelerates the object with a constant magnitude in an ever-changing direction. The object has a velocity, and will continue moving with this velocity unless acted on by the centripetal force, which is perpetually adding velocity towards the centre of the circle.

If you were to subject a stationary object to the centripetal force, it would simply fall. If you gave it a little bit of velocity, it would still fall, but it would not land directly beneath its starting position. If you kept increasing the velocity and dropping it, there would come a point when it would land infinitely far away - it would go into orbit. The relationship between this 'magic' velocity and the magnitude of the centripetal force is as follows:

$$F = \frac{mv^2}{r},$$

where m is the mass of the object in circular motion, v is the magnitude of its velocity, and r is the distance from the centre of the circle to the object. Since $F=ma$, the centripetal acceleration is:



$$a = \frac{v^2}{r}$$

The centripetal force may manifest itself as many things: the tension in a string, friction, gravity or even an electric or magnetic field. In all these cases we can equate the equation for centripetal force with the equation for the force it really is.

Angular Velocity

Velocity is the rate of change of displacement. Angular velocity is the rate of change of angle, commonly denoted ω and measured in radians per. second:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

In circular motion:

$$\omega = \frac{2\pi}{T} = 2\pi f,$$

where T is the time for one revolution and f is the frequency of rotation. However:

$$v = \frac{2\pi r}{T}$$

$$\frac{v}{r} = \frac{2\pi}{T}$$

Therefore, the relationship between velocity and angular velocity is:

$$\omega = \frac{v}{r}$$

If we substitute this into the formula for centripetal acceleration:

$$a = \frac{(\omega r)^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$$

Questions

1. A tennis ball of mass 10g is attached to the end of a 0.75m string and is swung in a circle around someone's head at a frequency of 1.5Hz. What is the tension in the string?
2. A planet orbits a star in a circle. Its year is 100 days, and the distance from the star to the planet is 70 Gm from the star. What is the mass of the star?
3. A 2000kg car turns a corner, which is the arc of a circle, at 20kmh^{-1} . The centripetal force due to friction is 1.5 times the weight of the car. What is the radius of the corner?
4. Using the formulae for centripetal acceleration and gravitational field strength, and the definition of angular velocity, derive an equation linking the orbital period of a planet to the radius of its orbit.

/Worked Solutions/

Astrophysics

Radar and Triangulation

Radar and triangulation are two relatively easy methods of measuring the distance to some celestial objects. Radar can also be used to measure the velocity of a celestial object relative to us.

Radar

Essentially, radar is a system which uses a radio pulse to measure the distance to an object. The pulse is transmitted, reflected by the object, and then received at the site of the transmitter. The time taken for all this to happen is measured. This can be used to determine the distance to a planet or even the velocity of a spaceship.

Distance

The speed of electromagnetic waves (c) is constant in a vacuum: $3 \times 10^8 \text{ ms}^{-1}$. If we fire a pulse of radio waves to a planet within the Solar System, we know that:

$$d = ct$$

where d is the distance to the planet, and t is the time taken for the pulse to get there. However, the pulse has to get both there and back, so:

$$2d = ct$$

$$d = \frac{ct}{2}$$

where d is the distance to the planet, and t is the time taken for the pulse to return.

Velocity

The velocity of an object can be found by firing two radar pulses at an object at different times. Two distances are measured. When asked to calculate the relative velocity of an object in this way, use the following method:

1. Calculate the distance to the object at both times:

$$d_1 = \frac{c\Delta t_1}{2}$$

$$d_2 = \frac{c\Delta t_2}{2}$$

2. Calculate the distance the object has travelled between the two pulses. This is the difference between the two distances previously calculated:

$$\Delta d = d_2 - d_1$$

3. Calculate the time between the transmission (or reception, but not both) of the two pulses:

$$\Delta t = t_2 - t_1$$

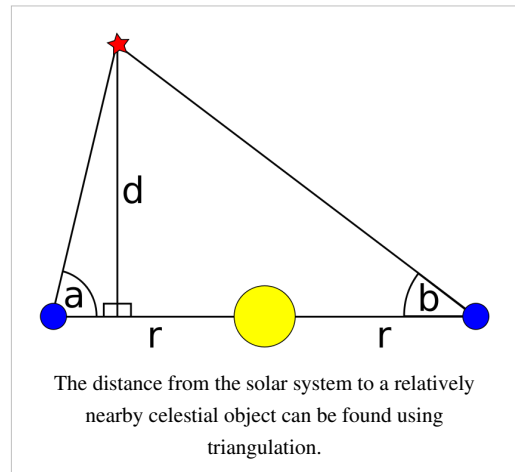
4. Divide the distance calculated in step 2 by the time calculated in step 3 to find the average velocity of the object between the transmission of the two pulses:

$$v = \frac{\Delta d}{\Delta t}$$

Triangulation

We know that the Earth is, on average, about 150 Gm away from the Sun. If we measure the angle between the vertical and the light from a nearby star 6 months apart (ie. on opposite sides of the Sun), we can approximate the distance from the Solar System to the star.

Let r be the radius of the Earth's orbit (assumed to be constant for simplicity's sake), a and b be the angles to the star (from the horizontal) when the Earth is on either side of the Sun, and let d be the perpendicular distance from the plane of the Earth's orbit to the star, as shown in the diagram on the right. By simple trigonometry:



$$2r = \frac{d}{\tan a} + \frac{d}{\tan b} = \frac{d(\tan a + \tan b)}{\tan a \tan b}$$

Therefore:

$$d = \frac{2r \tan a \tan b}{\tan a + \tan b}$$

Questions

1. A radar pulse takes 8 minutes to travel to Venus and back. How far away is Venus at this time?
2. Why can't a radar pulse be used to measure the distance to the Sun?
3. Radar is used to measure the velocity of a spacecraft travelling between the Earth and the Moon. Use the following data to measure this velocity:

Pulse	Transmission Time	Reception Time
1	3:26:45.31213	3:26:45.51213
2	3:26:46.32742	3:26:46.52785

4. The angles between the horizontal and a star are measured at midnight on January 1 as 89.99980° and at midnight on July 1 as 89.99982° . How far away is the star?
5. Why can't triangulation be used to measure the distance to another galaxy?

/Worked Solutions/

Large Units

The distances in space are so large that we need some very large units to describe them with.

Light Years

A light year is the distance that light travels in one year. The velocity of light is constant ($3 \times 10^8 \text{ms}^{-1}$), so 1 light year is:

$$3 \times 10^8 \times 365.24 \times 24 \times 60 \times 60 \approx 9.46 \times 10^{15} \text{ m}$$

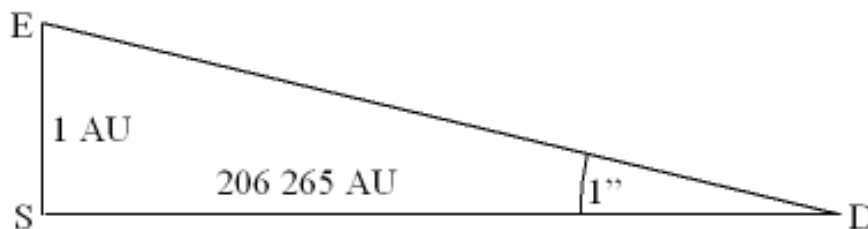
Light seconds, light minutes, light hours and light days are less commonly used, but may be calculated in a similar fashion.

Astronomical Units

1 astronomical unit (denoted AU) is the mean average distance from the Earth to the Sun. This is approximately $150 \times 10^9 \text{m}$.

Parsecs

Degrees can be divided into minutes and seconds. There are 60 minutes in a degree, and 60 seconds in a minute. This means that 1 second is $\frac{1}{3600}$ of a degree. A degree is denoted $^\circ$, a minute $'$ and a second $''$. The definition of a parsec uses a simplified form of triangulation. It assumes that the perpendicular to the plane of the Earth's orbit passes through the Sun and a celestial object. A parsec is the distance from the Sun to this celestial object if the angle between the perpendicular and the the line connecting the Earth to the celestial object. This gives us the following right-angled triangle (the distance from the Earth to the Sun is 1 AU):



$$\tan 1'' = \tan \left(\frac{1}{3600} ^\circ \right) = 4.85 \times 10^{-6} = \frac{1}{\text{parsec}}$$

Therefore, a parsec is 206,265 AU.

Questions

1. What is one parsec in m?
2. Convert 3 light days into km.
3. Convert 5.5 parsecs into light years.
4. The difference in angle of a star on the perpendicular to the plane of the Earth's orbit which passes through the Sun when viewed from either side of the Earth's orbit is 0.1° . How far away is the star in parsecs?

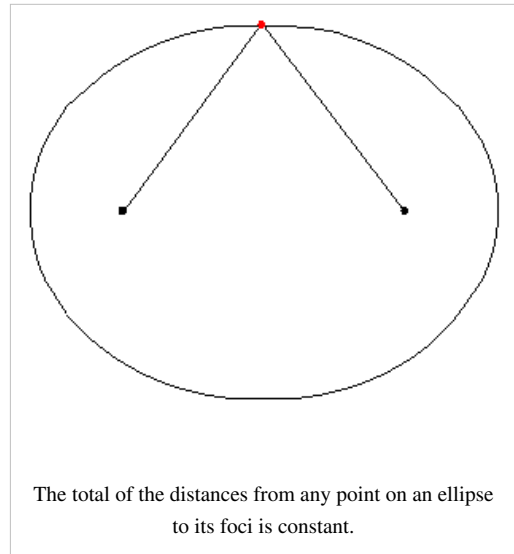
/Worked Solutions/

Orbits

Planets orbit a sun. Theoretically, their orbit may be circular. This case is dealt with under circular motion.

In reality, planets orbit in ellipses. An ellipse is a shape which has two foci (singular 'focus'). The total of the distances from any point on an ellipse to its foci is constant. All orbits take an elliptical shape, with the sun as one of the foci. As the planet approaches its sun, its speed increases. This is because gravitational potential energy is being converted into kinetic energy.

A circle is an ellipse, in the special case when both foci are at the same point.



Kepler's Third Law

Kepler's third law states that:



The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.







Mathematically, for orbital period T and semi-major axis R :

$$T^2 \propto R^3$$

This result was derived for the special case of a circular orbit as the fourth circular motion problem. The semi-major axis is the distance from the centre of the ellipse (the midpoint of the foci) to either of the points on the edge of the ellipse closest to one of the foci.

Questions

Planet	Mercury	Venus	Earth	Mars
Picture				
Mean distance from Sun (km)	57,909,175	108,208,930	149,597,890	227,936,640
Orbital period (years)	0.2408467	0.61519726	1.0000174	1.8808476

1. The semi-major axis of an elliptical orbit can be approximated reasonably accurately by the mean distance of the planet for the Sun. How would you test, using the data in the table above, that the inner planets of the Solar System obey Kepler's Third Law?
2. Perform this test. Does Kepler's Third Law hold?
3. If $T^2 \propto R^3$, express a constant C in terms of T and R .
4. Io, one of Jupiter's moons, has a mean orbital radius of 421600km, and a year of 1.77 Earth days. What is the value of C for Jupiter's moons?
5. Ganymede, another of Jupiter's moons, has a mean orbital radius of 1070400km. How long is its year?

/Worked Solutions/

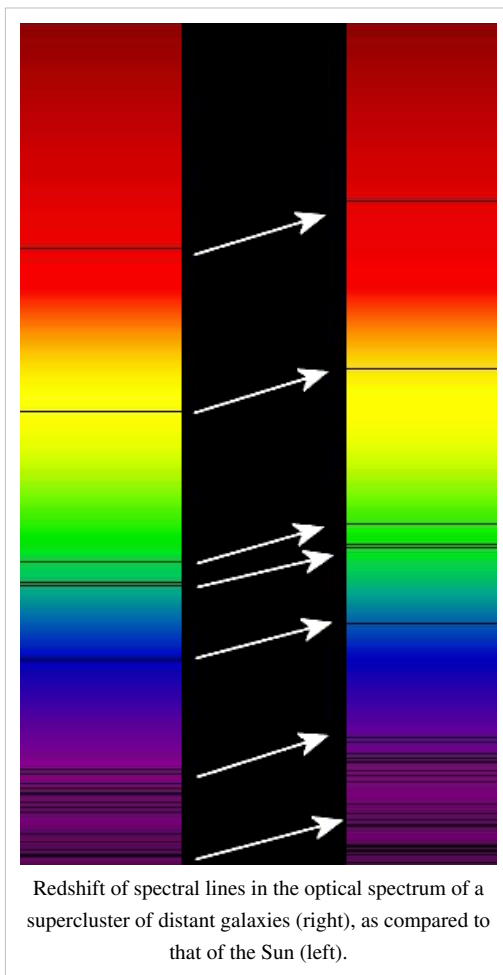
Doppler Effect

The Doppler effect is a change in the frequency of a wave which occurs if one is in a different frame of reference from the emitter of the wave. Relative to us, we observe such a change if an emitter of a wave is moving relative to us.

All waves travel in a medium. So, they have a velocity relative to this medium v . They also have a velocity relative to their source v_s and a velocity relative to the place where they are received v_r . The frequency at which they are received f is related to the frequency of transmission f_0 by the formula:

$$f = \left(\frac{v + v_r}{v + v_s} \right) f_0$$

The Doppler effect can be used to measure the velocity at which a star is moving away from or towards us by comparing the wavelength received λ with the wavelength we would expect a star of that type to emit λ_0 . Since the speed of light c is constant regardless of reference medium:



$$c = f\lambda = f_0\lambda_0$$

Therefore:

$$f = \frac{c}{\lambda} \text{ and } f_0 = \frac{c}{\lambda_0}$$

By substitution:

$$\frac{c}{\lambda} = \left(\frac{v + v_r}{v + v_s} \right) \frac{c}{\lambda_0}$$

$$\frac{1}{\lambda} = \left(\frac{v + v_r}{v + v_s} \right) \frac{1}{\lambda_0}$$

$$\lambda = \frac{\lambda_0(v + v_s)}{v + v_r}$$

In this case, v is the speed of light, so $v = c$. Relative to us, we are stationary, so $v_r = 0$. So:

$$\lambda = \frac{\lambda_0(c + v_s)}{c}$$

$$\frac{\lambda}{\lambda_0} = \frac{(c + v_s)}{c} = 1 + \frac{v_s}{c}$$

If we call the change in wavelength due to Doppler shift $\Delta\lambda$, we know that $\lambda = \lambda_0 + \Delta\lambda$. Therefore:

$$\frac{\lambda_0 + \Delta\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0} = 1 + \frac{v_s}{c}$$

So, the important result you need to know is that:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_s}{c} = z$$

This value is known as the red-shift of a star, denoted z . If z is positive, the star is moving away from us - the wavelength is shifted up towards the 'red' end of the electromagnetic spectrum. If z is negative, the star is moving towards us. This is known as blue shift. Note that we have assumed that v is much smaller than c . Otherwise, special relativity makes a significant difference to the formula.

Questions

1. M31 (the Andromeda galaxy) is approaching us at about 120kms^{-1} . What is its red-shift?
2. Some light from M31 reaches us with a wavelength of 590nm . What is its wavelength, relative to M31?
3. Some light has a wavelength, relative to M31, of 480nm . What is its wavelength, relative to us?
4. A quasar emits electromagnetic radiation at a wavelength of 121.6nm . If, relative to us, this wavelength is red-shifted 0.2nm , what is the velocity of recession of the quasar?

/Worked Solutions/

The Big Bang

Big Bang theory states that space-time began as a single point, and that, as time passed, space itself expanded.

Hubble's Law

Hubble's Law describes the expansion of the universe mathematically:

$$v = H_0 d,$$

where v is the velocity of recession of a celestial object, and d is the distance to the object. H_0 is the Hubble constant, where $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The '0' signifies that this is the Hubble constant now, not in the past or the future. This allows for the fact that the Hubble constant might be changing, but very slowly.

The Age of the Universe

Imagine a galaxy which flies out from the big bang at the speed of light (c). The distance it has travelled d is given by:

$$d = vt,$$

where t is the age of the universe, since the galaxy has been travelling since the beginning. If we substitute in Hubble's Law for v , we get:

$$d = H_0 dt$$

$$1 = H_0 t$$

$$t = \frac{1}{H_0}$$

So, the reciprocal of the Hubble constant is the age of the universe - but be careful with the units.

More Doppler Effect

We have already seen that red-shift z is given by:

$$z = \frac{\Delta\lambda}{\lambda_s} = \frac{v_s}{c},$$

where $\Delta\lambda$ is the amount by which radiation is red-shifted from a celestial object, λ_s is the wavelength of the radiation relative to the celestial object, v_s is the velocity of recession of the object, c is the speed of light, and v is much less than c . If λ is the wavelength of the radiation relative to us:

$$z = \frac{\lambda - \lambda_s}{\lambda_s} = \frac{\lambda}{\lambda_s} - 1$$

$$z + 1 = \frac{\lambda}{\lambda_s}$$

However, if it is actually space that is being stretched, then this is actually the ratio of the distances between us and the celestial object at two times: the time at which the radiation was emitted, and the time at which the radiation was received. We can apply this to any distance between any two stars:

$$\frac{R_{now}}{R_{then}} = z + 1$$

Evidence for the Big Bang

Red Shift

If we measure the red shift of celestial objects, we see that most of them are moving away from us - the light from them is red-shifted. This is not true of all celestial objects - the Andromeda galaxy, for example, is blue-shifted; it is moving towards us due to the gravitational attraction of the Milky Way. Some galaxies are partly red-shifted, and partly blue-shifted. This is due to their rotation - some parts of the galaxy are rotating towards us, while others are rotating away from us. However, the majority of celestial objects are moving away from us. If we extrapolate backwards, we find that the universe must have started at a single point. However, we are assuming that the universe has always expanded. Red shift provides evidence for a Big Bang, but does not prove it.

Cosmic Microwave Background Radiation

Models of the Big Bang show that, at the beginning of the universe, radiation of a relatively short wavelength would have been produced. Now, this radiation, due to the expansion of space, has been stretched - it has become microwave radiation. Cosmic microwave background radiation fits in extremely well with Big Bang theory, and so is strong evidence for it.

Questions

1. What is the Hubble Constant in s^{-1} ?
2. How old is the universe?
3. What effect might gravity have had on this figure?
4. Polaris is 132pc away. What is its velocity of recession, according to Hubble's Law?

/Worked Solutions/

Thermodynamics

Heat and Energy

Matter is made of particles. These particles are constantly moving. When we feel some matter, we feel something that we call 'heat'. This is just our impression of how fast the particles are moving. The higher the average speed of the particles, the hotter something is.

If we had some matter which was made of stationary particles, then we would not be able to make the particles more stationary. The concept is meaningless. When matter is in this state, it is at the coldest temperature possible. We call this temperature 0°K. This corresponds to -273.15°C. If the temperature rises by 1°K, the temperature rises by 1°C. The only difference between the two scales is what temperature is defined as 0 - in Kelvin, 0 is absolute zero. In Celsius, 0 is the freezing point of water. In both scales, 1° is one hundredth of the difference in temperature between the freezing and boiling points of water.

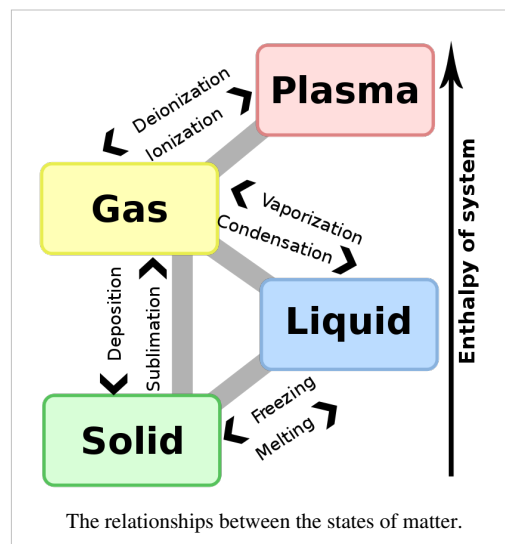
Some matter of temperature T consists of many particles. Their motion is essentially random - they are all moving at different speeds, and so they all have different kinetic energies. The temperature is related to the *average* energy per. particle E by the following *approximate* relationship:

$$E \approx kT,$$

where k is a constant known as the Boltzmann constant. $k = 1.38 \times 10^{-23} \text{JK}^{-1}$. T must always be measured in Kelvin.

Changes of State

Different substances change state at different temperatures. In other words, when the average energy per. particle reaches a certain level, the substance changes state. The situation complicates itself since, in order to change state, additional energy is required (or is given out). When liquid water reaches its boiling point, it will stay at its boiling point until it has all changed into water vapour since the energy being taken in is being used to change state, instead of to increase the temperature of the water. The average energy per. particle required to change state can be *approximated* using the formula above, where T is the temperature at which the substance changes state.



Activation Energy

Many things have an activation energy. In order for a chemical reaction to start, for example, the average energy per. particle must reach a certain level. However, most of the time, chemical reactions start at a lower average energy per. particle than the activation energy. This is because there is always a chance that some particles have the required activation energy, since the particles are moving at random. If the reaction is exothermic (this means that it gives out heat, raising the average energy per. particle), then, once one reaction has happened, more of the particles have the activation energy, and so the reaction accelerates until all the reagents are used up. The activation energy can be related to the temperature of the substance using the formula $E=kT$.

Questions

1. Carbon dioxide sublimates at 195°K. Roughly what energy per. particle does this correspond to?
2. A certain chemical reaction requires particles with mass of the order 10^{-26} to move, on average, at 10ms^{-1} . Roughly what temperature does this correspond to?
3. The boiling point of water is 100°C. Roughly what energy per. particle does this correspond to?
4. Thermionic emission from copper requires around 5eV of energy per. particle. How hot will the wire be at this energy level?

/Worked Solutions/

Specific Heat Capacity

It takes energy to heat things up, since heat is energy. If we heat a more massive thing up, it takes more energy, because we have to give more particles, on average, an energy kT . Some substances require more energy to heat up than others. This property is known as specific heat capacity. This gives us the formula:

$$\Delta E = mc\Delta\theta,$$

where ΔE is the energy put in to heating something up (in J), m is the mass of the thing we are heating up (in kg), c is the specific heat capacity (in $\text{Jkg}^{-1}\text{K}^{-1}$), and $\Delta\theta$ is the difference in temperature due to the work done on the substance (in degrees Celsius or Kelvin).

It should be noted that the specific heat capacity changes slightly with temperature, and more than slightly when the material changes state. A table of the specific heat capacities of various substances is given below:

Substance	State	Temperature (°C)	Specific Heat Capacity ($\text{kJkg}^{-1}\text{K}^{-1}$)
Air	gas	23	1.01
Aluminium	solid	25	0.90
Animal (and human) tissue	mixed	25	3.5
Argon	gas	25	0.52
Copper	solid	25	0.39
Glass	solid	25	0.84
Helium	gas	25	5.19
Hydrogen	gas	25	14.3
Iron	solid	25	0.45
Lead	solid	25	0.13
Nitrogen	gas	25	1.04
Oxygen	gas	25	0.92
Uranium	solid	25	0.12
Water	solid	-10	2.05
Water	liquid	25	4.18
Water	gas	100	2.08

Questions

1. How much energy would it take to heat 100kg of liquid water from 20°C to 36.8°C?
2. How much energy would it take to heat a well-insulated room from 15°C to 21°C, if the room is a cube with side length 10m, and the density of the air is 1.2kgm^{-3} ?
3. A 10kg block of iron at 80°C is placed in the room above once it has reached 21°C. If the iron cools by 40°C, what is the new temperature of the room?

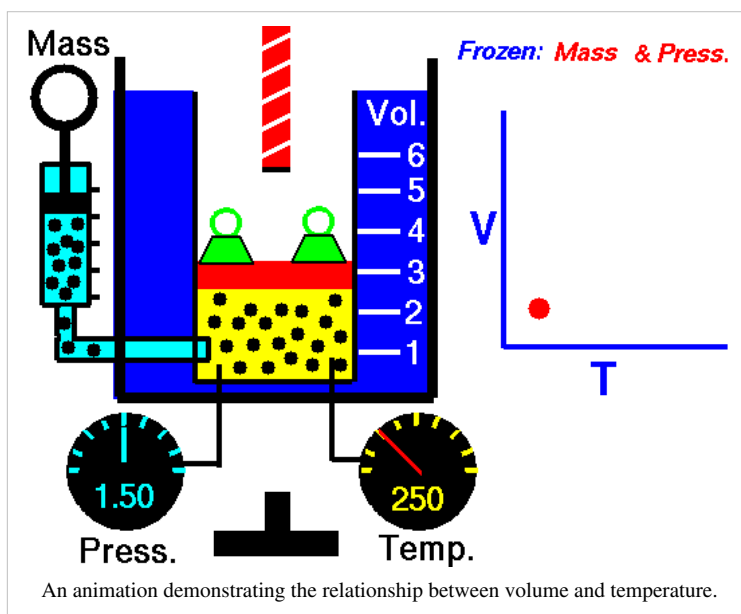
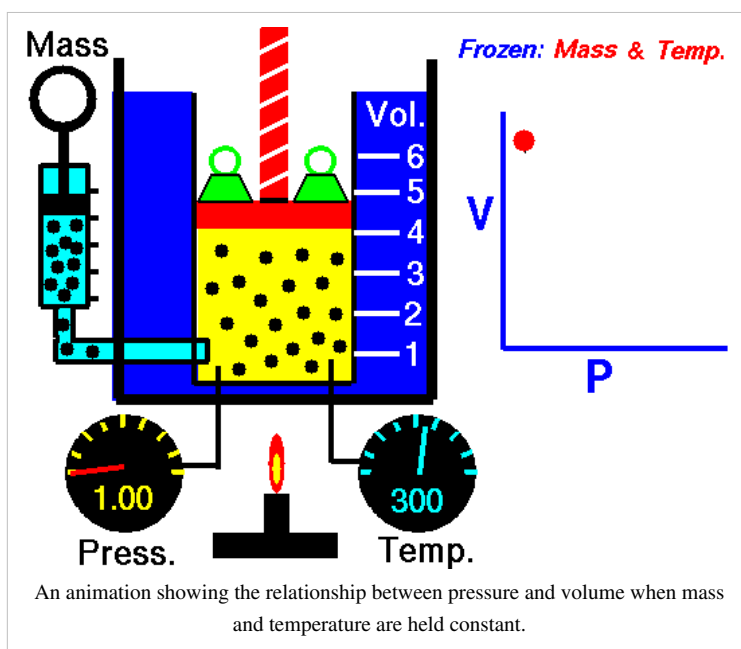
/Worked Solutions/

Ideal Gases

Real-world gases can be modelled as ideal gases. An ideal gas consists of lots of point particles moving at random, colliding with each other elastically. There are four simple laws which apply to an ideal gas which you need to know about:

Boyle's Law

Boyle's Law states that the pressure of an ideal gas is inversely proportional to its volume, assuming that the mass and temperature of the gas remain constant. If I compress an ideal gas into half the space, the pressure on the outside of the container will double. So:



$$p \propto \frac{1}{V}$$

Charles' Law

Charles' Law states that the volume of an ideal gas is proportional to its temperature:

$$V \propto T$$

T must be measured in kelvin, where $1^\circ\text{K} = 1^\circ\text{C}$, but $0^\circ\text{C} = 273^\circ\text{K}$. If we double the temperature of a gas, the particles move around twice as much, and so the volume also doubles.

Amount Law

This law states that the pressure of an ideal gas is proportional to the amount of gas. If we have twice the number of gas particles N, then twice the pressure is exerted on the container they are in:

$$p \propto N$$

A mole is a number of particles. 1 mole = 6.02×10^{23} particles. So, the pressure of a gas is also proportional to the number of moles of gas present n:

$$p \propto n$$

Pressure Law

The pressure law states that the pressure of an ideal gas is proportional to its temperature. A gas at twice the temperature (in $^\circ\text{K}$) exerts twice the pressure on the sides of a container which it is in:

$$p \propto T$$

These laws can be put together into larger formulae linking p, V, T and N.

Questions

1. I heat some argon from 250°K to 300°K . If the pressure of the gas at 250°K is 0.1 MPa, what is its pressure after heating?
2. The argon is in a 0.5m long cylindrical tank with radius 10cm. What volume does it occupy?
3. The argon is then squeezed with a piston so that it only occupies 0.4m of the tank's length. What is its new pressure?
4. What is its new temperature?
5. 25% of the argon is sucked out. What is its pressure now?

/Worked Solutions/

Kinetic Theory

One formula which sums up a lot of the kinetic theory of an ideal gas is the following:

$$pV = \frac{1}{3}Nm\bar{c}^2,$$

where p is the pressure of the gas, V is its volume, N is the number of molecules, m is the mass of each molecule, and \bar{c}^2 is the mean square speed of the molecules. If you knew the speeds of all the molecules, you could calculate the mean square speed by squaring each speed, and then taking the mean average of all the squared speeds.

Derivation

This formula can be derived from first principles by modelling the gas as a lot of particles colliding. The particles have a momentum $p = mc$. If we put them in a box of volume V and length l , the change in momentum when they hit the side of the box is:

$$\Delta p = m(c - (-c)) = 2mc$$

Every time the particle travels the length of the box (l) and back (another l), it hits the wall, so:

$$c = \frac{2l}{t},$$

where t is the time between collisions. Therefore:

$$t = \frac{2l}{c}$$

Each collision exerts a force on the wall. Force is the rate of change of momentum, so:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mc}{\frac{2l}{c}} = \frac{2mc^2}{2l} = \frac{mc^2}{l}$$

However, we have got N particles all doing this, so the total force on the wall is given by:

$$F = \frac{Nm\bar{c}^2}{l}$$

The molecules all have different velocities, so we have to taken an average - the mean square speed. This force is the force in all three dimensions. The force in only one dimension is therefore:

$$F = \frac{Nm\bar{c}^2}{3l}$$

Pressure, by definition, is:

$$p = \frac{F}{A} = \frac{Nm\bar{c}^2}{3Al}$$

But area multiplied by length is volume, so:

$$p = \frac{Nm\bar{c}^2}{3V}$$

Therefore:

$$pV = \frac{1}{3}Nm\bar{c}^2$$

Questions

- Five molecules are moving at speeds of 1,5,6,8, and 36ms^{-1} . What is their mean square speed?
- What is the mass of one molecule of N_2 (atomic mass 14, $1\text{u} = 1.66 \times 10^{-23}\text{kg}$)?
- Atmospheric pressure is 101,325 Pa. If one mole of Nitrogen takes up 2.3 m^3 at about 10°C , what is the mean square speed of the molecules in the air outside, assuming that the atmosphere is 100% nitrogen (in reality, it is only 78%)?
- What is the average speed of a nitrogen molecule under the above conditions?
- The particles in question 1 are duplicated 3000 times. If they have a completely unrealistic mass of 1g, what is their pressure when they are crammed into a cube with side length 0.5m?

/Worked Solutions/

Boltzmann Factor

Particles in a gas lose and gain energy at random due to collisions with each other. On average, over a large number of particles, the proportion of particles which have at least a certain amount of energy ϵ is constant. This is known as the Boltzmann factor. It is a value between 0 and 1. The Boltzmann factor is given by the formula:

$$\frac{n}{n_0} = e^{\frac{-\epsilon}{kT}},$$

where n is the number of particles with kinetic energy above an energy level ϵ , n_0 is the total number of particles in the gas, T is the temperature of the gas (in kelvin) and k is the Boltzmann constant ($1.38 \times 10^{-23} \text{ JK}^{-1}$).

This energy could be any sort of energy that a particle can have - it could be gravitational potential energy, or kinetic energy, for example.

Derivation

In the atmosphere, particles are pulled downwards by gravity. They gain and lose gravitational potential energy (mgh) due to collisions with each other. First, let's consider a small chunk of the atmosphere. It has horizontal cross-sectional area A , height dh , molecular density (the number of molecules per. unit volume) n and all the molecules have mass m . Let the number of particles in the chunk be N .

$$n = \frac{N}{V} = \frac{N}{A dh}$$

Therefore:

$$V = A dh \text{ (which makes sense, if you think about it)}$$

By definition:

$$N = nV = nA dh$$

The total mass Σm is the mass of one molecule (m) multiplied by the number of molecules (N):

$$\Sigma m = mN = mnA dh$$

Then work out the weight of the chunk:

$$W = g\Sigma m = nmgA dh$$

The downwards pressure P is force per. unit area, so:

$$P = \frac{W}{A} = \frac{nmgA dh}{A} = nmg dh$$

We know that, as we go up in the atmosphere, the pressure decreases. So, across our little chunk there is a difference in pressure dP given by:

$$dP = -nmg \, dh \quad (1)$$

We also know that:

$$PV = NkT$$

So:

$$P = \frac{NkT}{V}$$

But:

$$n = \frac{N}{V}$$

So, by substitution:

$$P = nkT$$

So, for our little chunk:

$$dP = kT \, dn \quad (2)$$

If we equate (1) and (2):

$$dP = -nmg \, dh = kT \, dn$$

Rearrange to get:

$$\frac{dn}{dh} = \frac{-nmg}{kT}$$

Integrate between the limits n_0 and n :

$$h = \frac{-kT}{mg} \int_{n_0}^n \frac{1}{n} \, dn = \frac{-kT}{mg} [\ln n]_{n_0}^n = \frac{-kT}{mg} (\ln n - \ln n_0) = \frac{-kT}{mg} \ln \frac{n}{n_0}$$

$$\ln \frac{n}{n_0} = \frac{-mgh}{kT}$$

$$\frac{n}{n_0} = e^{\frac{-mgh}{kT}}$$

Since we are dealing with gravitational potential energy, $\epsilon = mgh$, so:

$$\frac{n}{n_0} = e^{\frac{-\epsilon}{kT}}$$

Questions

$$1u = 1.66 \times 10^{-27} \text{ kg}$$

$$g = 9.81 \text{ ms}^{-2}$$

1. A nitrogen molecule has a molecular mass of 28u. If the Earth's atmosphere is 100% nitrous, with a temperature of 18°C, what proportion of nitrogen molecules reach a height of 2km?
2. What proportion of the molecules in a box of hydrogen (molecular mass 2u) at 0°C have a velocity greater than 5ms⁻¹?
3. What is the temperature of the hydrogen if half of the hydrogen is moving at at least 10ms⁻¹?
4. Some ionised hydrogen (charge $-1.6 \times 10^{-19} \text{ C}$) is placed in an uniform electric field. The potential difference between the two plates is 20V, and they are 1m apart. What proportion of the molecules are at least 0.5m from the positive plate (ignoring gravity) at 350°K?

/Worked Solutions/

Magnetic Fields

Flux

A coil of wire creates magnetic flux. The amount of magnetic flux created is dependant on three things: the number of coils in the wire, the amount of current flowing through the wire, and the permeance of the object through which the flux is flowing. So:

$$\Phi = \Lambda NI,$$

where Φ is flux (in webers, denoted Wb), Λ is permeance (in WbA^{-1}) and I is current (in A). This is the total flux induced. NI is known as flux linkage. Permeance is related to permeability (a material property) by the following equation:

$$\Lambda = \frac{\mu A}{L},$$

where μ is permeability, A is cross-sectional area, and L is length. A permanent magnet is just like a coil, except that a current does not need to be generated to maintain the flux. Over smaller areas, we need to know the flux density B . This is the amount of flux per. unit area:

$$B = \frac{\Phi}{A}$$

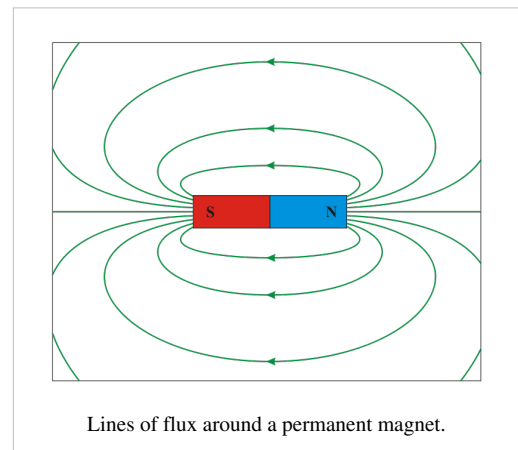
Therefore:

$$\Phi = AB$$

The flux around a coil of wire varies - ΛNI only gives the total flux, not the flux across a certain area. To show this, we use lines of flux. These obey the following rules:

1. Lines of flux go from the north pole of a permanent magnet to the south pole.
2. Lines of flux go clockwise about wires carrying current away from you.
3. Lines of flux never touch, intersect, or cross.

The direction of the flux is shown with an arrow. Flux is a bit like electricity in that it must have a complete circuit. The lines of flux always take the route of least permeance. An iron core has around 800 times as much permeability as some air. So, flux goes through the iron core, and not the air.



Questions

1. A circular steel core has a cross-sectional area of 9 cm^2 , and a length of 0.5m . If the permeability of steel is $875 \mu\text{NA}^{-2}$, what is the permeance of the core?
2. A coil of insulated wire is wrapped 60 times around the top of the core, and a 9A direct current is put through the coil. How much flux is induced?
3. Assuming that all the flux goes through the core, what is the flux density at any point in the core?
4. Draw a diagram showing the lines of flux within the core.

/Worked Solutions/

Induction

A magnetic field creates a current in a wire moving through it. This process is known as induction.

Flux Linkage

A magnetic field going through a coil of wire has a property known as flux linkage. This is the product of the flux Φ and the number of coils in the wire N .

Faraday's Law

Electric current is only induced in a coil of wire if the magnetic field is moving relative to the coil. Faraday's Law gives the electromotive force (emf) ϵ produced in a coil by a magnetic field:

$$\epsilon = - \frac{dN\Phi}{dt}$$

In other words, the emf (electric potential) induced in the coil is proportional to the rate of change of flux linkage. In practice, this means that if the coil is stationary relative to the magnetic field, no emf is induced. In order to induce emf, either the coil or the magnetic field must move. Alternatively, we may change the number of coils, for example, by crushing the coil, or pressing a switch which added more coils into the circuit, or moving more of the coils into the magnetic field.

Faraday's Law also works the other way. If we were to integrate both sides and rearrange the formula in terms of Φ , we would find that the flux depends on the integral of the voltage - not on its rate of change. If we put an emf across a coil, it produces a magnetic field - it induces a magnetic field. The flux does not depend on the rate of change of emf, but the emf does depend on the rate of change of the flux linkage.

Lenz's Law

Lenz's Law describes the direction of the current / emf induced by a change in magnetic flux. It states that current induced opposes the magnetic field. It does this by creating its own magnetic field. This explains the minus sign in Faraday's Law. This also means that the flux induced by a current (not a change in current) is proportional to the current, since the flux is produced in response to the current.

So, a change in flux induces a current and a voltage which is proportional to the rate of change of flux. This fits with Ohm's Law ($V = IR$). A current and a voltage in a coil induce a flux which is proportional to the current and the voltage.

Questions

1. What is the flux linkage of a 30cm coil of 0.5mm thick wire with a flux perpendicular to it of 10Wb?
2. If the above coil is crushed steadily over a period of 2s, what emf is maintained?
3. The flux in a flux circuit varies according to the equation $\Phi = \sin \omega t$. What is the equation for the emf induced?
4. Using a constant k, what is the equation for a current which could induce the flux in the flux circuit above?
5. Draw a graph of the flux, flux linkage, emf and current as deduced in the previous two questions.

/Worked Solutions/

Force

Magnetic fields exert a force on a charge when the charge is moving. If the charge is stationary, no force is exerted. This force is given by:

$$\vec{F} = q(\vec{v} \times \vec{B}),$$

where q is the charge on the point charge, v is its velocity and B is the magnetic field strength. This involves a vector cross product, which you don't need to know about for A-level. However, you do need to know a simplified version of this. The magnitude of this force F is given by:

$$F = Bqv \sin \theta,$$

where θ is the angle between the direction of motion of the point charge and the direction of the magnetic field. If the velocity and the magnetic field are in the same direction, the $\theta = 0$, so $\sin \theta = 0$ and $F = 0$. If the velocity and the magnetic field are perpendicular to each other, $\theta = \pi$, so $\sin \theta = 1$. This means that, in the special case where velocity is perpendicular to the magnetic field:

$$F = Bqv$$

If q is negative (for example, for an electron), the force is in the opposite direction.

Current

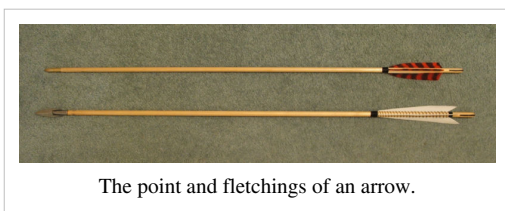
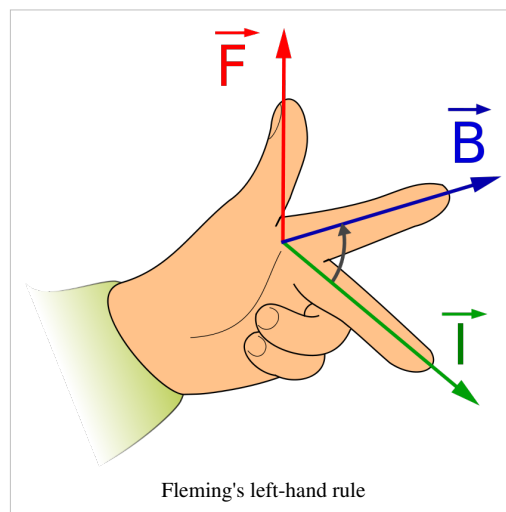
A current is just a flow of moving electrons, and so a magnetic field will exert a force on a wire with a current flowing through it. The case you need to know about is when the magnetic field is perpendicular to the wire. In this case, the magnitude of the force on the wire is given by:

$$F = BIl,$$

where I is current, and l is the length of the wire.

Direction

The direction of the force on either a point charge or on a wire can be worked out using Fleming's left-hand rule, as shown in the diagram on the right. The direction of the *thumb* is that of the force (or *thrust*), the direction of the *first* finger is that of the magnetic *field*, and the direction of the *second* finger is that of the current (or the motion of the point charge).

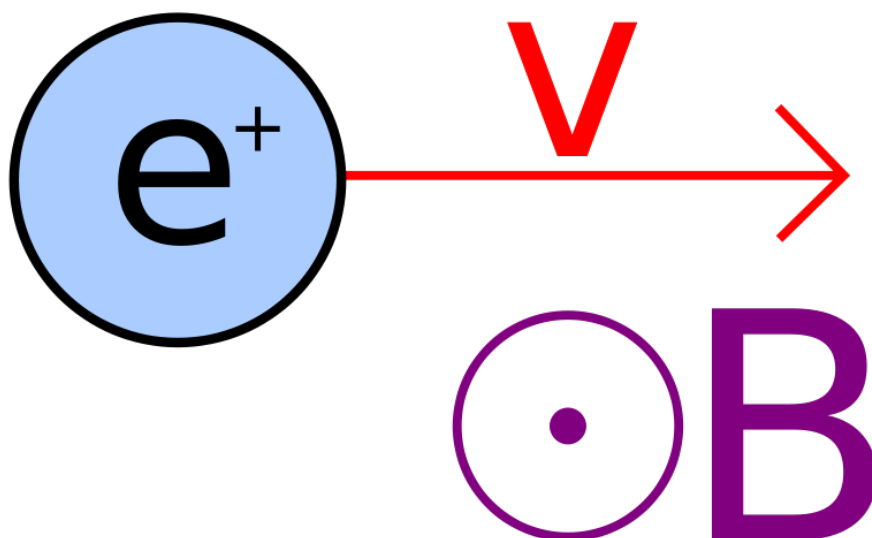


On a 2D diagram, the direction of a magnetic field is represented by one of two symbols, which resemble the point and fletchings of an arrow pointing in the direction of the magnetic field. The symbol \odot means that the field is pointing towards you (just as the arrow would be, if you were looking at the point). The symbol \otimes means that the field is pointing away from you (just as the

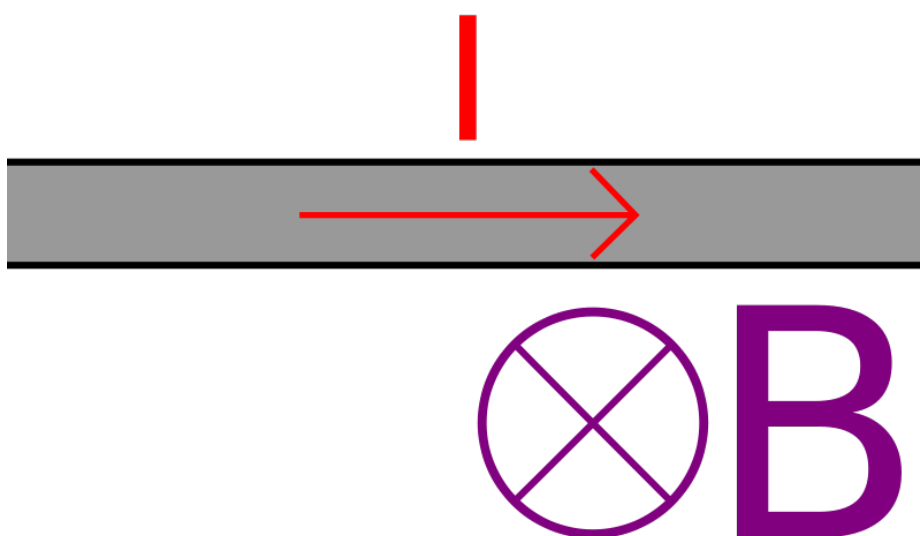
arrow would be, if you were looking at the fletching).

Questions

1. What force is exerted by a 1T magnetic field on an electron (of charge $-1.6 \times 10^{-19}\text{C}$) moving at 5% of the speed of light ($3 \times 10^8 \text{ms}^{-1}$)?
2. What force is exerted by a 5mT magnetic field on a 20cm wire with resistance $1\mu\Omega$ attached to a 9V battery?
3. The following diagram shows a positive charge moving through a magnetic field. Draw an arrow representing the direction of the force on the charge.



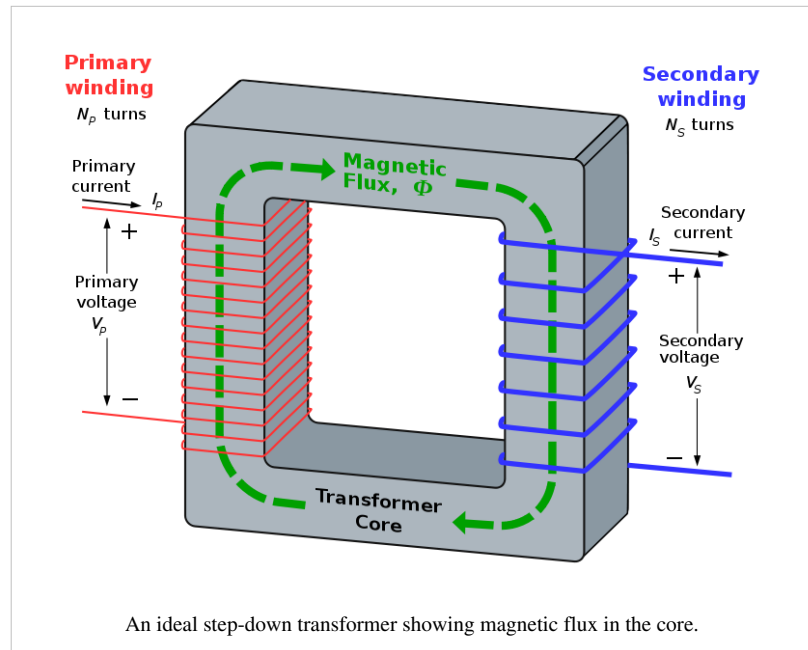
4. The following diagram shows a wire in a magnetic field. Draw an arrow representing the direction of the force on the wire.



/Worked Solutions/

Transformers

We have already seen that a change in flux induces an emf in a coil, given by Faraday's Law:



$$\epsilon = -N \frac{d\phi}{dt}$$

We have also seen that a voltage in a coil induces a magnetic flux inside the coil. If we were to connect two coils with the same core, the flux, and the rate of change of flux, would be exactly the same inside both coils. We would have created a kind of flux circuit known as a transformer. The ratio between the voltage at the primary coil V_p and the voltage at the secondary coil V_s would have to be (since ϕ is constant):

$$\frac{V_p}{V_s} = \frac{-N_p \frac{d\phi}{dt}}{-N_s \frac{d\phi}{dt}} = \frac{N_p}{N_s},$$

where N_p and N_s are the numbers of coils in the primary and secondary coils respectively.

In other words, we can change the voltage of some electricity by varying the number of coils in each coil. In order for this to work, the current used must be an alternating current (AC). This means that the current and voltage are constantly changing sinusoidally, and so there is a sinusoidal change in flux. This means that an emf is induced in the secondary coil. If the flux did not change (ie. we were using direct current), then no emf would be induced, and the transformer would be useless except as a magnet (since it would still have a flux circuit in it).

Ideal Transformers

An ideal transformer is one in which all the electrical energy put into one coil comes out of the other coil. An ideal transformer does not exist, but, since it makes the maths easy, we like to pretend that it does. In this case, the power in must equal the power out:

$$P = P_p = P_s = I_p V_p = I_s V_s,$$

where I_p and I_s are the currents in the primary and secondary coils, respectively. So:

$$V_p = \frac{P}{I_p} \text{ and } V_s = \frac{P}{I_s}$$

By substitution into the transformer equation for voltage:

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{I_s}{I_p} = \frac{1}{\frac{I_p}{I_s}} = \frac{I_s}{I_p}$$

So, in an ideal transformer, the ratio between the voltages is equal to the ratio between the numbers of coils, but the ratio between the currents is equal to the *reciprocal* of the ratio between the numbers of coils.

Eddy Currents

In reality, the electrical energy is not all conserved - a lot of it is converted into heat by eddy currents. In a transformer, the magnetic flux created by the primary coil induces a current in the core. This occurs in order to oppose the change that produced the magnetic flux (Lenz's Law). The currents flowing in the core are called eddy currents.

These currents produce heat, using up energy and so causing inefficiency. One way of minimising the effects of eddy currents is to make the core out of iron laminate. This is layers of iron separated by thin layers of an insulator such as varnish. The amplitude of the eddy currents produced is reduced as currents cannot flow through the layers of insulator. (Note: OCR B question papers tend to have a question on eddy currents.)

Questions

1. A step-down transformer has 300 coils on one coil, and 50 coils on the other. If 30 kV AC is put in, what voltage comes out?
2. A step-up transformer has 200 coils on one coil, and 980 coils on the other. If 25 kV AC comes out, what voltage was put in?
3. An ideal transformer transforms a 50A current into a 1A current. It has 40 coils on the primary coil. How many coils are in the secondary coil?
4. Transformers tend to vibrate. Why is this? What effect does this have on the efficiency of the transformer?
5. Air does have some permeability. What effect does this have on the efficiency of the transformer? Why?

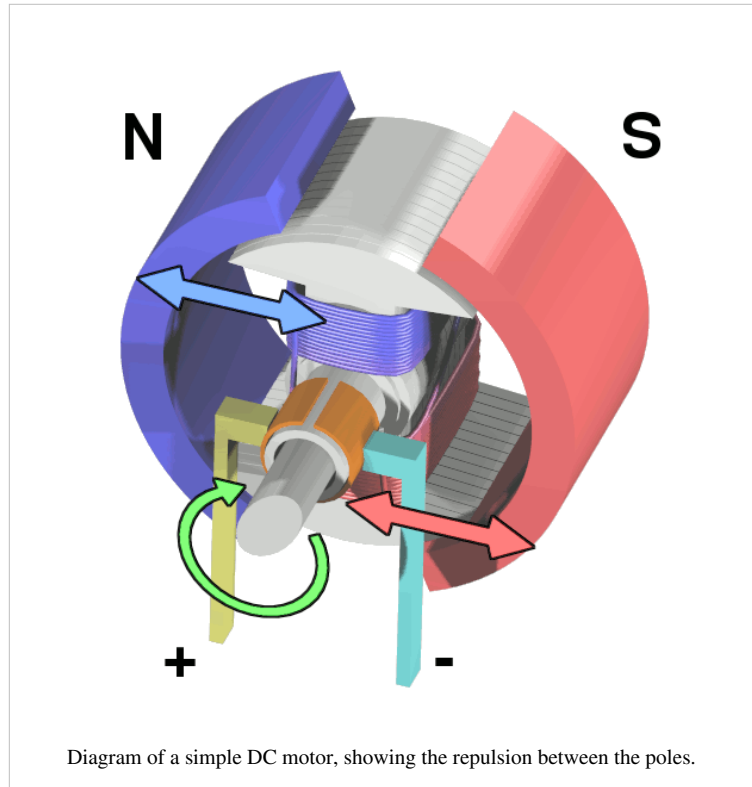
/Worked Solutions/

Motors

Just as a moving magnetic field induces current in conductors, a changing current in a magnetic field induces motion. When this motion is used to ensure that the current keeps changing relative to the magnetic field, the motion will continue, and so we have an electric motor. There are several types of electric motor.

Simple DC Motor

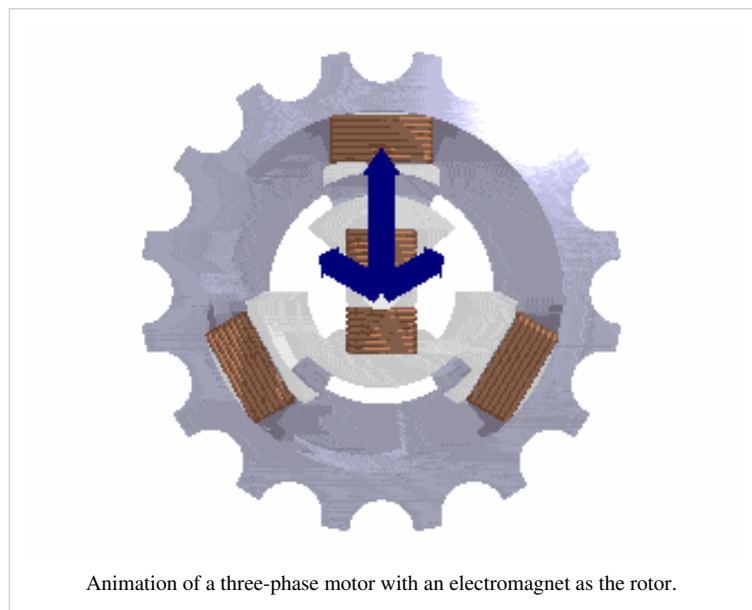
When a coil is placed inside a stationary magnetic field, and a direct current is run through the coil, the coil tries to align itself with the field since it becomes an electromagnet. This would be useless as a motor, since it would always move to the same position when you turned it on, and then stop. If, however, we use a split-ring commutator which changes the direction of the current every half-rotation, then the coil



would try to align itself in the opposite direction every half-rotation. This means that, once the rotor starts to move, it continues to move. This is a DC electric motor. The permanent magnets can be replaced with electromagnets as well. The main advantage of this type of motor is that the commutator works, regardless of the frequency of rotation.

Three-phase Motor

The three-phase power produced by a three-phase generator may be used to power a motor. Each phase of power is connected to one of three coils. This creates a magnetic field which rotates once for each cycle of the power. If a permanent magnet is placed in the middle, at any given time, its north pole will be attracted to a south pole in one of the coils, and will be repelled by a north pole in one of the coils. The converse would be true for its south pole. This means that the rotating magnetic field drags the magnet around with it, causing the magnet to rotate with the same frequency as the magnetic field. The disadvantage of this type of motor is that it goes at one frequency only - the frequency of the current.

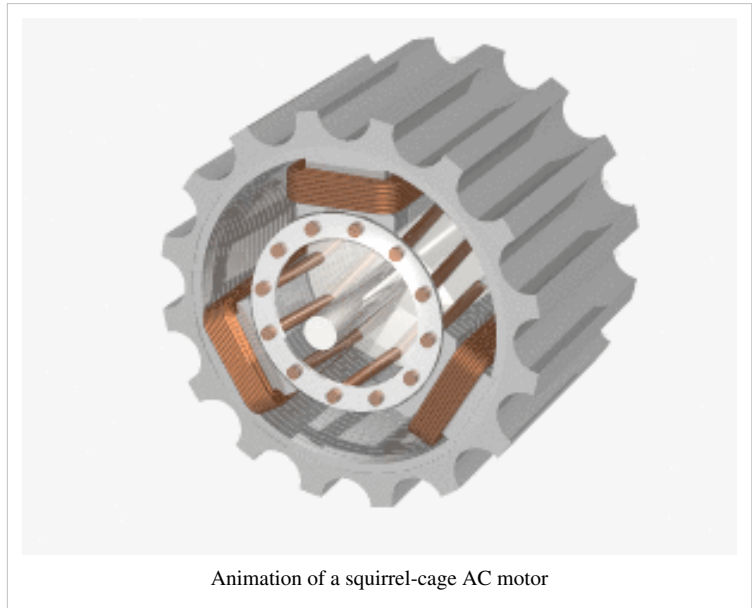


The permanent magnet can be replaced with a coil with direct current in it. This creates a magnetic field, the advantage being that there is no need for a permanent magnet which is expensive and heavy. The main disadvantages are that electricity must be used to power the electromagnet, and that a slip-ring commutator must be used to prevent

the coil getting tangled up and stopping the motor from running.

Squirrel Cage Motor

A squirrel cage motor works on a similar principle, except that the rotor is no longer a permanent magnet. Instead, a series of metal rods run through the rotor, connected to each other at either ends. The rods run perpendicular to the rotating magnetic field. Once the rotor starts to rotate, an electric current is created in the rods - eddy currents. This creates a magnetic field which is perpendicular to the rotating magnetic at all times. As the rotating magnetic field created by the stator rotates, it pulls the induced magnetic field around after it, causing the rotor to continue to rotate.



A squirrel cage motor relies on the fact that the two magnetic fields are rotating at different rates. If they were not, then there would be no change in flux in the rotor, and so no eddy currents would be induced.

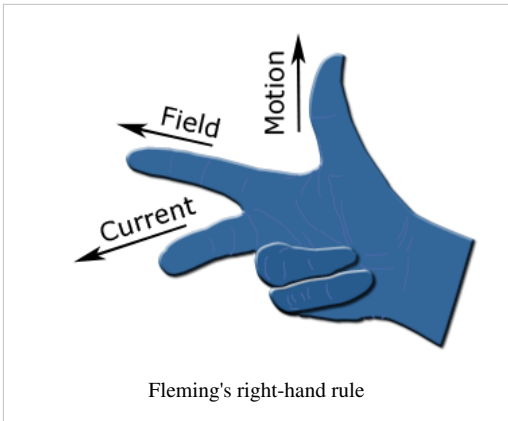
Questions

1. How could you adapt the simple DC motor to use AC?
2. Why does a three-phase motor have a constant angular velocity?
3. What is the difference between a split-ring and a slip-ring commutator?
4. How could the angular velocity of a three-phase motor be increased?
5. A squirrel-cage motor relies on eddy currents running along the rotor to function. However, if eddy currents run across the rotor, then the force on the rotor is reduced. How may these eddy currents be reduced without reducing the desired eddy currents?

/Worked Solutions/

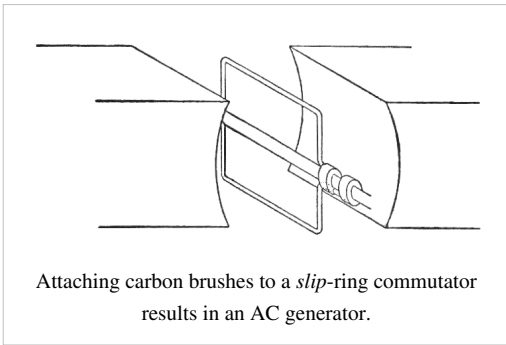
Generators

We have seen that a change in flux induces an electric current in a coil of wire. One way of changing the flux is to move the magnet. Alternatively, we can move the coil relative to the magnet. Generators work on this principle - a non-electrical source of energy is used to rotate something (known as the rotor), which induces an electric current in either the rotor or the stator (the stationary part of any electromagnetic machine). For a generator, the relationships between the directions of current, field and motion are given by Fleming's right-hand rule (right).



Moving Coil

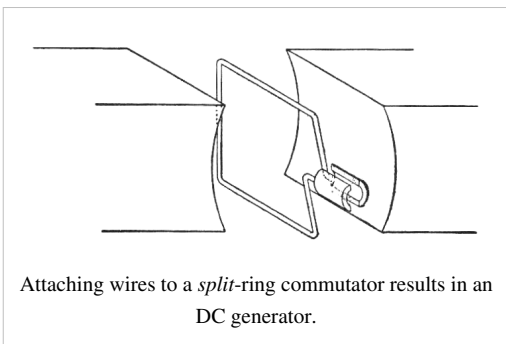
AC Generator



If a coil of wire is placed in a magnetic field and rotated, an alternating (sinusoidal) current is induced. As it rotates, sometimes it is 'cutting' through lots of flux, and so lots of current is induced. At other times, it is moving parallel to the flux, and so no flux is cut, and no current is induced. In between, some current is induced. This creates an alternating current.

Either end of the coil can be connected to wires outside of the generator in order to use the current elsewhere. This would be fine for the first few rotations, but after this, the wires would get tangled up and the generator would be useless. To avoid this, we use a commutator. In an AC generator, this is a pair of rotating conducting 'slip rings' attached to either end of the coil. Carbon brushes bring these into contact with the outside world.

DC Generator



If we replace the slip-ring commutator in an AC generator with a pair of brushes which the ends of the coil rotate inside, the generator creates direct current (DC) instead. Halfway through the rotation, the brushes come into contact with the other end of the coil, and so the AC changes direction every half a rotation. This approximates to a direct current. This direct current is not perfect since it consists of a series of positive-voltage pulses. These pulses can be smoothed out using a capacitor or a complex system of commutators.

Moving Magnet

Simple AC Generator

An alternative method of generating an alternating current is to rotate a permanent magnet in a gap between two coils. This has the advantage of not requiring a commutator (the coil is the stator), but often a coil is lighter than a magnet, and so it is more efficient to use a rotating coil.

Three-Phase Generator

If we place three pairs of coils, evenly spaced, around the rotating magnet, then three different alternating currents, with three different phases, will be generated. This is a more efficient method of generating electricity, since current is always being generated. The sum of all three currents is zero, so three different cables must be used to transport the currents. Three-phase power is often used in motors with three coils in the stator.

Questions

1. Draw diagrams of an alternating current, the 'direct current' produced by a DC generator, and this current once it has been smoothed with a capacitor.
2. What is the phase difference (in radians) between the voltages produced by a three-phase generator?
3. According to Faraday's law, what three things will increase the amplitude of the emf created by a generator?
4. If an albatross touched two power cables carrying AC in phase, what would happen?
5. What would happen if the two cables carried three-phase power?

/Worked Solutions/

Electric Fields

Force

Electric fields are caused by charge. This charge can be either positive or negative. Like charges repel each other, and opposite charges attract each other. If we have two point charges of charge Q and q respectively, and they are a distance r apart, the force on each of them is:

$$F_{electric} = \frac{kQq}{r^2} = \frac{Qq}{4\pi\epsilon_0 r^2},$$

where k and ϵ_0 are constants ($k = 8.99 \times 10^9 \text{ Nm}^2\text{C}^{-2}$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$). This means that, twice as far away from the point charge, the force on another charge decreases by a factor of 4. Electric force around a point charge is very similar to gravitational force around a point mass.

An uniform electric field consists of two conducting plates. These plates are oppositely charged, and infinitely wide. Obviously, infinitely wide conducting plates do not exist, so uniform electric fields do not exist. However, fields which approximate uniform electric fields do exist, provided we look towards the middle of the plates, and the plates are not too far apart - at the ends, the formulae for uniform fields no longer apply.

The force on a charge in an uniform electric field is given by:

$$F_{electric} = \frac{qV}{d},$$

where V is the potential difference between the two plates, q is the charge of the point charge upon which the force is acting, and d is the distance between the two plates. This force remains constant as the charge travels within the electric field.

Questions

$$e = 1.6 \times 10^{-19} \text{ C}$$

1. A positron (charge $+e$) is $1 \mu\text{m}$ from a lithium nucleus (charge $+3e$). What is the magnitude of the force acting on each of the particles? In what direction is it acting?
2. An electron is 1mm from the positively charged plate in an uniform electric field. The potential difference between the plates is 20V , and the plates are 10cm apart. What force is acting on the electron? In what direction?
3. The acceleration due to gravity around a point mass is constant, irrespective of the mass of the objects it is acting on. The acceleration due to electricity around a point charge is not. Use Newton's Second Law ($F=ma$) to explain this.
4. An insulator contains charged particles, even though the overall charge on the insulator is 0 . Why is the insulator attracted by a nearby charge?
5. Where in the charged conducting plates which create an uniform electric field would you expect to find the charge located? Why?

/Worked Solutions/

Field

Electric field E is the force per. unit charge caused by an electric field:

$$E_{electric} = \frac{F_{electric}}{q}$$

The unit of electric field is NC^{-1} or Vm^{-1} . In general, the electric field is the rate of change of electric potential (voltage) with respect to distance:

$$E_{electric} = -\frac{dV_{electric}}{dx}$$

Special Cases

There are two different types of field which you need to know about. Uniform fields occur between two plates with opposite charges. Here, the electric field is simply:

$$E_{electric} = \frac{V_{electric}}{d}$$

A charged sphere also has an electric field. To gain a formula for this, we divide the formula for force around a charged sphere by q , so:

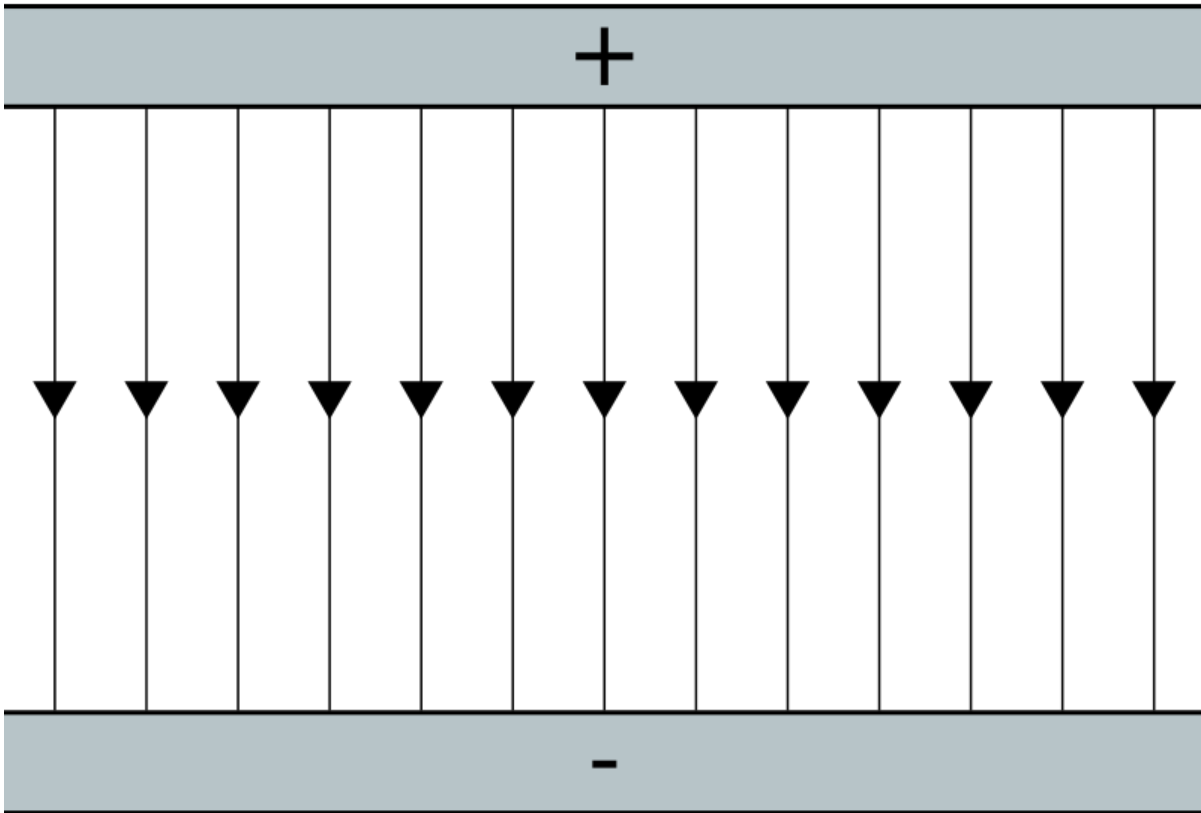
$$E_{electric} = \frac{Q}{4\pi\epsilon_0 r^2},$$

where $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2\text{N}^{-1}\text{m}^{-2}$.

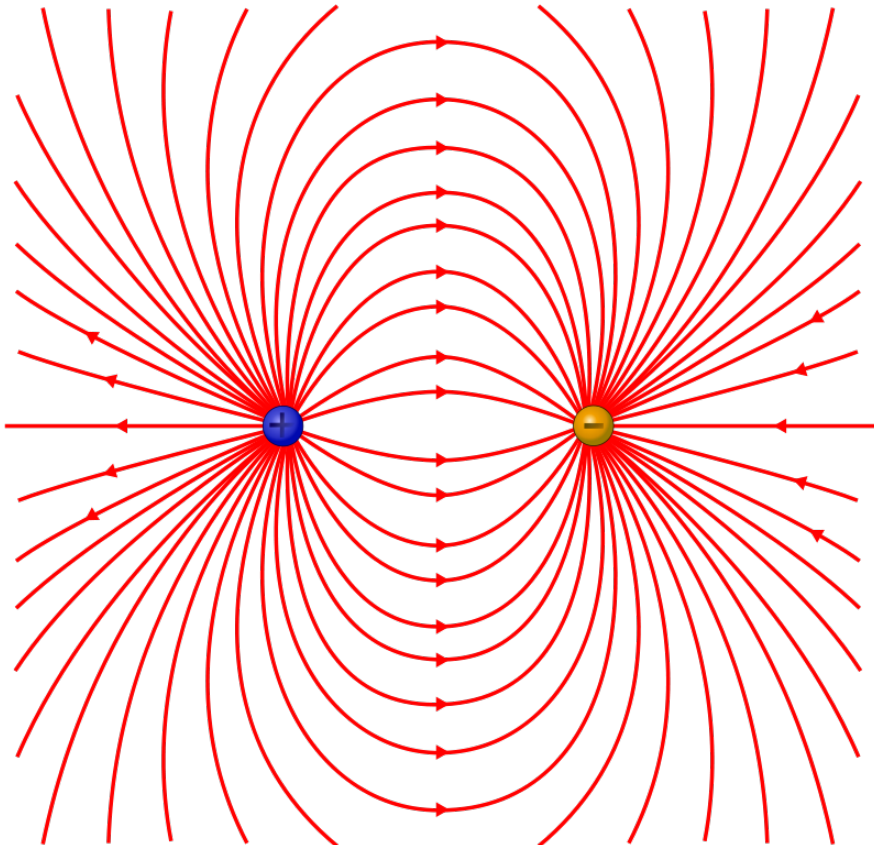
The electric field around a point charge is called a radial field. The field strength is highest at the centre and decreases as the distance from the centre increases. This is reflected in the above formula, which shows that $E_{electric}$ is proportional to $\frac{1}{r^2}$.

Field Lines

We can represent electric field using field lines. These go from positive charge to negative charge. They are more closely packed together when the electric field is stronger. In a uniform field, they look like the following:



Around two oppositely charged spheres (known as a dipole), they look like the following:



Questions

1. Two metal plates are connected to a 9V battery with negligible internal resistance. If the plates are 10cm apart, what is the electric field at either of the plates?
2. What is the electric field at the midpoint between the plates?
3. The charge on an electron is -1.6×10^{-19} C. What is the electric field $1\mu\text{m}$ from a hydrogen nucleus?
4. What is the direction of this field?
5. A 2C charge is placed 1m from a -1C charge. At what point will the electric field be 0?

/Worked Solutions/

Potential

Relationship to Electric Potential Energy

You will probably remember from AS (or even GCSE) that the energy U which flows along a wire is given by:

$$U = Vq,$$

where V is the potential difference between either end of the wire, and q is the amount of charge which flows. A simple rearrangement shows that:

$$V = \frac{U}{q}$$

This potential difference is the same thing as electric potential. In a wire, the electric field is very simple. There are other electric fields, and in these fields as well, the electric potential is the electric potential energy per. unit charge. Electric potential energy between two point charges Q and q is given by:

$$U = \frac{Qq}{4\pi\epsilon_0 r}$$

So, the electric potential at a distance r from any point charge Q (ignoring other charges) is:

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Relationship to Electric Field Strength

Electric potential is also the integral of electric field strength. This is why it is often called potential difference - it is an integral between two limits (two points in space) with respect to distance. So, the potential difference between two points a and b is:

$$V_{ab} = \int_a^b E \, dx = \int_a^b \frac{Q}{4\pi\epsilon_0 x^2} \, dx = \left[-\frac{Q}{4\pi\epsilon_0 x} \right]_a^b$$

But, if we define b as infinity and a as r :

$$V = \left[-\frac{Q}{4\pi\epsilon_0 x} \right]_r^\infty = -\frac{Q}{4\pi\epsilon_0 \infty} - \left(-\frac{Q}{4\pi\epsilon_0 r} \right) = \frac{Q}{4\pi\epsilon_0 r}$$

So, the area under a graph of electric field strength against distance, between two points, is the potential difference between those two points.

For a uniform electric field, E is constant, so:

$$V = \int_r^\infty E \, dx = Er$$

In other words, V is proportional to r . If we double the distance between us and a point, the potential difference between us and that point will also double in a uniform electric field.

Equipotentials

Equipotentials are a bit like contours on a map. Contours are lines which join up all the points which have the same height. Equipotentials join up all the points which have the same electric potential. They always run parallel to electric field lines. As the field lines get closer together, the equipotentials get closer together.

Questions

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$$

1. Draw a diagram of an uniform electric field between two plates, showing the field lines and the equipotentials.
2. Do the same for the electric field around a point charge.
3. The potential difference between two plates is 100V. What is the potential difference between a point halfway between the plates and one of the plates?
4. What is the electric potential at a point 0.2m from an alpha particle (charge on an electron = $-1.6 \times 10^{-19} \text{ C}$)?
5. What is the electric potential energy of an electron at the negative electrode of an electron gun if the potential difference between the electrodes is 10V?

/Worked Solutions/

Potential Energy

Just as an object at a distance r from a sphere has gravitational potential energy, a charge at a distance r from another charge has electrical potential energy ϵ_{elec} . This is given by the formula:

$$\epsilon_{elec} = V_{elec}q,$$

where V_{elec} is the potential difference between the two charges Q and q . In an uniform field, voltage is given by:

$$V_{elec} = E_{elec}d,$$

where d is distance, and E_{elec} is electric field strength. Combining these two formulae, we get:

$$\epsilon_{elec} = qE_{elec}d$$

For the field around a point charge, the situation is different. By the same method, we get:

$$\epsilon_{elec} = \frac{-kQq}{r}$$

If a charge loses electric potential energy, it must gain some other sort of energy. You should also note that force is the rate of change of energy with respect to distance, and that, therefore:

$$\epsilon_{elec} = \int F dr$$

The Electronvolt

The electronvolt (eV) is a unit of energy equal to the charge of a proton or a positron. Its definition is the kinetic energy gained by an electron which has been accelerated through a potential difference of 1V:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

For example: If a proton has an energy of 5MeV then in Joules it will be $= 5 \times 10^6 \times 1.6 \times 10^{-19} = 8 \times 10^{-13} \text{ J}$.

Using eV is an advantage when high energy particles are involved as in case of particle accelerators.

Summary of Electric Fields

You should now know (if you did the electric fields section in the right order) about four attributes of electric fields: force, field strength, potential energy and potential. These can be summarised by the following table:

Force $F_{elec} = \frac{-kQq}{r^2}$	$\rightarrow \text{integrate} \rightarrow$ with respect to r	Potential Energy $\epsilon_{elec} = \frac{-kQq}{r}$
$\downarrow \text{per. unit charge} \downarrow$		
Field Strength $E_{elec} = \frac{-kQ}{r^2}$	$\rightarrow \text{integrate} \rightarrow$ with respect to r	Potential $V_{elec} = \frac{-kQ}{r}$

This table is very similar to that for gravitational fields. The only difference is that field strength and potential are per. unit charge, instead of per. unit mass. This means that field strength is not the same as acceleration. Remember that integrate means 'find the area under the graph' and differentiate (the reverse process) means 'find the gradient of the graph'.

Questions

$$k = 8.99 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

- Convert $5 \times 10^{-13} \text{ J}$ to MeV.
- Convert 0.9 GeV to J.
- What is the potential energy of an electron at the negatively charged plate of an uniform electric field when the potential difference between the two plates is 100V?
- What is the potential energy of a 2C charge 2cm from a 0.5C charge?
- What is represented by the gradient of a graph of electric potential energy against distance from some charge?

/Worked Solutions/

Particle Physics

The Standard Model

The standard model of particle physics attempts to explain everything in the universe in terms of fundamental particles. A fundamental particle is one which cannot be broken down into anything else. These fundamental particles are the building blocks of matter, and the things which hold matter together.

The standard model is usually represented by the following diagram:

Three Generations of Matter (Fermions)

	I	II	III	
mass→	2.4 MeV	1.27 GeV	171.2 GeV	0
charge→	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin→	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name→	u up	c charm	t top	γ photon
	d down	s strange	b bottom	g gluon
Quarks				
	<2.2 eV	<0.17 MeV	<15.5 MeV	91.2 GeV
	0	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z⁰ weak force
	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV
	-1	-1	-1	± 1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Leptons	e electron	μ muon	τ tau	W[±] weak force
				Bosons (Forces)

The particles in the standard model can be put into two groups: fermions and bosons. Fermions are the building blocks of matter. They all obey the Pauli exclusion principle. Bosons are force-carriers. They carry the electromagnetic, strong, and weak forces between fermions.

Bosons

There are four bosons in the right-hand column of the standard model. The photon carries the electromagnetic force - photons are responsible for electromagnetic radiation, electric fields and magnetic fields. The gluon carries the strong nuclear force - they 'glue' quarks together to make up larger non-fundamental particles. The W^+ , W^- and Z^0 bosons carry the weak nuclear force. When one quark changes into another quark, it gives off one of these bosons, which in turn decays into fermions.

Fermions

Fermions, in turn, can be put into two categories: quarks and leptons. Quarks make up, amongst other things, the protons and neutrons in the nucleus. Leptons include electrons and neutrinos. The difference between quarks and leptons is that quarks interact with the strong nuclear force, whereas leptons do not.

Generations

Fermions are also divided into three generations. The first generation contains the fermions which we are made of - electrons, the up and down quarks, and the neutrino. The first generation particles have less mass than the second, and the second generation particles have less mass than their respective third generation particles. The second generation (the μ generation) contains two leptons: the muon and the muon-neutrino. It also contains the charm and strange quarks. The third generation (the τ generation) contains another two leptons: the tau and the tau-neutrino. Its quarks are the top and bottom quarks.

Antiparticles

Every fermion has its antiparticle. An antiparticle has the same mass as a particle, but the opposite charge. So, the standard model contains 12 quarks, 12 leptons, and the bosons (which are even more complex).

Questions

1. The third generation top quark was the last quark in the Standard Model to have its existence proven experimentally (in 1995). It is also the most massive of the quarks. Why was it so difficult to observe a top quark?
2. What observable phenomena does the Standard Model not explain?
3. How much more massive is an up quark than an electron?
4. How many fermions are there in the Standard Model?
5. The antiparticle of the electron (e^-) is the positron. What is the charge and rest mass of a positron?

/Worked Solutions/

Quarks

Quarks (pronounced like 'orcs' with a 'qu' on the front) are a subset of the fermions - they make up part of matter, most notably the nuclei of atoms. Quarks interact with all four of the fundamental forces: gravity, electromagnetism, and the weak and strong nuclear forces.

Generations

There are four quarks in each of the three generations of fermions. The first contains the up quark (u), down quark (d), antiup quark (\bar{u}) and antidown quark (\bar{d}). The second generation contains the charm quark (c), strange quark (s), anticharm quark (\bar{c}) and anti strange quark (\bar{s}). The third generation contains the top quark (t), bottom quark (b), antitop quark (\bar{t}) and antibottom quark (\bar{b}).

Charge

The up, charm and top quarks have a charge of $+\frac{2}{3}e$, and so their respective antiparticles have a charge of $-\frac{2}{3}e$. The down, strange and bottom quarks have a charge of $-\frac{1}{3}e$, and so their respective antiparticles have a charge of $+\frac{1}{3}e$.

Hadrons

When quarks are combined, they form larger particles, which are not fundamental. These larger particles are known as hadrons and are held together by the strong nuclear force. There are two types of hadrons: baryons and mesons.

Baryons

Baryons are hadrons which are made up of three quarks. The two most common baryons are the proton and the neutron. Protons are made up of two up quarks and one down quark, giving them a total charge of $+1e$. Neutrons are made up of one up quark and two down quarks, giving them net charge of 0.

Mesons

Mesons are hadrons which are made up of a quark and an antiquark. For example, pions are made up of two first generation quarks - the π^0 is made up of either an up quark and an antiup quark, or a down quark and an antidown quark. The π^+ is made up of an up quark and an antidown quark (total charge $+1e$), and The π^- is made up of a down quark and an antiup quark (total charge $-1e$).

Questions

1. The Δ^{++} baryon is made up of up quarks. What is its total charge?
2. The Δ^- baryon has a total charge of $-1e$. Given that it is made up of only one type of first generation quark, what is this quark?
3. What is an antiproton made of? What is its charge?
4. A K^+ meson is made of an up quark and an anti strange quark. What is its total charge?
5. Lambda (Λ) baryons are made up of an up quark, a down quark, and another quark (not an antiquark). The Λ^0 is neutral, and contains a second generation quark. What is this quark?

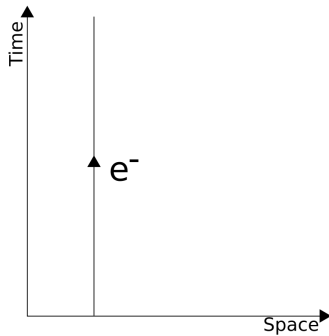
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Bosons

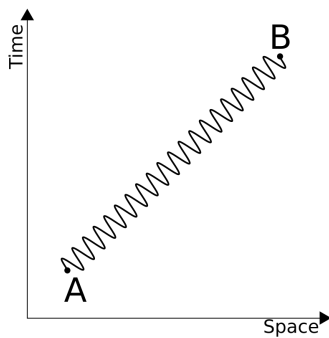
Bosons are particles which carry force. Different types of bosons carry different forces.

Feynman Diagrams

One way of representing these interactions is the Feynman diagram. This is a graph with time on the vertical axis, and space on the horizontal axis showing the paths of particles through space and time as lines. So, a stationary electron looks like this:



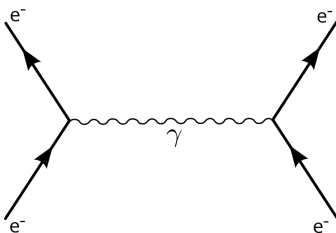
It is often useful to define our units of space and time in such a way that, if something is travelling at the speed of light, it makes a 45° angle. Bosons are virtual particles, so they are given wavy lines. So, a photon travelling at the speed of light from A to B looks like the following:



Different particles can, of course, interact with each other. These interactions must take place at a definite point in space-time. They can be represented by a certain point on a Feynman diagram, with lines coming in and out of the point representing the velocities of particles which take part in the interaction.

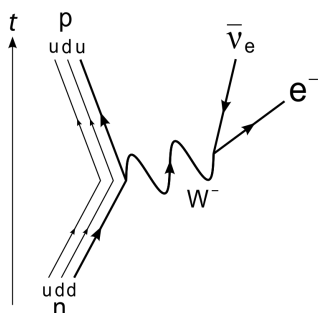
Photons

Photons carry the electromagnetic force. They are 'given off' by one particle, causing it to change its velocity. They are then 'received' by another particle, causing it too to change its velocity. This can be represented on a Feynman diagram in the following way:



W and Z Bosons

W and Z bosons carry the weak nuclear force between particles. This occurs, for example, in β decay, which actually takes place in two stages. First, a proton turns into a neutron (or vice versa), emitting a W boson. Then, the W boson 'turns into' an electron / positron and an (anti-) neutrino. This is shown in the following Feynman diagram:



Gluons

Gluons carry the strong nuclear force between quarks, holding hadrons together. Quarks have a property known as 'colour', as do gluons. The gluons carry colour between the quarks, mediating the strong force. You probably won't be asked about them in the exam.

Questions

1. A stationary light source emits single photons at regular intervals. Draw a Feynman diagram to represent this.
2. Write two equations (including a W^+ boson) which describe positron emission.
3. What is the charge on a W^- boson?
4. Read Richard Feynman's excellent book, "QED - the Strange Theory of Light and Matter", ISBN 978-0-140-12505-4.

/Worked Solutions/

Leptons

Leptons are particles which interact with all the fundamental forces except for the strong nuclear force. There are two types of leptons: electrons and neutrinos.

Electrons

Electrons are particles with a charge of $-1.6 \times 10^{-19}\text{C}$. They are responsible, amongst other things, for the whole of chemistry since, as they occupy the quantum states around the nucleus. There are three types of electrons: the electron (e^-), the muon (μ^-), and the tauon (τ^-), one for each generation. These electrons have antiparticles, each with a charge of $+1.6 \times 10^{-19}\text{C}$: the positron (e^+), the antimuon (μ^+), and the antitauon (τ^+), respectively.

Neutrinos

Neutrinos are chargeless, and almost massless. Loads of them travel around the universe and through you at speeds close to the speed of light. The symbol for a neutrino is the greek letter nu (ν), with its generation (e, μ or τ) in subscript. If it is an antineutrino, the symbol has a bar above it. So, the symbol for a muon-antineutrino is $\bar{\nu}_\mu$.

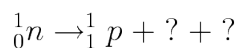
Lepton Number

All leptons have a lepton number of 1. All antileptons have a lepton number of -1. In a nuclear reaction, the lepton number before the reaction must equal the lepton number after the reaction. This necessitates the existence of neutrinos. When a nucleus gives off a beta particle (electron), the lepton number before the emission is 0. Without neutrinos, the lepton number after the emission would be 1, not 0. In reality, an electron-antineutrino is also emitted, with a lepton number of -1, and so the total lepton number both sides of the reaction is 0.

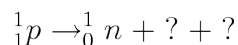
The situation is actually slightly more complicated, as the lepton number from each generation of particles must also be conserved. The lepton number from the beta particle cannot be balanced out by a tauon-antineutrino, since this is from a different generation.

Questions

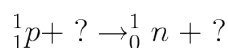
1. An electron is produced by a nuclear reaction, but an electron-antineutrino is not produced. What other particle is produced?
2. Why do electrons not make up part of the nucleus?
3. Why did it take until the 1950s to detect the first antineutrino?
4. Complete the following equation for the emission of a beta particle from a nucleus:



5. Complete the following equation for the emission of an antielectron from a nucleus:



6. Complete the following equation for the capture of an electron by a nucleus:

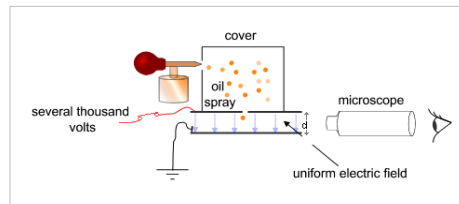


/Worked Solutions/

Millikan's Experiment

Electrons have a finite charge, which is approximately $1.6 \times 10^{-19}\text{C}$. This was first proven by Robert Millikan in 1909. Millikan sprayed drops of oil which were then charged (ionised) either by friction as they were sprayed, or with x-rays. They were then allowed to fall into an uniform electric field.

Once in the uniform electric field, the strength of the field was adjusted in order to keep an oil drop stationary. This was done by hand, looking through a microscope. In a stationary position, the gravitational force and the electric force were balanced - there was no net force on the oil drop. So, at this point:



$$\frac{qV}{d} = mg$$

The electric field strength was adjusted by changing the voltage between the two plates. The voltage at which the drops were stationary was measured. The charge on each drop was then calculated. Millikan found that these charges were all multiples of $1.6 \times 10^{-19}\text{C}$, thus showing that the charge of each drop was made up of smaller charges with a charge of $1.6 \times 10^{-19}\text{C}$.

Questions

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$g = 9.81 \text{ ms}^{-2}$$

1. Rearrange the formula above in terms of q .
2. The mass of an oil drop cannot be measured easily. Express the mass of an oil drop in terms of its radius r and its density ρ , and, by substitution, find a more useful formula for q .
3. An oil droplet of density 885kgm^{-3} and radius $1\mu\text{m}$ is held stationary in between two plates which are 10cm apart. At what potential differences between the plates is this possible?
4. If the X-rays used to ionise the oil are of wavelength 1nm , how much energy do they give to the electrons? Why does this mean that the oil drops are ionised?
5. In reality, the oil drops are moving when they enter the uniform electric field. How can this be compensated for?

/Worked Solutions/

Pair Production and Annihilation

Pair Production

Sometimes, a photon turns into a particle and its antiparticle, for example, an electron and a positron. It could not turn into just an electron, since this would leave the lepton number unbalanced. The photon must have enough energy to create the masses of the two particles. The energy required to create one of the particles is given by:

$$E = mc^2,$$

where m is the mass of the particle, and c is the speed of light ($3 \times 10^8 \text{ ms}^{-1}$). However, two particles must be created. Since the two particles are each other's antiparticle, they have identical masses. So, the total energy required is:

$$E = 2mc^2$$

Annihilation

When a particle meets its antiparticle, the two annihilate each other to form a photon with energy equivalent to the total mass-energy of both particles.

Sometimes, a pair of particles annihilates, but then the photon produces another pair of particles. Also, a photon could produce a pair of particles which then annihilate each other.

Questions

$$h = 6.63 \times 10^{-34} \text{ Js}$$

1. The mass of an electron is $9.11 \times 10^{-31} \text{ kg}$. What is the minimum amount of energy a photon must have to create an electron?
2. A 1.1 MeV electron annihilates with a 1.1 MeV positron. What is the total energy of the photon produced?
3. What is its frequency?
4. What is its wavelength?
5. What classical physical conditions might cause a newly produced electron-positron pair to annihilate almost immediately?

/Worked Solutions/

Particle Accelerators

Modern experimental particle physics requires particles to be accelerated to very high energies. This is accomplished by passing them through an electric field multiple times, in a similar fashion to an electron gun. Types of particle accelerator include linear accelerators and cyclotrons.

Linear Accelerators

In a linear accelerator, particles pass through a series of tubes. At either end of each tube are electrodes. An alternating current is used. This means that, when particles pass an electrode to which they are being attracted, the electrode switches charge, and starts to repel the particle. The distances between electrodes increase as you go along the accelerator, since, as the particles accelerate, they travel further per. oscillation of the current.

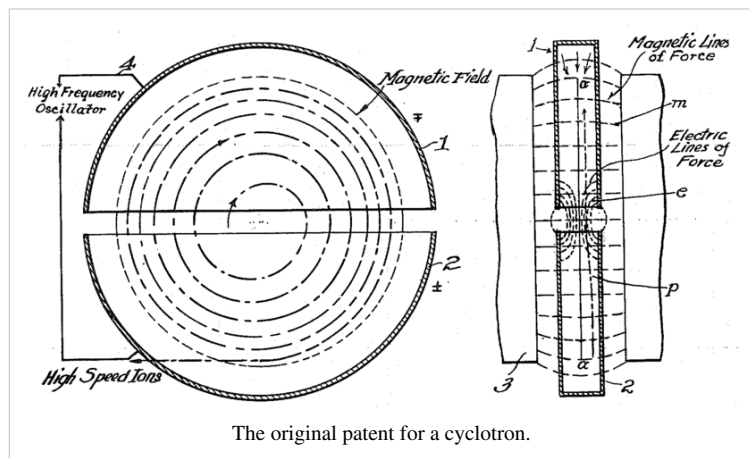


Aerial photo of the Stanford Linear Accelerator Center, with detector complex at the right (east) side

Cyclotrons

A cyclotron is like a linear accelerator, except that, instead of using lots of different electrodes, it uses the same two over and over again. The particles move around in a circle due to a magnetic field. The radius of this circle depends on the velocity of the particles. The orbits of the particles are enclosed by two semi-cylindrical electrodes. An alternating current is used to accelerate the particles. When the particles enter one half of the cyclotron, they are pulled back to the other half.

When they reach the other half, the current switches over, and they are pulled back to the first half. All the time, the magnetic field keeps them moving in circles. As they gain energy from the electric field, the radii of their orbits increase, and their velocities increase, until the radius is as large as the cyclotron.



The original patent for a cyclotron.

Questions

1. Use the formula for centripetal force to show that the radius of motion depends on the speed of the moving object.
2. A cyclotron with a diameter of 1.5m is used to accelerate electrons (mass 9.11×10^{-31} kg). The maximum force exerted on an electron is 2.4×10^{-18} N. What is the maximum velocity of the electrons?
3. What are the problems involved in constructing a large cyclotron?
4. Why don't particles stick to the electrodes when passing through them?

/Worked Solutions/

Cloud Chambers and Mass Spectrometers

Cloud Chambers

The magnitude of the magnetic force on a moving charged particle is given by:

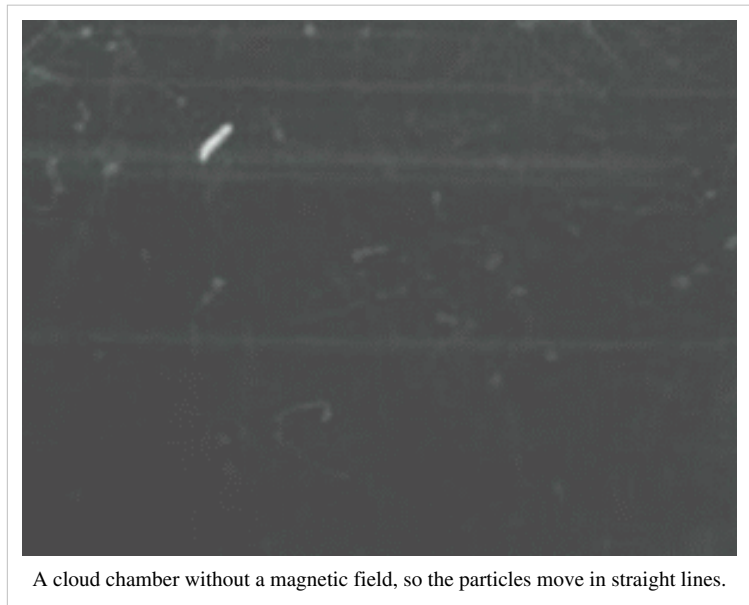
$$F = qvB,$$

where B is the magnetic field strength, v is the speed of the particle and q is the charge on the particle. This force is exerted in a direction perpendicular to both the magnetic field and the direction of motion. If a charged particle enters an uniform magnetic field which is perpendicular to its velocity, then it will move in a circle, since there will be a force of constant magnitude acting on it in a direction perpendicular to its motion. Using the equation for centripetal force, we can derive a formula for the radius of this circle:

$$\begin{aligned} \frac{mv^2}{r} &= qvB \\ \frac{mv}{r} &= qB \\ r &= \frac{mv}{qB} = \frac{p}{qB}, \end{aligned}$$

where p is the momentum of the particle, and m is the mass of the particle. This equation makes sense. If the particle has a higher momentum, then its circle of motion will have a larger radius. A stronger magnetic field strength, or a larger charge, will make the radius smaller.

In a cloud chamber, particles enter a magnetic field, and also a liquid which they ionise. This ionisation causes the paths of the particles to become visible. When the particle loses its charge, its track ceases. When the particle loses momentum, the radius of the circle decreases, and so, particles spiral inwards. The direction of this spiralling depends on the direction of the magnetic field. If the direction of the magnetic field causes a positively charged particle to spiral clockwise, then it will cause a negatively charged particle to spiral anticlockwise. Cloud chambers can, therefore, be used to identify particles by their charge and mass.



A cloud chamber without a magnetic field, so the particles move in straight lines.

Mass Spectrometers

Mass spectrometers work on a similar principle. Particles to be identified (such as nuclei) are accelerated using an electric field. Then, a velocity selector is used to ensure all the nuclei are at a known velocity - all the rest are discarded. These nuclei enter an uniform magnetic field where they move in a circle. However, they are only allowed to move half a circle, since they are collected at this point, and the number of particles arriving at each point is measured.

Velocity Selector

In the velocity selector, both an uniform electric field and an uniform magnetic field act on the particle. The only way a particle can travel through the velocity selector in a straight line is if the electric force on it is equal and opposite to the magnetic force on it. If this is not the case, the particle's path is bent, and so it does not get out of the velocity selector into the rest of the mass spectrometer. If we equate these two forces, we get:

$$qE = qvB ,$$

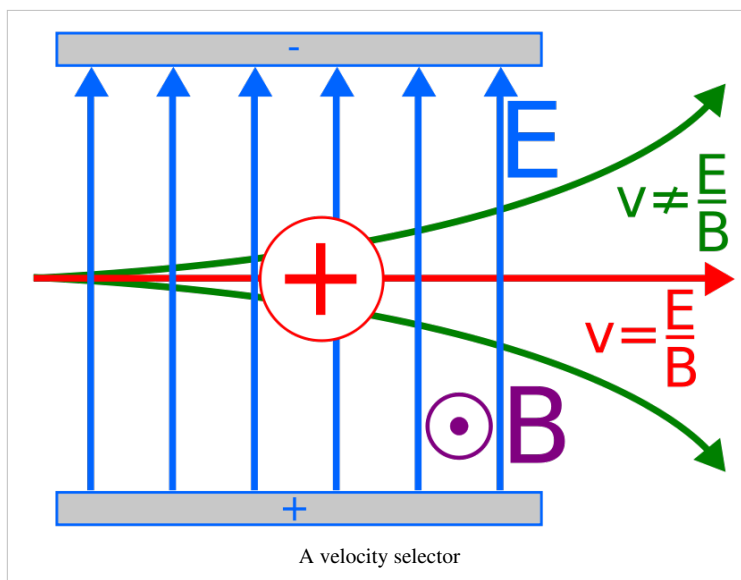
where q is the charge on the particle, E is the strength of the uniform electric field, v is the velocity of the particle, and B is the strength of the uniform magnetic field. The charge may be eliminated from both sides:

$$E = vB$$

Therefore:

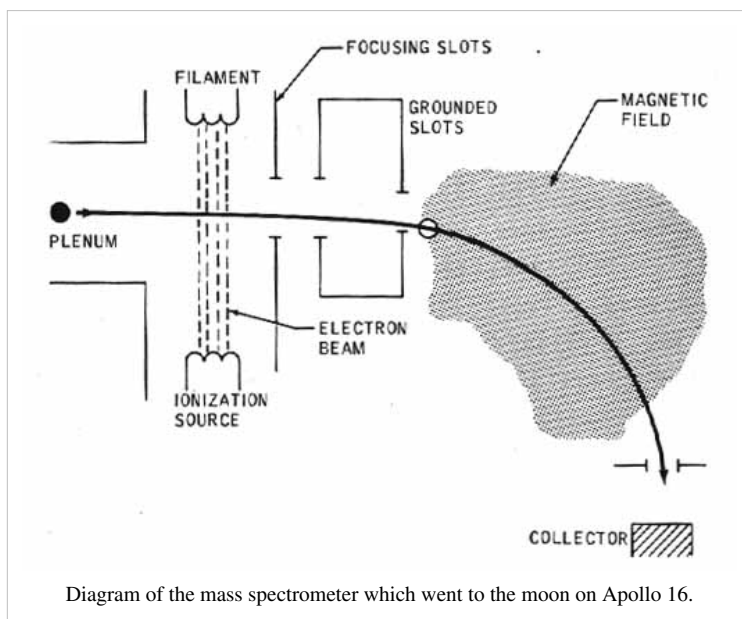
$$v = \frac{E}{B}$$

This means that, by adjusting the strengths of the electric and magnetic fields, we can choose the velocity at which particles emerge from the velocity selector.



Finding Mass

The particles then move at a speed v into another uniform magnetic field. Here, as in the cloud chamber, the radius of the circle in which the particle moves is given by:



$$r = \frac{mv}{qB} = \frac{mE_{selector}}{qBB_{selector}}$$

If we know the charge on the particle (for example, we know what element it is), we can measure the radius of the circle, and find the mass of the particle (ie. what isotope it is, since neutrons have no charge) using the formula:

$$m = \frac{qBr}{v} = \frac{qrBB_{selector}}{E_{selector}}$$

If we do not know the charge, then we can find the mass to charge ratio:

$$\frac{m}{q} = \frac{rBB_{selector}}{E_{selector}}$$

Questions

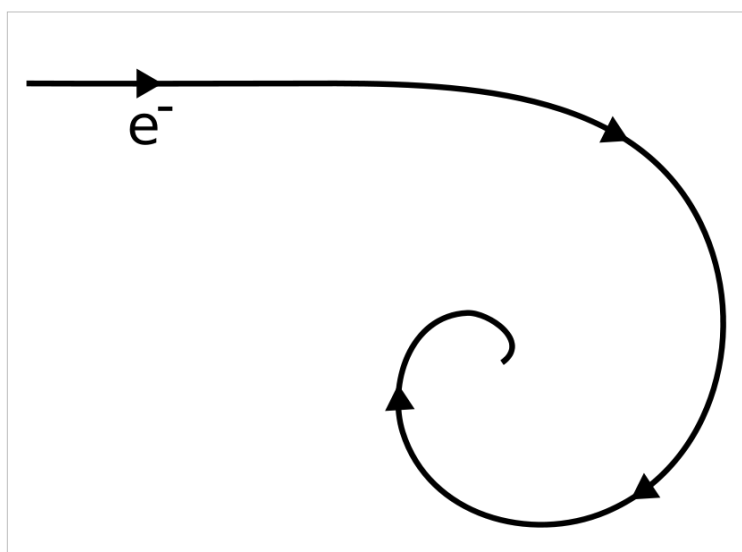
Charge of electron = $-1.6 \times 10^{-19} \text{C}$

Mass of electron = $9.11 \times 10^{-31} \text{kg}$

$u = 1.66 \times 10^{-27} \text{kg}$

1. An electron enters a cloud chamber, passing into a 0.1T magnetic field. The initial curvature (the reciprocal of its radius) of its path is 100m^{-1} . At what speed was it moving when it entered the magnetic field?

2. The electron spirals inwards in a clockwise direction, as show in the diagram on the right. What would the path of a positron, moving with an identical speed, look like?



3. Using a 2T magnetic field, what electric field strength must be used to get a velocity selector to select only particles which are moving at 100ms^{-1} ?

4. Some uranium (atomic number 92) ions (charge $+3e$) of various isotopes are put through the velocity selector described in question 3. They then enter an 0.00002T uniform magnetic field. What radius of circular motion would uranium-235 have?

/Worked Solutions/

Nuclear Physics

Quantum Principles

There are two principles which you do not need to know for the exam, but may be helpful in understanding some of the concepts in the course.

Heisenberg Uncertainty Principle

The Heisenberg uncertainty principle states that the momentum and position of an object are limited. Within a certain uncertainty, when we measure a quantum's position, it does not have a definite momentum. When we measure its momentum, it ceases to have a definite position. If we try and measure both, the uncertainty in both will be limited. If we let the uncertainty in our knowledge of momentum be Δp , and the uncertainty in our knowledge of position be Δx :

$$\Delta x \Delta p = \frac{h}{4\pi},$$

where h is Planck's constant (6.63×10^{-34} Js). The Heisenberg uncertainty principle explains what happens when electrons occupy energy levels - within these levels, they are limited to a certain range of momentums and positions, but it is meaningless to say which exact momentum and position they occupy.

Pauli Exclusion Principle

The Pauli exclusion principle states that no two particles may occupy the same quantum state as each other. In layman's inaccurate terms, this means that, although two particles can be in the same place as each other, if they are, they will be moving at different velocities and so will shortly no longer be in the same place as each other.

This is why, for example, electrons appear to have 'shells' - there is only a limited number of quantum states that the electrons can occupy, so some have to occupy a different 'shell'. Also, without the Pauli exclusion principle, matter would collapse in on itself - the attractive forces between particles are greater than the repulsive forces. However, the moment they try and do this, then they must be moving at different velocities, and so no longer be collapsing in on each other.

Radioactive Emissions

'Radioactivity' is a catch-all term for several different emissions from the nuclei of 'radioactive' atoms. There are three main types of radiation: alpha (α), beta (β) and gamma (γ). When radiation occurs, four things must be conserved:

- Mass
- Charge
- Lepton Number
- Baryon Number

In formulae, mass and charge are shown next to the symbol of the particle. For example, a neutron with mass 1u and no charge is 1_0n . The charge on a nucleus is equal to the number of protons in the nucleus (electrons can be ignored). The lepton and baryon numbers may be obtained by counting the number of leptons and baryons on either side of the equation, remembering that antiparticles have negative lepton and baryon numbers.

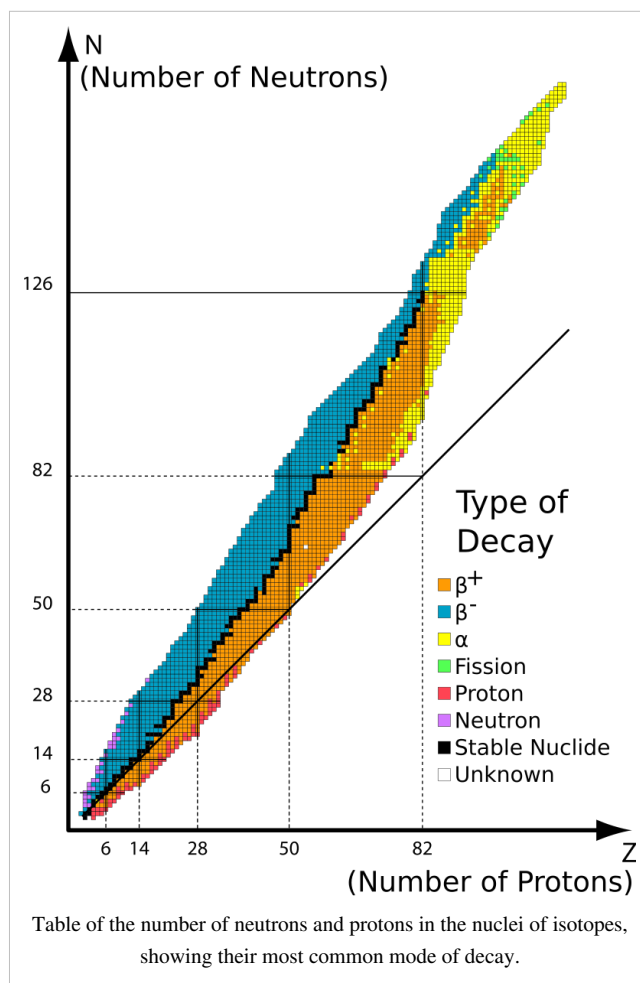
α Radiation

Unstable nuclei with a mass greater than 82u emit α radiation. This consists of an Helium nucleus (4_2He). The alpha particle simply splits off from the nucleus. Since the particle has no electrons, it has a charge of +2e. This, combined with its relatively large mass, means that it reacts easily with other particles, ionising them, meaning that it cannot penetrate more than a few centimetres of air.

β Radiation

Unstable nuclei with a mass below 82u emit β radiation. There are two types of β radiation. β^- radiation consists of an electron (${}^0_{-1}e$). This is produced by nuclei with many more neutrons than protons. A neutron changes into a proton, emitting an electron and an antineutrino in order to balance the lepton number. β^+ radiation consists of a positron (0_1e). This is produced by nuclei with roughly the same number of neutrons as protons. A proton changes into a neutron, emitting a positron and a neutrino.

β particles also ionise particles, but since they have less charge and mass, they do this less easily, and so they travel further (on average). Both α and β radiation result in the nucleus which emitted them being changed into another element.



γ Radiation

The binding energies of nuclei are quantized - they can only take on certain values. When a nucleus jumps down an energy level, this energy has to go somewhere - it takes the form of a γ photon. The structure of the nucleus is not changed by γ radiation. γ radiation is ionising, but only at the right frequency - the resonant frequency of the things it ionises. γ radiation travels very far, and only a good thick layer of lead can stop it.

Questions

You will need a periodic table.

1. Americium-241 is an α emitter. What element, and what isotope, is produced by this decay?
2. Iodine-129 is a β^- emitter. What element, and what isotope, is produced by this decay?
3. Gamma rays are used to kill microbes in food. Why doesn't the food become radioactive?
4. Plutonium-244 decays by emitting an α particle. It does this twice, emits a β^- particle, and then emits a further two α particles. The nucleus becomes a different element each time. What element is produced at the end?
5. Carbon-11 changes into Boron-11 by a radioactive emission. What was emitted?

/Worked Solutions/

Energy Levels

As an electron approaches a nucleus from infinity, it becomes 'bound' - it is attached to the nucleus, if you like. In this bound state, the electron occupies what is called an energy level. A nucleus has a discrete number of energy levels, and so electrons bound to a certain nucleus can only take on certain potential energies. These energies are negative by convention.

The lowest (most negative) energy level is denoted $n=1$, the next lowest $n=2$, and so on. The values of these can be found using formulae which you don't need to know about. Alternatively, they may be determined experimentally.

At random, electrons jump between energy levels. If they jump to a lower energy level (more negative), they release energy in the form of a photon. If they jump to a higher energy level, they must absorb a photon of the appropriate energy. The energies of these photons can be calculated using the following formulae, which you should already know from AS:

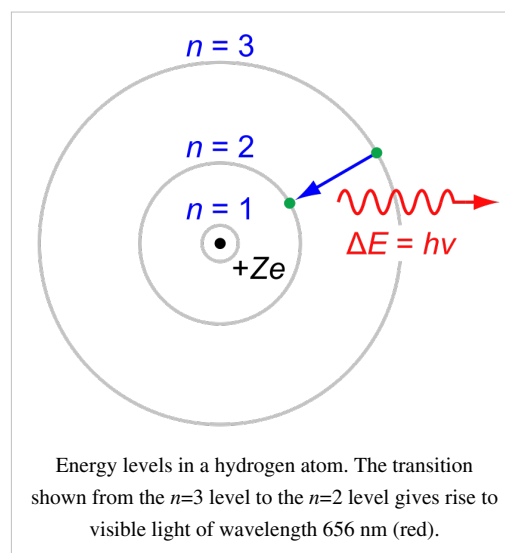
$$E = hf$$

$$c = \lambda f,$$

where E is energy, h is Planck's constant (6.63×10^{-34} J s), f is frequency, c is the speed of light, and λ is wavelength.

The energy levels of different nuclei are different. Evidence for these energy levels comes from the emission and absorption spectra of atoms. An emission spectrum can be obtained by heating a sample of an element. This gives the electrons energy, so they jump up the energy levels. At random, they then jump down again, giving off photons with measurable frequencies. The formulae above can be used to calculate the difference in energy between the levels between which the electrons have jumped.

An absorption spectrum can be found by passing light through (for example) a gas, and observing the frequencies of light which are absorbed. These frequencies correspond to jumps between energy levels which electrons have



undergone when they absorb the photons, gaining energy.

It should be noted that electrons do not always jump to the next-door energy level - they can, in principle, jump to any energy level. They cannot jump to an energy which is not that of an energy level.

Questions

The following table gives the wavelengths of light given off when electrons change between the energy levels in hydrogen as described in the first row:

Transition of n	3→2	4→2	5→2	6→2	7→2	8→2	9→2	∞ →2
Wavelength (nm)	656.3	486.1	434.1	410.2	397.0	388.9	383.5	364.6
Color	Red	Blue-green	Violet	Violet	(Ultraviolet)	(Ultraviolet)	(Ultraviolet)	(Ultraviolet)

1. Calculate the potential energy of an electron at level $n=2$.
2. Calculate the difference in potential energy between levels $n=2$ and $n=3$.
3. What is the potential energy of an electron at level $n=3$?
4. If an electron were to jump from $n=7$ to $n=5$, what would the wavelength of the photon given off be?

/Worked Solutions/

Fission

Nuclear fission is the splitting of the nucleus of a massive atom into smaller nuclei. This is used to produce energy in power stations and nuclear bombs.

Chain Reaction

In order to start nuclear fission, one nucleus must be made to split apart. This is achieved by getting the nucleus to absorb a slow-moving neutron. When the nucleus splits, it releases energy, two components, and possibly some more neutrons. If at least one neutron is released, then a chain reaction occurs. This neutrons goes on to make another nucleus unstable, which splits, and produces more neutrons, and so on.

If this chain reaction is uncontrolled, a massive amount of energy is released very fast. This is an atomic explosion, which is used in nuclear bombs. In order to use nuclear fission in a power station, the number of neutrons released must be controlled by inserting a substance such as boron into the reactor, which absorbs the neutrons, preventing them from going on to make more nuclei split.

Binding Energy

The reason nuclear fission produces energy is that the binding energy of the original nucleus is greater than the binding energy of the products of the fission reaction. This difference in binding energy is the amount of energy released as photons (some of which are infra-red). This energy is used to heat up steam, pressurizing it, and enabling it to turn a turbine, producing electricity.

Neutron Moderator

Neutrons have to be moving slowly in order to cause a nucleus to become unstable and split. If they are moving too fast, then they simply bounce off. A neutron moderator (such as graphite or heavy water) is used to slow them down.

Questions

1. A neutron is fired at some Uranium-235. Barium-141 and Krypton-92 are produced:



How many neutrons are produced (ie. what is the value of N)?

2. What proportion of the neutrons produced must be absorbed in order to make the reaction stable?

3. What would happen if too many neutrons were absorbed?

4. Alternatively, Uranium-235 can split into Xenon-140, two neutrons and another element. What is this element? (You will need to use a periodic table.)

/Worked Solutions/

Fusion

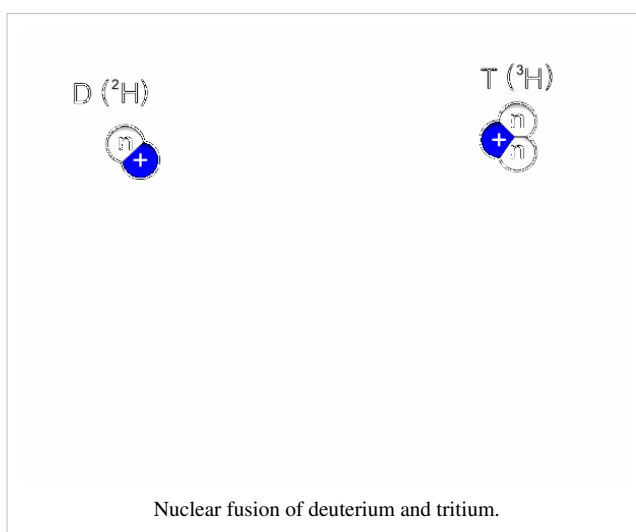
Nuclear fusion is the joining together of atomic nuclei to form a larger nucleus, and possibly some other products, including energy. It occurs naturally in stars, where hydrogen is fused together into larger isotopes of hydrogen and then into helium, releasing energy along the way.

Forces

Nuclei repel each other due to the electromagnetic force, since they have the same charge. In order for two nuclei to fuse, they must be sufficiently close enough together that the attractive force between the baryons due to the strong nuclear force is greater than the repulsive force due to the electromagnetic force. If this is the case, then the two nuclei will become a new, larger, nucleus.

Uses

Nuclear fusion was used by humans for the first time in the hydrogen bomb, since the fusion of hydrogen produces more energy than nuclear fission. At the time of writing (2009), commercially viable fusion power has not yet been achieved. However, research is under way to bring a fusion reaction under control so that it can be used to generate electricity. This would have the advantage of minimal nuclear waste, since the main product would be non-radioactive helium, with some tritium, which has a relatively short 12-year half-life.



Binding Energy

The fusion of nuclei smaller than Iron-56 releases energy. This is because, if we were to take all the baryons of both the nuclei apart, and then stick them all back together as one, we would do less work than would be required to stick them together as the two separate nuclei. The difference in binding energy is the energy which is released by a fusion reaction. This energy might be given to the 'real' particles which are given off, or to a 'virtual' particle such as a photon.

Questions

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

1. In the Sun, two tritium nuclei (${}^3_1\text{H}$) are fused to produce helium-4 (${}^4_2\text{He}$). What else is produced, apart from energy?
2. In larger stars, carbon-12 (${}^{12}_6\text{C}$) is fused with protium (${}^1_1\text{H}$). What single nucleus does this produce?
3. In this reaction, 1.95MeV of energy is released. What difference in binding energy does this correspond to?
4. If all this energy was emitted as a photon, what would its frequency be?
5. In order to contain a fusion reaction, electromagnetism may be used. What other force could be used? Why is this not being used for fusion reactors on Earth?

/Worked Solutions/

Binding Energy

It takes energy to pull nuclei apart. The amount of work (energy) which must be done in order to pull all of the neutrons and protons in a nucleus infinitely far apart from each other is known as the binding energy of the nucleus. Practically, pulling them all apart far enough to stop them interacting will do.

If energy must be put in to a nucleus to break it apart, where does this energy go? The answer lies in the fact that if you add up the masses of all the protons and neutrons in a nucleus individually, it is a little bit more than the actual mass of the nucleus. The binding energy put in to break the nucleus apart has 'become' mass in the individual baryons. So, the binding energy of a nucleus can be calculated using the following formula:

$$E_b = (n_N m_N + n_Z m_Z - M)c^2,$$

where n_N and n_Z are the numbers of neutrons and protons in the nucleus, m_N and m_Z are the masses of neutrons and protons, M is the mass of the nucleus and c is the speed of light ($3 \times 10^8 \text{ ms}^{-1}$).

The Unified Atomic Mass Unit

The unified atomic mass unit, denoted u , is roughly equal to the mass of one proton or neutron. $1 u = 1.660538782 \times 10^{-27} \text{ kg}$. They are useful since 1 mole of atoms with a mass of 1 u each will weigh exactly 1 gram. However, when dealing with binding energy, you must **never** use atomic mass units in this way. The mass defect is so small that using atomic mass units will result in a completely wrong answer. If you want to use them with lots of decimal places, then you will save writing in standard form.

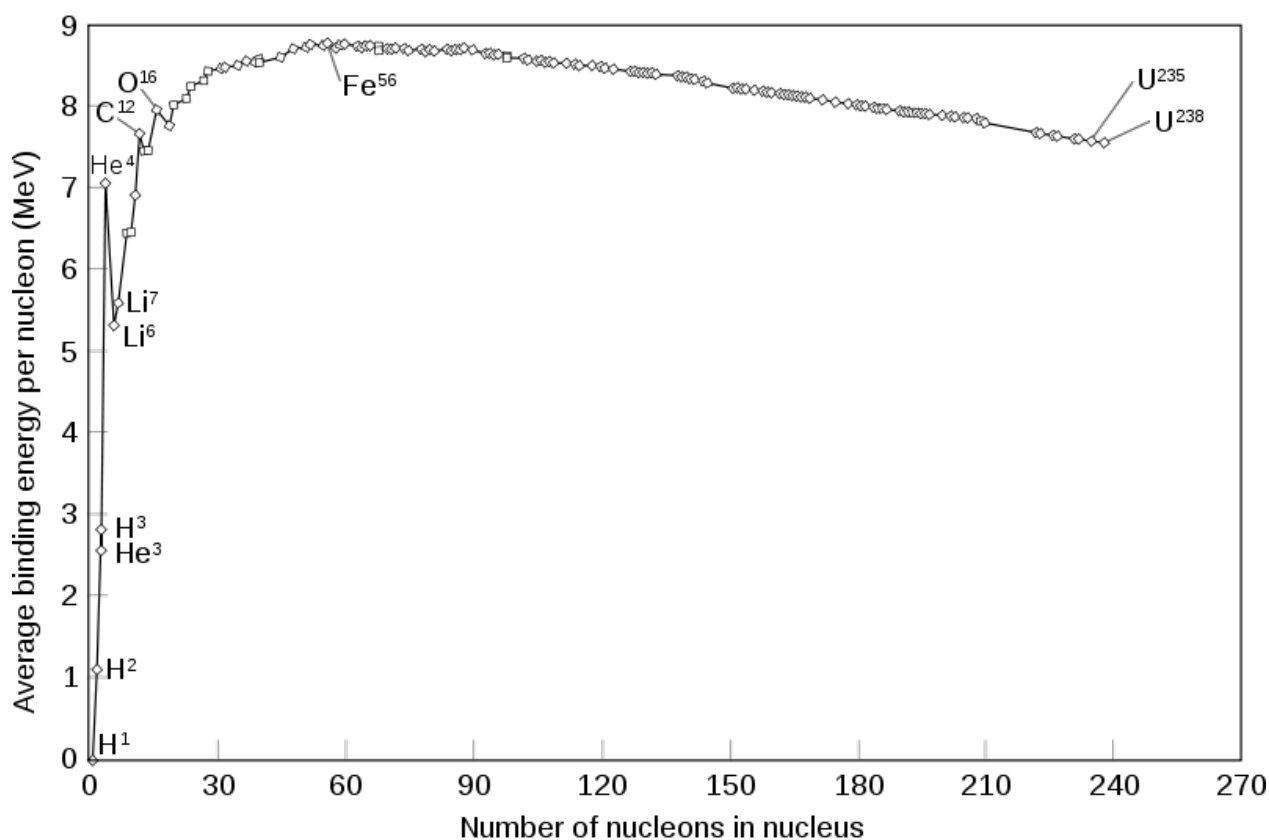
Data

The following table gives the masses in kg and u of the proton and the neutron:

Name	Mass (kg)	Mass (u)
Proton	$1.67262164 \times 10^{-27}$	1.00727647
Neutron	$1.67492729 \times 10^{-27}$	1.00866492

The Binding Energy Curve

Different nuclei have different binding energies. These are determined by the combination of protons and neutrons in the nucleus. These are shown in the following graph:



The position of Iron-56 at the top is important. If you take two nuclei completely apart, you do work. If you then put all the baryons back together again as one nucleus, you will get energy back out. Sometimes, the energy you get back out will be more than the work you had to do to take the nuclei apart. Overall, you release energy by fusing the nuclei together. This happens to nuclei which are smaller than Iron-56. Nuclei which are larger than Iron-56 will give out less energy when fused than you had to put in to take them apart into their constituent baryons in the first place. To the right of Iron-56, nuclear fusion, overall, requires energy.

If you take only one nucleus apart you still do work. If you stick its protons and neutrons back together, but this time in two lumps, you will get energy out. Again, sometimes this energy will be greater than the work you had to do to take them apart in the first place. Nuclear fission will be releasing energy. This occurs when the nucleus is larger than Iron-56. If the energy released is less than the initial work you put in, then nuclear fission, overall, requires energy. This happens when the nucleus is smaller than Iron-56.

This can be summarized in the following table:

Type of Nucleus	Nuclear Fusion	Nuclear Fission
smaller than Iron-56	releases energy	absorbs energy
Iron-56	≈ no energy change	≈ no energy change
Larger than Iron-56	absorbs energy	releases energy

Questions

1. Deuterium (an isotope of Hydrogen with an extra neutron) has a nuclear mass of 2.01355321270 u. What is its binding energy?
2. Uranium-235 has a nuclear mass of 235.0439299 u. It contains 92 protons. What is its binding energy?
3. How would you expect H-2 and U-235 to be used in nuclear reactors? Why?

/Worked Solutions/

Risks, Doses and Dose Equivalents

Risk

Radioactivity results in risk - this could be a risk of death, or a risk of developing cancer. In physics, risk is what you expect, on average, to happen:

$$\text{risk} = \text{probability} \times \text{consequence}$$

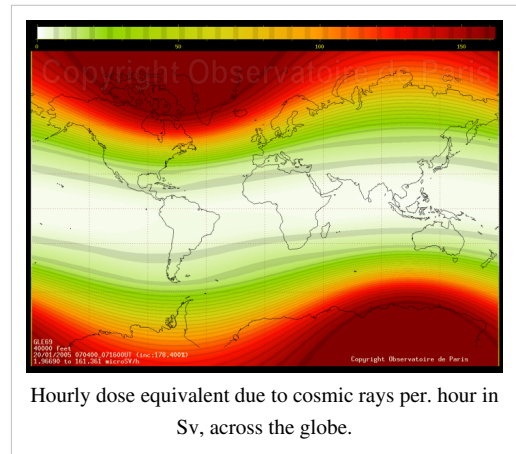
So, if there is a 1 in 500 chance that someone gets run over by a car when crossing the road, the risk involved in allowing 500 people to cross the road is one death.

Absorbed Dose

Absorbed dose is measured in grays, commonly denoted Gy. One gray is defined as one joule absorbed per kilogram. You may be expected to use the equation $E = mc^2$ to calculate absorbed dose in terms of numbers of particles with a given mass. If someone is exposed to a certain activity (particles per second) over a period of time, the absorbed dose accumulates.

Dose Equivalent

Absorbed dose does not give a full picture of the potential harm radioactivity can do to you. Different types of radiation do different amounts of damage. Absorbed dose equivalent, measured in sieverts (denoted Sv) attempts to compensate. To calculate the dose equivalent, multiply the dose in grays by the quality factor of the particles absorbed. These quality factors are given in the table below.



Radiation Type	Quality Factor
photons and leptons	1
neutrons with an energy > 10 MeV or < 10keV, protons	5
neutrons with an energy between 10 and 100 keV, or between 2 and 20 MeV	10
neutrons with an energy between 100 keV and 2 MeV and atomic nuclei	20

Questions

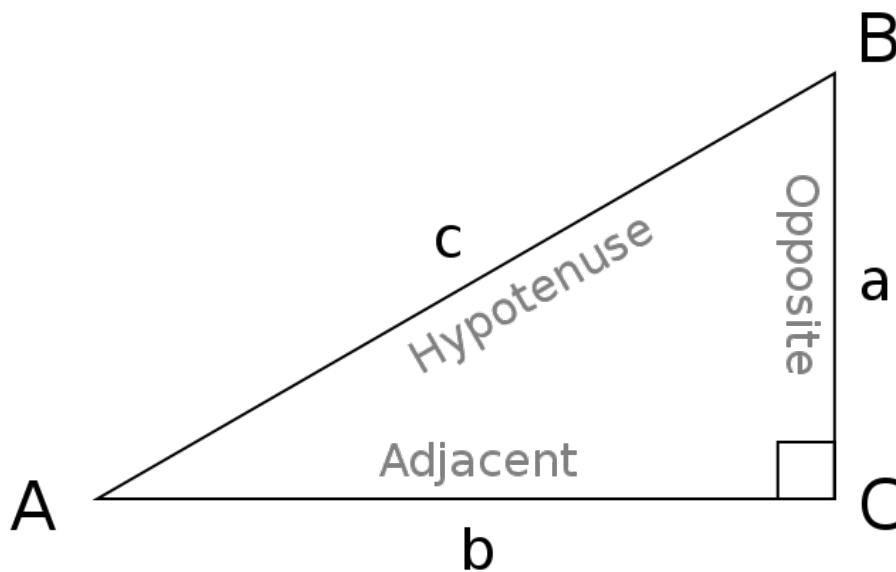
1. A mobile phone emits electromagnetic radiation. 1.2 watts of power are absorbed per. kilogram. Assuming that the radiation is absorbed uniformly across a 5kg head, what dose of radiation would be delivered to the head when making a 10-minute telephone call?
2. What dose equivalent does this correspond to?
3. How many nuclei are there in 1 mg of Americium-241?
4. An ham sandwich becomes contaminated with 1 μg of Americium-241, and is eaten by an 80kg person. The half-life of Americium-241 is 432 years. Given that Americium-241 gives off 5.638 MeV alpha particles, how long would it be before a dose equivalent of 6 Sv is absorbed, making death certain?
5. What assumptions have you made?

/Worked Solutions/

Appendices

Trigonometry

Trigonometry is the study of the ratios between sides on a right-angled triangle. Consider the following diagram:



Depending on the value of the angle BAC, the ratios between the three sides of the triangle will vary. These three sides are generally given the names 'hypotenuse' (H), 'opposite' (O) and 'adjacent' (A). The opposite side is opposite the angle being considered. The hypotenuse is the side opposite the right angle, and the adjacent side is the remaining side, which is also next to the angle being considered.

The three basic trigonometric ratios, in relation to an acute angle θ , are the sine, cosine and tangent ratios, as follows:

$$\sin \theta = \frac{O}{H}$$

$$\cos \theta = \frac{A}{H}$$

$$\tan \theta = \frac{O}{A}$$

It is often helpful to remember them by using the mnemonic 'SOH-CAH-TOA'.

These ratios can be calculated using the appropriate buttons on a scientific calculator. In addition, if you already know the ratio between two sides, and wish to calculate the angle from it, you can use the \sin^{-1} , \cos^{-1} and \tan^{-1} buttons to use the inverse function. These functions are technically known as arcsin, arccosin and arctan.

Logarithms

That formula involved logarithms. If $a^b = c$, then:

$$\log_a c = b$$

In other words, a logarithm is a way of asking the question, "a to the power of what is c?" a is known as the base of the logarithm. log a is shorthand for $\log_{10} a$, and can be calculated using the 'log' button on a scientific calculator.

Just as with indices, there are some laws of logarithms. At present, we are only concerned with this one:

$$\log_a (c^n) = n \log_a c$$

If we apply this to the formula for calculating the number of possible values a pixel can take on ($v = 2^b$), we can rearrange it to get a formula for b by taking the logarithm to the base 10 of both sides:

$$\log v = \log(2^b)$$

$$\log v = b \log 2$$

$$b = \frac{\log v}{\log 2}$$

We can then use a scientific calculator to calculate $\log 2$, giving us the formula:

$$b \approx \frac{\log v}{0.3}$$

Delta - 'difference in'

The Greek letter delta means 'difference in'. So, in the case of current, it represents the difference in charge divided by the difference in time. If we write it as we would in calculus:

$$I = \frac{dQ}{dt}$$

Without using calculus, we can only take an approximation of I from values as close together as possible on either side of the point at which we wish to calculate the current. By using calculus, we can calculate the exact value of the current at a given point. For example, if $Q = 2t$ (ie. 1 coulomb every 2 seconds):

$$I = \frac{dQ}{dt} = 1 \times 2t^{1-1} = 2t^0 = 2 \text{ A}$$

Sigma - 'sum of'

That's the Greek letter Sigma. It means 'sum of', so Kirchoff's First Law is stating that the sum of all the currents going into a point equals the sum of all the currents leaving the point. So:

$$I_{in1} + I_{in2} + I_{in3} + \dots + I_{inn-2} + I_{inn-1} + I_{inn} = I_{out1} + I_{out2} + I_{out3} + \dots + I_{outn-2} + I_{outn-1} + I_{outn}$$

Derivation of Equations for Simple Harmonic Motion

The following second-order differential equation describes simple harmonic motion:

$$\frac{d^2x}{dt^2} = \frac{-kx}{m}$$

$$\frac{d^2x}{dt^2} + \frac{kx}{m} = 0$$

Since we have a second derivative, we cannot separate the variables, so let:

$$x = e^{zt}$$

Therefore:

$$\frac{dx}{dt} = ze^{zt}$$

$$\frac{d^2x}{dt^2} = z^2e^{zt}$$

By substitution:

$$z^2e^{zt} + \frac{ke^{zt}}{m} = 0$$

$$e^{zt} \left(z^2 + \frac{k}{m} \right)$$

e^{zt} is asymptotic at 0, so e^{zt} cannot equal 0, and we can therefore get away with dividing by e^{zt} :

$$z^2 + \frac{k}{m} = 0$$

$$z^2 = \frac{-k}{m}$$

$$z = \pm \sqrt{\frac{-k}{m}} = \pm i \sqrt{\frac{k}{m}}$$

Therefore:

$$x = Pe^{it\sqrt{\frac{k}{m}}} + Qe^{-it\sqrt{\frac{k}{m}}},$$

where P and Q are constants of integration. At this point, it is useful to clean things up a bit by letting:

$$\omega^2 = \frac{k}{m}$$

$$x = Pe^{i\omega t} + Qe^{-i\omega t}$$

It has been proven elsewhere (de Moivre's Theorem) that, when n is a constant:

$$e^{ni\theta} = \cos n\theta + i \sin n\theta \text{ and } e^{-ni\theta} = \cos n\theta - i \sin n\theta$$

Therefore:

$$x = P(\cos \omega t + i \sin \omega t) + Q(\cos \omega t - i \sin \omega t)$$

$$x = (P + Q) \cos \omega t + (P - Q)i \sin \omega t$$

Let: $R = P + Q$

$$S = P - Q$$

So, the general solution of the differential equation is:

$$x = R \cos \omega t + Si \sin \omega t$$

This describes what the simple harmonic oscillator will do given any possible situation. However, we don't want an equation which will cover anything and everything. We want to give our oscillator a starting position - let's say, at a position where $x = A$ at $t = 0$:

$$A = R \cos (\omega \times 0) + Si \sin (\omega \times 0)$$

$$A = R \cos 0 + Si \sin 0$$

Therefore, $R = A$ and $S = 0$.

So, the specific solution is:

$$x = A \cos \omega t$$

Worked Solutions

Lenses

1. A lens has a focal length of 10cm. What is its power, in dioptres?

Always use SI units, so 10cm = 0.1m.

$$P = \frac{1}{f} = \frac{1}{0.1} = 10 \text{ D}$$

2. Light reflected off a cactus 1.5m from a 20D lens forms an image. How many metres is it from the other side of the lens?

u must be negative, as per. the Cartesian convention, so $u = -1.5$.

$$\frac{1}{v} = \frac{1}{-1.5} + 20 = \frac{58}{3} \text{ D}$$

$$v = \frac{3}{58} = 0.0517 \text{ m} = 51.7 \text{ mm}$$

3. A lens in an RGB projector causes an image to focus on a large screen. What sort of lens is it? Is its power positive or negative?

The wavefronts are being caused to spread out (diverge) more by the lens. Hence, it is a diverging lens. The wavefronts are losing curvature, so the lens has a negative power.

4. What is the focal length of a 100D lens?

$$100 = \frac{1}{f}$$

$$f = \frac{1}{100} = 0.01 \text{ m} = 10 \text{ mm}$$

5. The film in a camera is 5mm from a lens when automatically focussed on someone's face, 10m from the camera. What is the power of the lens?

$$5\text{mm} = 0.005\text{m}$$

$$\frac{1}{0.005} = \frac{1}{-10} + P$$

$$200 = P - 0.1$$

$$P = 200.1 \text{ D}$$

Since lenses are generally made to have nice round powers, this was probably a 200D lens. However, this is in real life, and this an hypothetical question with numbers picked out of thin air by me.

6. The light from a candle is enlarged by a factor of 0.5 by a lens, and produces an image of a candle, 0.05m high, on a wall. What is the height of the candle?

$$0.5 = \frac{0.05}{h_1}$$

$$2 = \frac{h_1}{0.05}$$

$$h_1 = 100 \text{ mm}$$

Refraction

1. A ray of light is reflected from a mirror. Its angle to the normal when it reaches the mirror is 70° . What is its angle of reflection?

The angles of reflection on both sides of the normal are always equal, so the angle of reflection is 70° .

2. The speed of light in diamond is 1.24×10^8 m/s. What is its refractive index?

$$n = \frac{v_1}{1.24 \times 10^8}$$

Assuming the speed of light in the reference medium is 3×10^8 :

$$n = \frac{3 \times 10^8}{1.24 \times 10^8} = \frac{3}{1.24} \approx 2.42$$

3. The refractive index of ice is 1.31. What is the speed of light in ice?

$$1.31 = \frac{3 \times 10^8}{v_2}$$

$$v_2 = \frac{3 \times 10^8}{1.31} \approx 2.29 \times 10^8 \text{ ms}^{-1}$$

4. A ray of light passes the boundary between air and a transparent material. The angle of reflection is 20° , and the angle of incidence is 10° . What is the speed of light in this material? Why is it impossible for this material to exist?

$$\frac{3 \times 10^8}{v_2} = \frac{\sin 10}{\sin 20} \approx 0.508$$

$$v_2 \approx \frac{3 \times 10^8}{0.508} \approx 5.91 \times 10^8$$

This is greater than the speed of light in a vacuum (3×10^8), and so is impossible.

5. What is the critical angle of a beam of light leaving a transparent material with a refractive index of 2?

$$\sin C = \frac{1}{2}$$

$C = 30^\circ$ (using the \sin^{-1} button on a scientific calculator - this means 'inverse sine', often denoted arcsine or asin for short.)

Digital Storage

1. An image transmitted down a SVGA video cable is 800 pixels wide, and 600 pixels high. How many pixels are there in the image?

$$800 \times 600 = 480000 \text{ pixels}$$

2. A grayscale image is encoded using 3 bits. How many possible values can each pixel have?

$$2^3 = 8$$

3. The characters in a text document are numbered from 0 - 255. How many bits should each character be encoded with?

There are 256 possible values.

$$b = \frac{\log 256}{\log 2} = 8$$

4. A page contains 30 lines of text, with an average of 15 characters on each line. Each character is represented by 4 bits. How many megabytes of uncompressed storage will a book consisting of 650 pages like this fill on a computer's hard disk?

$$\text{Total number of characters} = 650 \times 30 \times 15 = 292\,500$$

$$\text{Total amount of information} = 292500 \times 4 = 1\,170\,000 \text{ bits}$$

$$1170000 \text{ bits} = \frac{1170000}{8} \text{ bytes} = 146250 \text{ bytes} = \frac{146250}{1024^2} \text{ Mbytes} \approx 0.14 \text{ Mbytes}$$

5. An 10cm wide square image is scanned into a computer. Each pixel is encoded using 3 channels (red, green and blue), and each channel can take on 256 possible values. One pixel is 0.01 mm wide. How much information does the scanned image contain? Express your answer using an appropriate unit.

$$10\text{cm} = \frac{0.1}{0.01 \times 10^{-3}} \text{ pixels} = 10000 \text{ pixels}$$

$$\text{Total number of pixels} = 10\,000^2 = 100\,000\,000 \text{ pixels}$$

$$b = \frac{\log 256}{\log 2} = 8 \text{ bits per. channel per. pixel} = 8 \times 3 \text{ bits per. pixel} = 24 \text{ bits per. pixel}$$

$$\text{Total information} = 24 \times 100\,000\,000 = 2\,400\,000\,000 \text{ bits} = 300\,000\,000 \text{ bytes}$$

$$\frac{300000000}{1024^2} \approx 286 \text{ Mbytes (This is why we usually compress images before storage, or at least use fewer bits per. pixel.)}$$

Digital Processing

1. How could the above methods be applied to a digital sound sample?

By taking the median or mean of the sample and the values on either side of it. This would be unreliable, as sound is a wave, so the samples need to vary quite widely. However, something similar could be used when comparing repeating patterns in a waveform.

2. Which of the above methods would be suitable for smoothing sharp edges? Why?

Mean smoothing - median smoothing would not blur the edges.

3. Use median smoothing to remove noise from the following image of a white cat in a snowstorm (the black pixels have a value of 255):

Before				After			
000	■	000	000	000	000	000	000
000	000	000	■	000	000	000	000
■	000	000	000	000	000	000	000
000	000	■	000	000	000	000	000

4. Why would mean sampling not be appropriate for smoothing the image given in question 3?

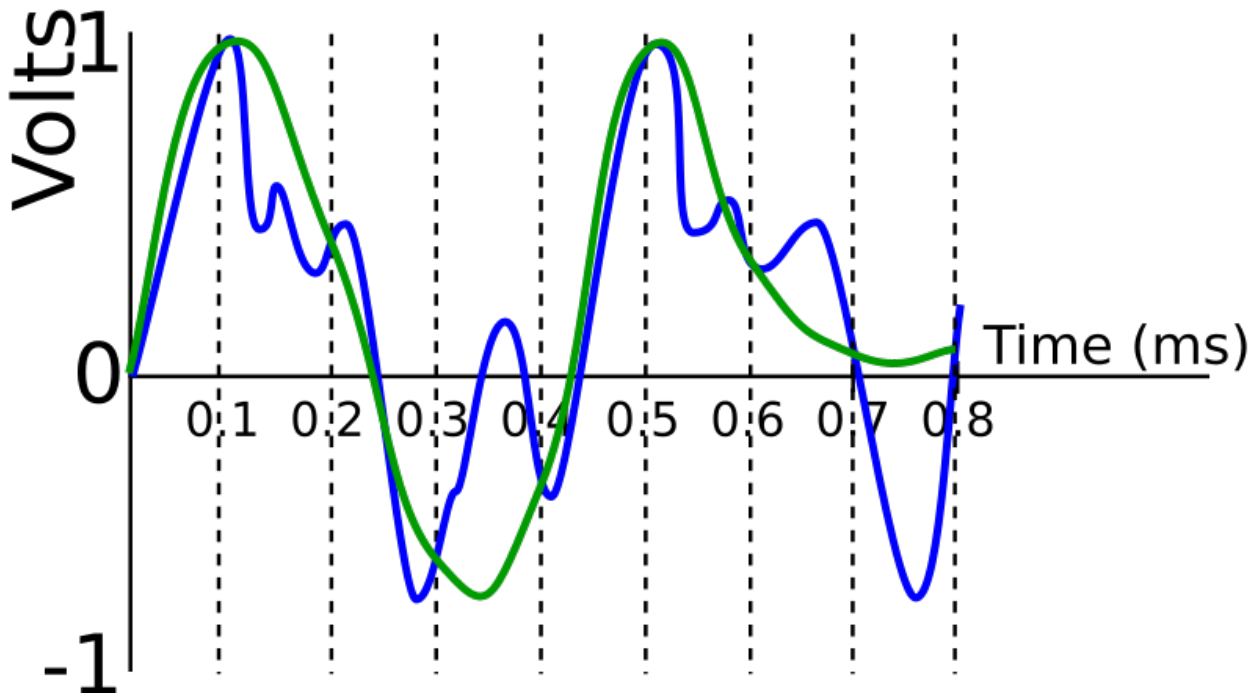
It would produce a really blurred mess, instead of an image, as the noise is too dense.

5. Use mean smoothing to remove noise from the following image of a black cat in a coal cellar:

Before	After		
000	213	213	191
	227	227	213
	213	213	191

Digitisation

1. Take samples for the signal below every 0.1ms, and then produce a reconstructed signal. How does it differ from the original?



The high frequency elements of the signal have been lost.

2. A signal is sampled for 5 seconds at a sampling rate of 20 kHz. How many samples were taken?

$$20 \times 10^3 = \frac{\text{No. of samples}}{5}$$

$$\text{No. of samples} = 20 \times 10^3 \times 5 = 100000$$

3. Most sounds created by human speech except for 'ss' and 'ff' have a maximum frequency of 4 kHz. What is a suitable sampling rate for a low-quality telephone?

$$4 \times 2 = 8 \text{ kHz}$$

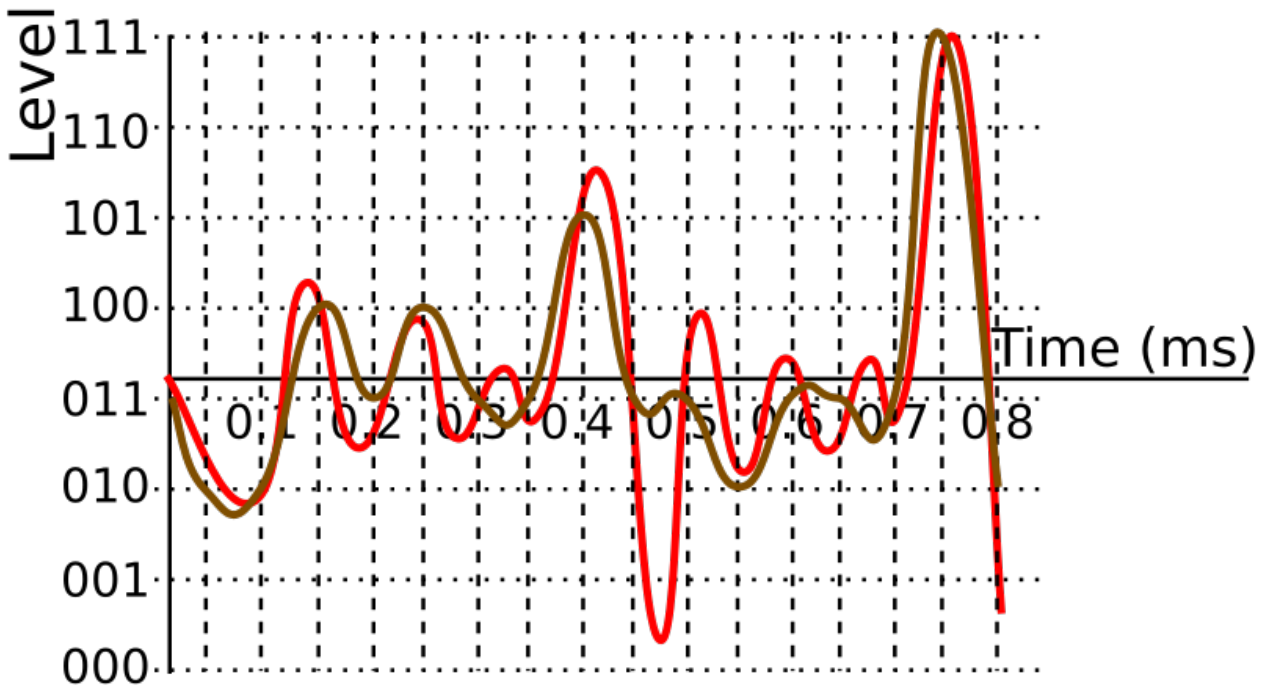
4. Using a sampling rate of 20 kHz and 3 bits, sample the following signal, and then produce a reconstructed signal. What is the maximum frequency that can be perfectly reconstructed using this sampling rate?

First, calculate the length of each sample, by letting the number of samples equal 1:

$$20 \times 10^3 = \frac{1}{\text{Length of one sample}}$$

$$\text{Length of one sample} = \frac{1}{20 \times 10^3} = 0.00005 \text{ s} = 0.05 \text{ ms}$$

Then, we can sample the waveform and create a reconstructed signal:



To calculate the maximum frequency that can be perfectly reconstructed using this sampling rate (20 kHz):

$$\frac{20}{2} = 10 \text{ kHz}$$

Signal Frequencies

1. What is the frequency of an X-ray (wavelength 0.5nm)?

X-rays are electromagnetic waves, so they travel at the speed of light ($3 \times 10^8 \text{ ms}^{-1}$).

$$3 \times 10^8 = f \times 0.5 \times 10^{-9}$$

$$f = \frac{3 \times 10^8}{0.5 \times 10^{-9}} = 600 \times 10^{15} \text{ Hz} = 600 \text{ PHz}$$

2. A sound wave, with a frequency of 44 kHz, has a wavelength of 7.7mm. What is the speed of sound?

$$v = 44 \times 10^3 \times 7.7 \times 10^{-3} = 338.8 \text{ ms}^{-1}$$

3. What is the fundamental frequency of the following signal?

The big spike on the left is at approximately 750 Hz, so this is the fundamental frequency.

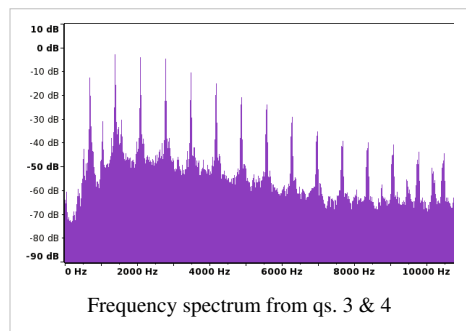
4. Approximately how many harmonics does it contain?

There are 14 other big spikes, plus a few other spikes which may be large enough to be harmonics.

5. The three sine waves $\sin x^\circ$, $4\sin(2x-50)^\circ$ and $0.5\sin(3x+120)^\circ$ are added together to form a signal. What are the frequencies of each of the waves? What is the signal's fundamental frequency? Assume that the waves are travelling at the speed of light, and that $60^\circ = 1\text{mm}$

$\sin x^\circ$ has a wavelength of 360° . Using this, we can calculate the wavelengths of the other two waves, since $f(ax)$ stretches $f(x)$ by the reciprocal of a on the x axis. The frequency of each wave is given by the formula:

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{\lambda \times 10^{-3}}$$



Wave (y=)	Wavelength (°)	Wavelength (mm)	Frequency (GHz)
$\sin x^\circ$	360	6	50
$4\sin(2x-50)^\circ$	180	3	100
$0.5\sin(3x+120)^\circ$	120	2	150

$\sin x^\circ$ has the lowest frequency, so 50 GHz is the fundamental frequency of the signal.

Bandwidth

1. A broadband internet connection has a bit rate of 8Mbit s^{-1} when downloading information. What is the minimum bandwidth required to carry this bit rate?

$$8 \times 10^6 = 2B$$

$$B = \frac{8 \times 10^6}{2} = 4 \times 10^6 \text{ Hz} = 4 \text{ MHz}$$

2. The same connection has a bandwidth of 100 kHz reserved for uploading information. What is the maximum bit rate that can be attained when uploading information using this connection?

$$b = 2 \times 100 \times 10^3 = 200 \times 10^3 \text{ bits / second} = 24.4 \text{ kbytes / second}$$

3. A lighthouse uses a flashing light and Morse Code to communicate with a nearby shore. A 'dash' consists of the light being on for 2s. The light is left off for 1s between dots and dashes. What is the bandwidth of the connection?

One bit takes 3 seconds to transmit.

$$B = \frac{1}{3} \text{ Hz} \approx 333 \text{ mHz}$$

4. The broadband connection in question two is used to upload a 1Mbyte image to a website. How long does it take to do this?

The bit rate is 200×10^3 bits / second.

$$1 \text{ Mbyte} = 1048576 \text{ bytes} = 8388608 \text{ bits.}$$

$$b = \frac{\text{no. of bits}}{\text{no. of seconds}}$$

$$200 \times 10^3 = \frac{8388608}{\text{no. of seconds}}$$

$$\text{no. of seconds} = \frac{8388608}{200 \times 10^3} \approx 42 \text{ s}$$

Charge

1. How much charge do 1234 electrons carry?

$$1234 \times 1.6 \times 10^{-19} = 1.9744 \times 10^{-16} \text{ C} = 197.44 \text{ aC (attocoulombs)}$$

2. How many electrons does it take to carry 5 C of charge?

$$5 = 1.6 \times 10^{-19} \times n$$

$$n = \frac{5}{1.6 \times 10^{-19}} = 31.25 \times 10^{18} \text{ electrons}$$

3. The total charge on 1 mole of electrons (6×10^{23} particles) is equal to 1 faraday of charge. How many coulombs of charge are equal to 1 faraday?

$$1 \text{ faraday} = 6 \times 10^{23} \times 1.6 \times 10^{-19} \text{ coulombs} = 96000 \text{ coulombs} = 96 \text{ kC}$$

Current

1. 10 coulombs flow past a point in a wire in 1 minute. How much current is flowing through the point?

$$I = \frac{10}{60} \approx 0.17 \text{ A}$$

2. How long does it take for a 2A current to carry 5C?

$$2 = \frac{5}{t}$$

$$t = \frac{5}{2} = 2.5 \text{ s}$$

3. In the diagram on the right, $I = 9\text{A}$, and $I_1 = 4.5\text{A}$. What is the current at I_2 ?

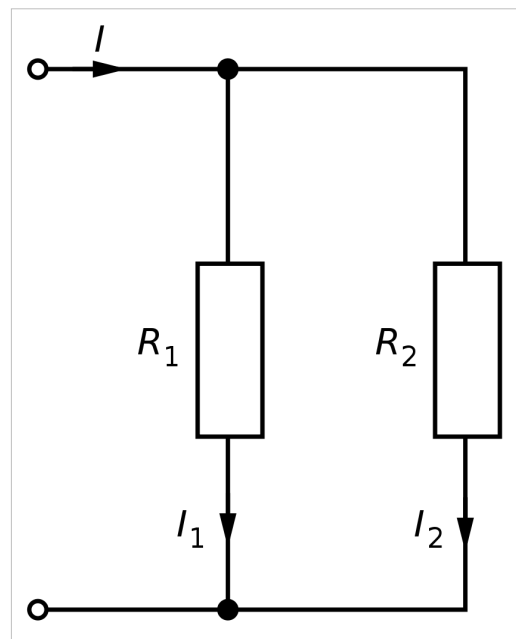
$$9 = 4.5 + I_2$$

$$I_2 = 4.5 \text{ A}$$

4. What would I equal if $I_1 = 10\text{A}$ and $I_2 = 15\text{A}$?

$$I = 10 + 15 = 25\text{A}$$

5. In the diagram on the right, in 5 seconds, 5C of charged particles flow past I_1 , and 6.7C flow past I_2 . How long does it take for 10C to flow past I ?



$$\frac{10}{t} = \frac{5}{5} + \frac{6.7}{5} = 2.34$$

$$t = \frac{10}{2.34} \approx 4.27 \text{ s}$$

Voltage

1. A battery has an EMF of 5V. What is the total potential difference across all the components in the circuit?

They are the same! The voltage decreases across the circuit from the voltage at one end of the battery to the other.

2. The voltages (relative to the voltage of the battery) on either side of a resistor are -6V and -5V. What is the potential difference across the resistor?

$$-5 - (-6) = -5 + 6 = 1\text{V}$$

3. At a given point in a circuit, 5C of charge have 10 kJ of potential energy. What is the voltage at this point?

$$V = \frac{10 \times 10^3}{5} = 2 \text{ kV}$$

4. Why do the electrons move to a point 1cm further along the wire?

The point further down the wire has a lower voltage (ie. less energy per. coulomb) - the electrons will be at a lower potential energy at this point, and fall to it, just as objects fall to lower potential energies due to gravity.

Power

1. The potential difference across a 9W light bulb is 240V. How much current is flowing through the light bulb?

$$9 = 240I$$

$$I = \frac{9}{240} = 0.0375 \text{ A}$$

2. How much energy is dissipated by a 10W component in 1 hour?

$$10 = \frac{E}{60^2}$$

$$E = 60^2 \times 10 = 36000 \text{ J} = 36 \text{ kJ}$$

3. The potential difference across a top-notch kettle, which can hold up to 1 litre of water, is 240V, and the current is 12.5 A. 4.2 kJ of energy is required to heat a litre of water to 100°C. Assuming 100% efficiency, how long does it take to boil 1 litre of water?

$$12.5 \times 240 = 3000 = \frac{4.2 \times 10^3}{t}$$

$$t = \frac{4.2 \times 10^3}{3000} = 1.4 \text{ s}$$

The reason this is unrealistically small is that kettles are not 100% efficient as they lose lots of heat into the air surrounding the kettle.

4. How much energy is dissipated by a 100Ω resistor in 10 seconds if 2A of current are flowing?

$$\frac{E}{10} = 2^2 \times 100 = 400$$

$$E = 4000 \text{ J} = 4 \text{ kJ}$$

5. The charge on an electron is -1.6×10^{-19} C. How long does it take for a mole (6×10^{23} particles) of electrons to flow through a 40W light bulb on a 240V ring main?

$$\text{Total charge} = 1.6 \times 10^{-19} \times 6 \times 10^{23} = 96000 \text{ C}$$

$$40 = \frac{96000 \times 240}{t} = \frac{23040000}{t}$$

$$t = \frac{23040000}{40} = 576000 \text{ s} = 9600 \text{ min} = 160 \text{ h} = 6 \text{ days } 16 \text{ hours}$$

Resistance and Conductance

1. The potential difference across a resistor is 4V, and the current is 10A. What is the resistance of the resistor?

$$R = \frac{V}{I} = \frac{4}{10} = 0.4 \Omega$$

2. What is the conductance of this resistor?

$$G = \frac{1}{R} = \frac{1}{0.4} = 2.5 \text{ S}$$

3. A conductor has a conductance of 2S, and the potential difference across it is 0.5V. How much current is flowing through it?

$$\frac{1}{2} = \frac{0.5}{I} = 0.5$$

$$I = 1 \text{ A}$$

4. A graph is drawn of potential difference across an Ohmic conductor, and current. For every 3cm across, the graph rises by 2cm. What is the conductance of the conductor?

$$\text{Gradient} = \frac{2}{3} = \text{Resistance}$$

$$G = \frac{1}{\frac{2}{3}} = \frac{3}{2} = 1.5 \text{ S}$$

5. On another graph of potential difference and current, the graph curves so that the gradient increases as current increases. What can you say about the resistor?

The resistor is not an Ohmic conductor.

6. 3 resistors, wired in series, have resistances of 1kΩ, 5kΩ and 500Ω each. What is the total resistance across all three resistors?

$$\Sigma R = (1 \times 10^3) + (5 \times 10^3) + 500 = 6500 \Omega = 6.5 \text{ k}\Omega$$

7. 2 conductors, wired in parallel, have conductances of 10S and 5S. What is the total resistance of both branches of the parallel circuit?

$$G = 10 + 5 = 15 \text{ S} = \frac{1}{R}$$

$$R = \frac{1}{15} \approx 0.0667 \Omega$$

8. The circuit above is attached in series to 1 10Ω resistor. What is the total conductance of the circuit now?

$$\Sigma R = 10 + \frac{1}{15} = \frac{151}{15} \Omega$$

$$G = \frac{15}{151} \approx 0.0993 \text{ S}$$

Internal Resistance

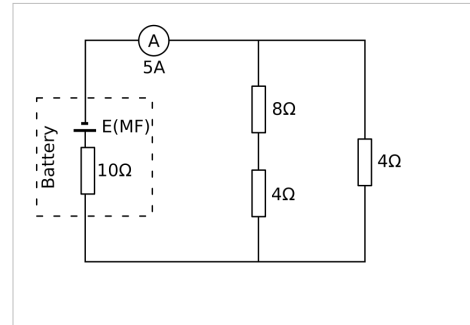
1. A 9V battery is short-circuited. The potential difference across the battery is found to be 8V, and the current is 5A. What is the internal resistance of the battery?

$$8 = 9 - 5R_{\text{internal}}$$

$$5R_{\text{internal}} = 1$$

$$R_{\text{internal}} = 0.2$$

2. What is the EMF of the battery in the following circuit?



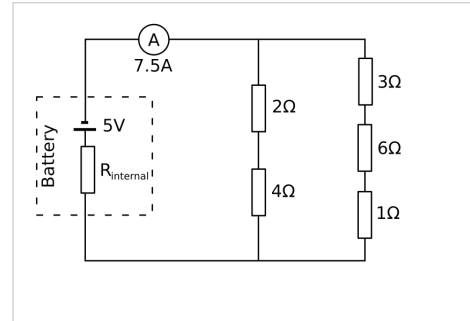
$$\sum R_{\text{external}} = \left(\frac{1}{8+4} + \frac{1}{4} \right)^{-1} = \left(\frac{1}{3} \right)^{-1} = 3 \Omega$$

$$V_{\text{external}} = 5 \times 3 = 15 \text{ V}$$

$$15 = E - (5 \times 10)$$

$$E = 50 + 15 = 65 \text{ V}$$

3. What is the internal resistance of the battery in the following circuit?



$$\sum R_{\text{external}} = \left(\frac{1}{2+4} + \frac{1}{3+6+1} \right)^{-1} = \left(\frac{4}{15} \right)^{-1} = 3.75 \Omega$$

$$V_{\text{external}} = 7.5 \times 3.75 = 28.125$$

Which actually makes this question impossible because the current is too high for the resistance supplied to the circuit.

Potential Dividers

1. A 12 k Ω resistor and a 20 k Ω resistor are connected to a 9V battery. A voltmeter is connected across the 12 Ω resistor. What is the reading on the voltmeter? (Assume negligible internal resistance.)

$$V_{12k\Omega} = 9 \times \frac{12}{12 + 20} = 3.375V$$

2. A potential divider consists of 100 5 Ω resistors, with a wiper which moves on one resistor for every 3.6° a handle connected to it turns. The wiper is connected to a voltmeter, and the circuit is powered by a 120V power source with negligible internal resistance. What is the reading on the voltmeter when the handle turns 120°?

First, work out the number (n) of resistors between the terminals of the voltmeter:

$$n = \frac{120}{3.6} = 33.\bar{3}$$

The handle, then, has not turned enough to reach the 34th resistor, so the voltmeter is connected just after the 33rd resistor.

$$V = 120 \times \frac{33 \times 5}{100 \times 5} = 39.6 \text{ V}$$

3. A 9V battery with internal resistance 0.8 Ω is connected to 3 resistors with conductances of 3, 2 and 1 Siemens. A voltmeter is connected across the 3 and 2 Siemens resistors. An ammeter is placed in the circuit, between the battery and the first terminal of the voltmeter, and reads 2A. What is the reading on the voltmeter?

First, work out the resistances of the resistors:

$$R = \frac{1}{G} = \frac{1}{3}, \frac{1}{2}, 1 \Omega$$

Then, work out the external potential difference (ie. excluding the potential difference lost due to the battery's internal resistance):

$$V_{\text{external}} = E - IR_{\text{internal}} = 9 - (2 \times 0.8) = 7.4V$$

$$V_{3\Omega, 2\Omega} = 7.4 \times \frac{\frac{1}{3} + \frac{1}{2}}{\frac{1}{3} + \frac{1}{2} + 1} \approx 6.17 \text{ V}$$

Sensors

Note to the reader: I am not entirely happy with the answers given on this page. --Sjlegg (talk) 15:01, 19 December 2007 (UTC)

An LDR's resistance decreases from a maximum resistance of 2kΩ to a minimum resistance of 0Ω as light intensity increases. It is used in a distance sensing system which consists of a 9V power supply, a 1.6 kΩ resistor, the LDR and a multimeter which displays voltage to 2 decimal places measuring the potential difference across one of the two resistors.

1. Across which resistor should the multimeter be connected in order to ensure that, as the distance from the light source to the sensor increases, the potential difference recorded increases?

As light intensity increases, distance decreases. So, as distance increases, light intensity decreases. As light intensity decreases, the LDR's resistance increases, as does the potential difference across it. So, as distance increases, the potential difference increases. Since we want the potential difference to change in the same direction as the distance, the multimeter must go across the LDR, not the resistor.

2. In complete darkness, what voltage is recorded on the multimeter?

The voltage is split between the 1.6kΩ resistor and the LDR, which currently has a resistance of 2kΩ. Therefore, the potential difference across the LDR is:

$$V = \frac{2}{2 + 1.6} \times 9 = 5V$$

3. When a light source moves 0.5m away from the sensor, the voltage on the multimeter increases by 2V. What is the sensitivity of the sensing system when using this light source, in V m⁻¹?

$$S = \frac{2}{0.5} = 4 \text{ Vm}^{-1}$$

4. When the same light source is placed 0m from the sensor, the potential difference is 0V. When the light source is 1m away, what voltage is displayed on the multimeter?

Assuming a linear relationship between distance and potential difference, we know that:

$$0.25 = \frac{m}{V}$$

Hence:

$$V = \frac{m}{0.25} = \frac{1}{0.25} = 4V$$

5. What is the resolution of the sensing system?

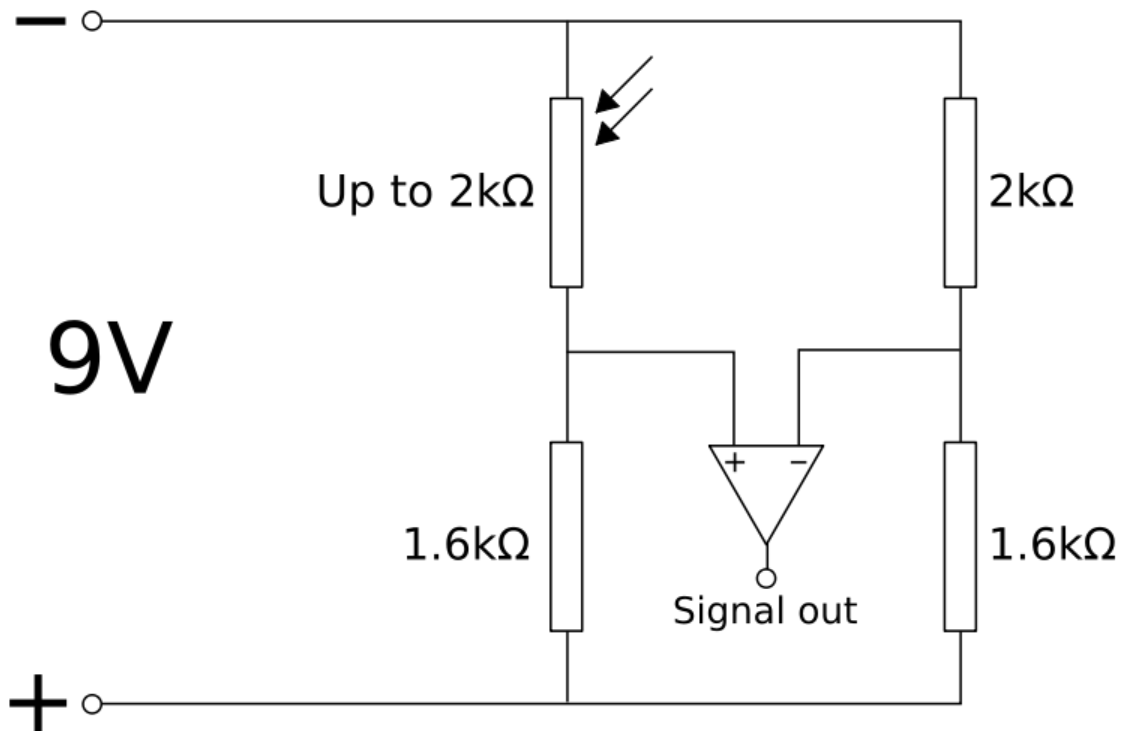
The multimeter measures to 2 decimal places, so the smallest measurable voltage is 0.01V.

$$0.25 = \frac{m}{V}$$

Hence:

$$m = 0.25 \times V = 0.25 \times 0.01 = 0.0025 \text{ m} = 2.5 \text{ mm}$$

6. Draw a circuit diagram showing a similar sensing system to this, using a Wheatstone bridge and amplifier to improve the sensitivity of the system.



7. What is the maximum potential difference that can reach the amplifier using this new system (ignore the amplification)?

The maximum potential difference will occur when the LDR has no resistance. This will result in 9V on the left-hand side of the voltmeter, and 5V on the right-hand side. The difference is 4V.

8. If this signal were to be amplified 3 times, would it exceed the maximum voltage of the system? What would the limits on the signal be?

$$4 \times 3 = 12V > 9V$$

The signal would be limited to the range $-9V < V < 9V$.

Resistivity and Conductivity

1. A material has a conductivity of 10^6 S m^{-1} . What is its resistivity?

$$\rho = \frac{1}{\sigma} = \frac{1}{10^6} = 10^{-6} \Omega \text{ m} = 1 \mu\Omega \text{ m}$$

2. A pure copper wire has a radius of 0.5mm, a resistance of 1 M Ω , and is 4680 km long. What is the resistivity of copper?

$$\rho = \frac{10^6 \times \pi \times (0.5 \times 10^{-3})^2}{4680 \times 10^3} \approx 168 \times 10^{-9} \Omega \text{ m} = 168 \text{ n}\Omega \text{ m}$$

3. Gold has a conductivity of 45 MS m^{-1} . What is the resistance of a 0.01m across gold connector, 0.05m long?

First, work out resistivity:

$$\rho = \frac{1}{45 \times 10^6} = 22.\bar{2} \times 10^{-9} \Omega \text{ m}$$

Then, substitute everything possible into the resistivity formula:

$$22.\bar{2} \times 10^{-9} = \frac{(\pi \times (0.5 \times 0.01)^2)R}{0.05} \approx 1.57 \times 10^{-3} R$$

$$R \approx \frac{22.\bar{2} \times 10^{-9}}{1.57 \times 10^{-3}} \approx 14.2 \times 10^{-6} \Omega = 14.2 \text{ m}\Omega$$

4. A strand of metal is stretched to twice its original length. What is its new resistance? State your assumptions.

The material does not change, so resistivity is constant. Length doubles, and we know that volume must be constant.

$$V = AL$$

$$A = V/L$$

$$\rho = \frac{RA}{L} = \frac{R(V/L)}{L} = \frac{RV}{L^2}$$

$$R_{old} = \frac{\rho}{1} \times \frac{L^2}{V} = \frac{\rho L^2}{V}$$

When we double L, we get:

$$R_{new} = \frac{\rho(2L)^2}{V} = \frac{4\rho L^2}{V} = 4 \times R_{old}$$

We are assuming that ρ and V are constant.

5. Which has the greater resistivity: a plank or a piece of sawdust, made from the same wood?

Sawdust and a plank are artefacts, not materials. Hence, they do not have a resistivity. Even if they did, they are made of the same thing, so they would have equal resistivity.

Semiconductors

1. What is the resistivity of silicon, at room temperature?

$$\rho = \frac{1}{\sigma} = \frac{1}{435 \times 10^{-6}} \approx 2300 \Omega\text{m} = 2.3\text{k}\Omega\text{m}$$

2. What sort of variable resistor would a semiconductor be useful in?

A thermistor, as the resistance of a semiconductor decreases as heat increases (but, assuming use of a potential divider, the voltmeter would have to be on the other resistor).

3. If positive ions are added to silicon (doping it), how does its conductivity change?

Positive ions mean more free electrons, and so greater conductivity.

Stress, Strain & the Young Modulus

1. 10N of force are exerted on a wire with cross-sectional area 0.5mm^2 . How much stress is being exerted on the wire?

$$0.5\text{mm}^2 = 0.5 \times (10^{-3})^2\text{m}^2 = 0.5 \times 10^{-6}\text{m}^2$$

$$\sigma = \frac{10}{0.5 \times 10^{-6}} = 20000000 \text{ Pa} = 20 \text{ MPa}$$

2. Another wire has a tensile strength of 70MPa, and breaks under 100N of force. What is the cross-sectional area of the wire just before breaking?

$$70 \times 10^6 = \frac{100}{A}$$

$$A = \frac{100}{70 \times 10^6} \approx 1.43 \times 10^{-6} \text{ m}^2 = 1.43 \text{ mm}^2$$

3. What is the strain on a Twix bar (original length 10cm) if it is now 12cm long?

$$E = \frac{12 - 10}{10} = \frac{2}{10} = 0.2$$

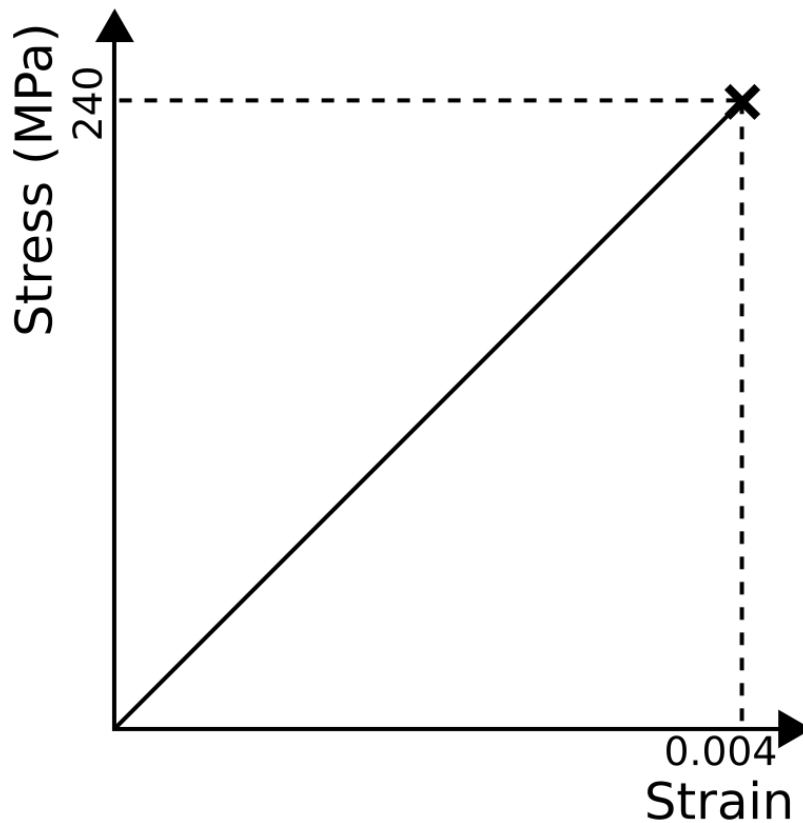
4. What is this strain, expressed as a percentage?

$$0.2 \times 100 = 20\%$$

5. 50N are applied to a wire with a radius of 1mm. The wire was 0.7m long, but is now 0.75m long. What is the Young's Modulus for the material the wire is made of?

$$Y = \frac{\left(\frac{50}{\pi \times (1 \times 10^{-3})^2}\right)}{\left(\frac{0.75 - 0.7}{0.7}\right)} \approx \frac{16000000}{0.0714} \approx 224000000 \text{ Pa} = 224 \text{ MPa}$$

6. Glass, a brittle material, fractures at a strain of 0.004 and a stress of 240 MPa. Sketch the stress-strain graph for glass.



7. (Extra nasty question which you won't ever get in an exam) What is the toughness of glass?

Toughness equals the area under the above graph. Since glass is brittle, we can assume that the gradient of the graph is constant, and, since the graph passes through the origin, it is a triangle. So:

$$A_{triangle} = \frac{\text{base} \times \text{height}}{2} = \text{Toughness} = \frac{\text{strain} \times \text{stress}}{2} = \frac{0.004 \times 240 \times 10^6}{2} = 480000 \text{ Jm}^{-3} = 480$$

Metals

1. Would you expect a metal to have more or less conductivity than a semiconductor? Why?

A metal has more conductivity than a semiconductor because a metal has more ions than a semiconductor, and hence more free (delocalized) electrons.

2. How can the stress-strain graph for a metal be explained in terms of ions in a sea of electrons?

In the elastic region, the ions are held together by the charge between them and the electrons, and they can move apart when under stress. Then, once the ions are too far apart, the bonds aren't strong enough to pull them back together, so the metal extends under the stress. Eventually, the bonds cannot maintain the structural integrity of the metal any more, so the metal 'necks' in one place, and then the metal fractures.

3. As a metal heats up, what happens to its conductivity? Why?

As a metal heats up, both ions and electrons vibrate more. This means that the collision rate between ions and electrons goes up, so it is harder for electrons to travel through the metal. So, as a metal heats up, its conductivity goes down.

Polymers

1. Different crystalline structures have different refractive indexes. Why does this mean that a polycrystalline polymer is translucent?

The refractive index of each area is different, so the angle of refraction is different for each area. As a result, the image is blurred.

2. What sort of polymer is a pane of perspex?

Crystalline

3. What sort of polymer does the pane of perspex become when shattered (but still in one piece)?

Polycrystalline

4. What sort of polymer is a rubber on the end of a pencil?

Amorphous

5. What happens to the translucency of an amorphous polymer when it is put under stress?

The polymer becomes more translucent, as it becomes more polycrystalline as the chains are stretched and straightened.

What is a wave?

1. Through what medium are sound waves propagated?

Usually the air, but most material objects will carry sound.

2. What aspects of the behaviour of light make it look like a wave?

Superposition and diffraction (and others).

3. What aspects of the behaviour of light make it look like a particle?

Photoelectric effect

4. Consider the diagram on the right. White light is partially reflected by the transparent material. Some of the light, however,

is refracted into the transparent material and reflected back by the opaque material. The result is two waves travelling in the same place at the same time at the same polarisation (the light is not a single beam). Why does, say, the red light disappear?

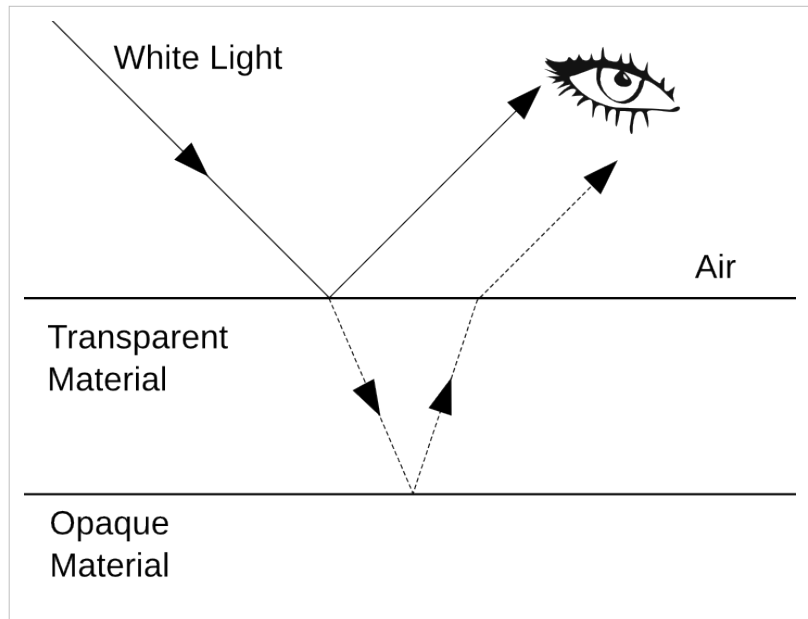
If the refractive index and width of the transparent material are correct, then the two waveforms leaving it will be half a wavelength apart, as one will have travelled further. Destructive interference will cause the red light to become 'invisible'.

5. What is the wavelength of green light?

500nm

6. The lowest frequency sound wave humans can hear has a frequency of approximately 20Hz. Given that the speed of sound in air is 343ms^{-1} , what is the wavelength of the lowest frequency human-audible sound?

$$\lambda = \frac{v}{f} = \frac{343}{20} = 17.15\text{m}$$



Phasors

1. A sine wave with wavelength 0.1m travels through a given point on the surface of the sea. A phasor arrow representing the effect of this wave on this point rotates 1000°. How many wavelengths have gone past in the time taken for the phasor to rotate this much?

0.1m is irrelevant information.

$$\frac{1000}{360} = 2.78\lambda$$

2. A sine wave has a maximum amplitude of 500nm. What is its amplitude when the phasor has rotated 60° from its start position?

$$a \sin \theta = 500 \times 10^{-9} \times \sin 60 \approx 433\text{nm}$$

3. Two waves have a phase difference of 45°. When the first wave is at its minimum amplitude of -0.3m, what is the total amplitude of the superposed waveforms?

The first wave has an amplitude of -0.300m. The second is at an angle of $270 - 45 = 225^\circ$.

$$a \sin \theta = 0.3 \times \sin 225 \approx -0.212\text{m}$$

$$-0.300 + -0.212 = -0.512\text{m}$$

Standing Waves

1. The air in a 3m organ pipe is resonating at the fundamental frequency. Organ pipes are effectively open at both ends. What is the wavelength of the sound?

$$3 = \frac{\lambda}{2}$$

$$\text{so } \lambda = 2 \times 3 = 6\text{m.}$$

2. A string is vibrating at the second harmonic frequency. How many wavelengths long is the standing wave created?

$$(2 + 1) \times \frac{\lambda}{2} = \frac{3\lambda}{2}$$

3. Express, in terms of λ , the length of a pipe which is closed at one end, where λ is the length of one wave at the fundamental frequency.

$\frac{1}{4}\lambda$ (this is the distance between a node and an antinode)

Young's Slits

1. A 2-slit experiment is set up in which the slits are 0.03m apart. A bright fringe is observed at an angle 10° from the normal. What sort of electromagnetic radiation was being used?

$$\lambda = d \sin \theta = 0.03 \times \sin 10 \approx 5.21 \times 10^{-3} \text{m}$$

So, microwaves were being used.

2. Light, with a wavelength of 500nm, is shone through 2 slits, which are 0.05m apart. What are the angles to the normal of the first three dark fringes?

$$n \times 500 \times 10^{-9} = 0.1 \times \sin \theta, \text{ where } n \text{ is an odd integer.}$$

$$\theta = \arcsin \frac{n \times 500 \times 10^{-9}}{0.1} = \arcsin (n \times 500 \times 10^{-8})$$

Then, substitute in $n = 1, 3$ and 5 to gain corresponding values of θ :

n	θ (°)
1	286×10^{-6}
3	859×10^{-6}
5	1430×10^{-6}

3. Some X-rays, with wavelength 1nm, are shone through a diffraction grating in which the slits are 50µm apart. A screen is placed 1.5m from the grating. How far are the first three light fringes from the point at which the normal intercepts the screen?

$$n \times 10^{-9} = \frac{50 \times 10^{-6} \times x}{1.5} \approx 33.3 \times 10^{-6} \times x, \text{ where } n \text{ is an integer.}$$

$$x = \frac{n \times 10^{-9}}{33.3 \times 10^{-6}} = 30 \times 10^{-6} \times n$$

Then, substitute in $n = 1, 2$ and 3 to gain corresponding values of x :

n	x (m)
1	30×10^{-6}
2	60×10^{-6}
3	90×10^{-6}

Diffraction

1. What is the width of the central bright fringe on a screen placed 5m from a single slit, where the slit is 0.01m wide and the wavelength is 500nm?

$$\frac{W}{L} \approx \frac{\lambda}{d}$$

$$\frac{W}{5} \approx \frac{500 \times 10^{-9}}{0.01}$$

$W = 250\mu\text{m}$, but since this is only half the beam, the beam is $500\mu\text{m}$ wide.

Light

1. How much energy does a photon with a frequency of 50kHz carry?

$$E = hf = 6.626 \times 50 \times 10^3 \times 10^{-34} = 3.313 \times 10^{-29} \text{ J}$$

2. A photon carries 10^{-30} J of energy. What is its frequency?

$$f = \frac{E}{h} = \frac{10^{-30}}{6.626 \times 10^{-34}} \approx 1509 \text{ Hz}$$

3. How many photons of frequency 545 THz does a 20W bulb give out each second?

First calculate the amount of energy given out per. second:

$$P = \frac{\Delta E}{t}$$

$$\Delta E = Pt = 20 \times 1 = 20 \text{ J}$$

Then, calculate the amount of energy carried by each photon:

$$E = hf = 6.626 \times 545 \times 10^{12} \times 10^{-34} \approx 3.61 \times 10^{-19} \text{ J}$$

Then divide the former by the latter to give the number of photons n :

$$n = \frac{20}{3.61 \times 10^{-19}} \approx 5.54 \times 10^{19} \text{ photons}$$

4. In one minute, a bulb gives out a million photons of frequency 600 THz. What is the power of the bulb?

First calculate the energy carried by one photon:

$$E = hf = 6.626 \times 10^{-34} \times 600 \times 10^{12} \approx 3.98 \times 10^{-19} \text{ J}$$

Then work out the energy carried by 1,000,000 photons:

$$E = 10^6 \times 3.98 \times 10^{-19} = 3.98 \times 10^{-13} \text{ J}$$

Then work out the power of the bulb:

$$P = \frac{\Delta E}{t} = \frac{3.98 \times 10^{-13}}{60} = 6.63 \times 10^{-15} \text{ W}$$

...maybe its a nanobulb.

5. The photons in a beam of electromagnetic radiation carry 2.5μJ of energy each. How long should the phasors representing this radiation take to rotate?

First calculate the frequency of each photon:

$$f = \frac{E}{h} = \frac{2.5 \times 10^{-6}}{6.626 \times 10^{-34}} \approx 3.77 \times 10^{27} \text{ Hz (That's one nasty gamma ray.)}$$

Then calculate the time taken for one 'wavelength' to go by:

$$f = \frac{1}{t}$$

$$t = \frac{1}{f} = \frac{1}{3.77 \times 10^{27}} \approx 2.65 \times 10^{-28} \text{s}$$

Electron Behaviour

1. An electron moves at $30,000 \text{ ms}^{-1}$. What is its de Broglie wavelength?

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{0.9 \times 10^{-27} \times 30000} \approx 2.45 \times 10^{-11} \text{m}$$

2. What is its frequency?

$$f = \frac{E_{kinetic}}{h} = \frac{0.5 \times 0.9 \times 10^{-27} \times 30000^2}{6.626 \times 10^{-34}} \approx 6.11 \times 10^{14} \text{ Hz}$$

3. What is its kinetic energy, in eV?

From the top half of the fraction in the previous question:

$$E_{kinetic} = 0.5 \times 0.9 \times 10^{-27} \times 30000^2 = 4.05 \times 10^{-19} \text{ J} = \frac{4.05 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \approx 2.53 \text{ eV}$$

4. Given that it is travelling out of an electron gun, what was the potential difference between the anode and the cathode?

2.53 eV - that's why we use eV!

5. An electron is accelerated by a potential difference of 150V. What is its frequency?

$$E_{kinetic} = 1.6 \times 10^{-19} \times 150 = 2.4 \times 10^{-17} \text{ J}$$

$$f = \frac{E_{kinetic}}{h} = \frac{2.4 \times 10^{-17}}{6.626 \times 10^{-34}} \approx 3.62 \times 10^{16} \text{ Hz}$$

Vectors

1. Which of the following are vectors?

- **20 cm** is scalar, not a vector - it does not have a directional component.
- **9.81 ms⁻² towards the centre of the earth** is a vector.
- **5 km south-east** is a vector.
- **500 ms⁻¹ on a bearing of 285.3°** is a vector.

2. A displacement vector a is the resultant vector of two other vectors, 5 m north and 10 m south-east. What does a equal, as a displacement and a bearing?

Displacement (m)	Bearing (°)	Horizontal Displacement (m)	Vertical Displacement (m)
5	000	0	5
10	135	$\sin 135 = \frac{s}{10}$, so $s = \frac{10}{\sqrt{2}}$	$\cos 135 = \frac{s}{10}$, so $s = -\frac{10}{\sqrt{2}}$

So, for the resultant vector:

$$\text{Horizontal displacement} = 0 + \frac{10}{\sqrt{2}} \approx 7.07 \text{ m}$$

$$\text{Vertical displacement} = 5 - \frac{10}{\sqrt{2}} \approx -2.07 \text{ m}$$

This gives us a right-angled triangle. So:

$$\theta = \arctan \frac{2.07}{7.07} \approx 16.3^\circ, \text{ so the bearing equals } 90^\circ + 16.3^\circ = 106.3^\circ.$$

$$\text{Resultant displacement} = \sqrt{7.07^2 + (-2.07)^2} \approx 7.4 \text{ m}$$

3. If I travel at a velocity of 10 ms⁻¹ on a bearing of 030°, at what velocity am I travelling north and east?

$$\text{Vertical velocity (North)} = 10 \times \cos 30 = 5\sqrt{3} \text{ ms}^{-1} \approx 8.66 \text{ ms}^{-1}$$

$$\text{Horizontal velocity (East)} = 10 \times \sin 30 = 5 \text{ ms}^{-1}$$

4. An alternative method of writing vectors is in a column, as follows:

$$\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix},$$

where **x** and **y** are the vertical and horizontal components of the vector respectively. Express **|a|** and the angle between **a** and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in terms of **x** and **y**.

By Pythagoras' theorem:

$$(|\mathbf{a}|)^2 = x^2 + y^2$$

$$|\mathbf{a}| = \sqrt{x^2 + y^2}$$

Let θ be the angle between **a** and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$\tan \theta = \frac{y}{x}$$

$$\theta = \arctan \frac{y}{x}$$

This angle θ is known as the argument of **a**.

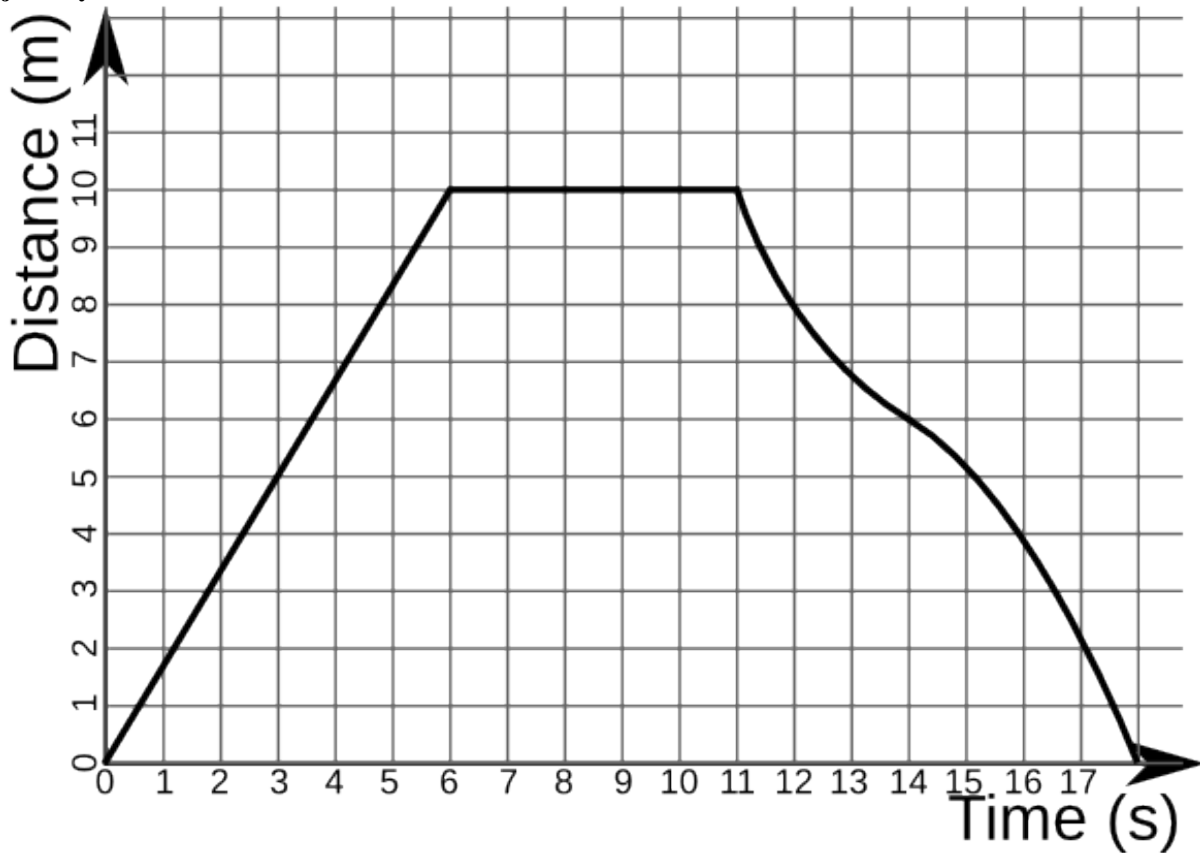
5. A more accurate method of modelling the trajectory of a ball is to include air resistance as a constant force F . How would this be achieved?

Once the arrow representing the acceleration due to gravity has been added on, add an horizontal arrow pushing against the motion of the ball. Since $F = ma$, the magnitude of this acceleration is F divided by m .

Note that this model is still not perfect. In fact, F is not constant - it depends on the horizontal component of the ball's velocity.

Graphs

1. In the following distance-time graph, what is the velocity 4 seconds after the beginning of the object's journey?



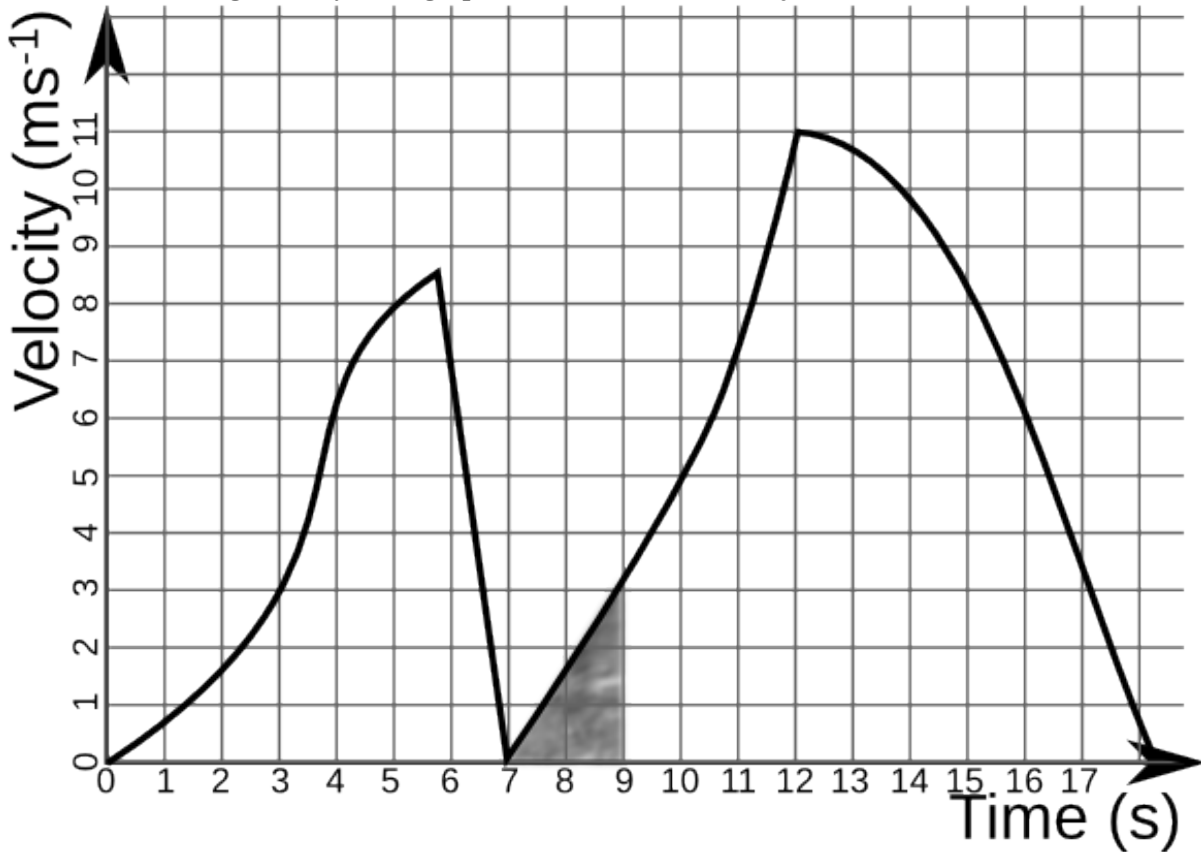
At this point, the graph is a straight line. So:

$$v = \frac{ds}{dt} = \frac{10 - 0}{6 - 0} = \frac{5}{3} \approx 1.67\text{ms}^{-1}$$

2. What is the velocity at 12 seconds?

Approximately, there is a rough straight line at this point. The object is travelling towards home 2 ms^{-1} each second (look at the previous second, for example). So, its velocity is -2 ms^{-1} .

3. In the following velocity-time graph, how far does the object travel between 7 and 9 seconds?



Distance travelled is equal to the area under the graph. Between 7 and 9 seconds, this is the shaded area of the graph. So, calculate the area of the triangle:

$$A = \frac{bh}{2} = \frac{2 \times 3}{2} = 3\text{m}$$

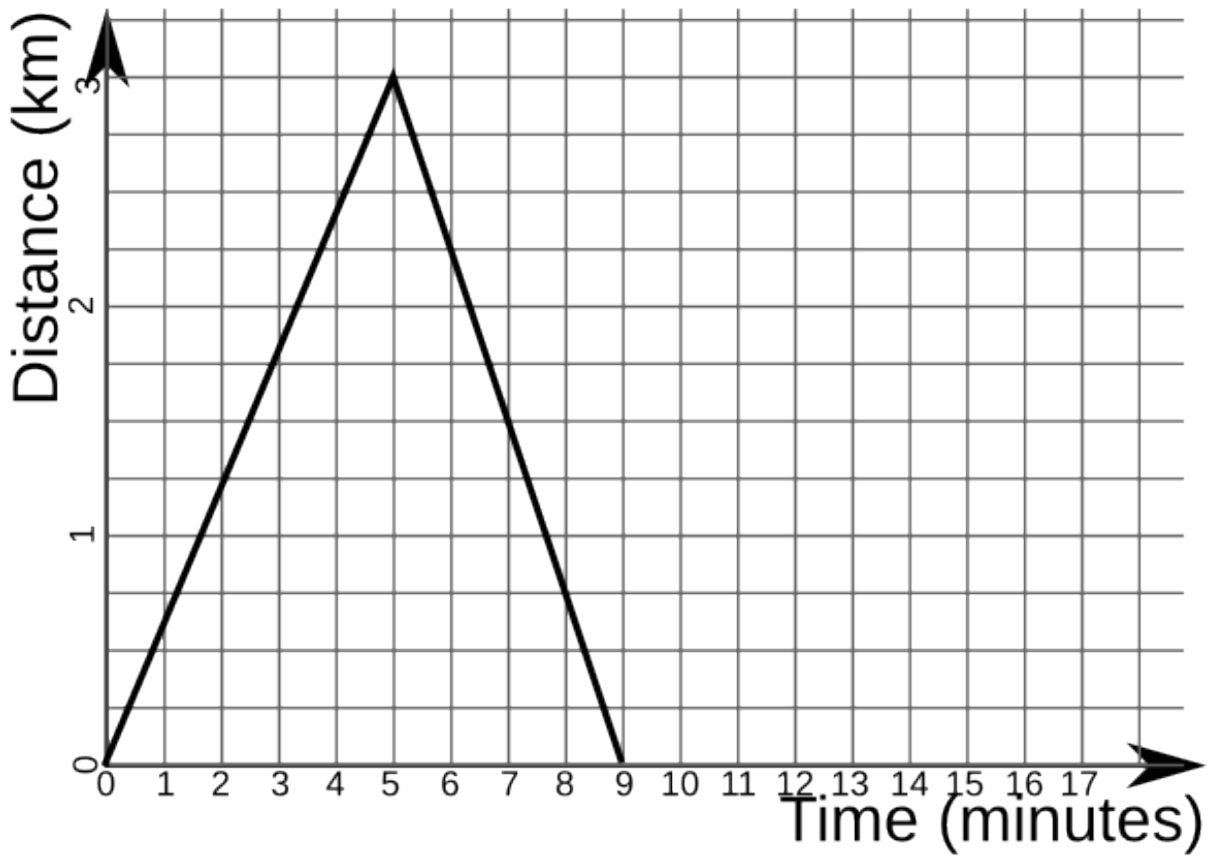
4. What is the object's acceleration at 8 seconds?

There is a straight line between 7 and 9 seconds which we can use to answer this question. The acceleration is equal to the gradient of the graph, so:

$$a = \frac{3}{2} = 1.5\text{ms}^{-2}$$

5. A car travels at 10ms^{-1} for 5 minutes in a straight line, and then returns to its original location over the next 4 minutes, travelling at a constant velocity. Draw a distance-time graph showing the distance the car has travelled from its original location.

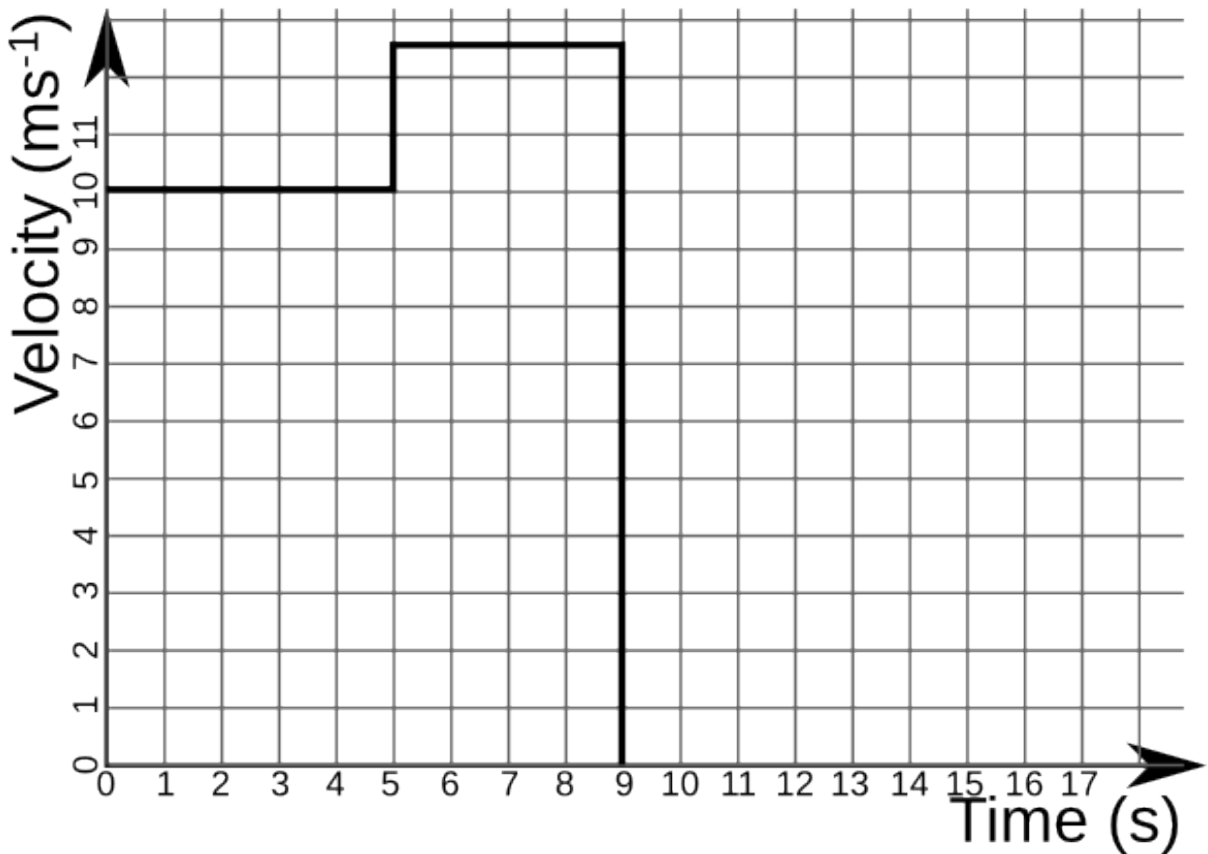
$$10\text{ms}^{-1} = 600 \text{ metres / minute.}$$



6. Draw the velocity-time graph for the above situation.

The velocity from 0-5s is 10ms^{-1} . The velocity from 5-9s is:

$$v = \frac{3 \times 10^3}{4} = 750 \text{ metres/minute} = 12.5\text{ms}^{-1}$$



7. The velocity of a ball is related to the time since it was thrown by the equation $v = 30 - 9.8t$. How far has the ball travelled after 2 seconds?

$$s = \int_{t_1}^{t_2} f(t) dt = \int_0^2 30 - 9.8t dt = [30t - 4.9t^2]_0^2 = 30(2) - 4.9(2^2) - 30(0) + 4.9(0^2) = 60 - 19.6 = 40.4 \text{ m}$$

Kinematics

1. A person accelerates from a speed of 1 ms^{-1} to 1.7 ms^{-1} in 25 seconds. How far has he travelled in this time?

$$u = 1, v = 1.7, t = 25$$

$$s = \frac{u + v}{2}t = \frac{1 + 1.7}{2} \times 25 = 1.35 \times 25 = 33.75 \text{ m}$$

2. A car accelerates at a rate of 18 kmh^{-2} to a speed of 60 kmh^{-1} , travelling 1 km in the process. How fast was the car travelling before it travelled this distance?

$$a = 18, v = 60, s = 1$$

$$60^2 = u^2 + (2 \times 18 \times 1)$$

$$3600 = u^2 + 36$$

$$u^2 = 3564$$

$$u = 59.7 \text{ kmh}^{-1} \text{ (to 3 significant figures)}$$

3. A goose in flight is travelling at 4 ms^{-1} . It accelerates at a rate of 1.5 ms^{-2} for 7 seconds. What is its new speed?

$$u = 4, a = 1.5, t = 7$$

$$v = u + at = 4 + (1.5 \times 7) = 14.5 \text{ ms}^{-1}$$

4. How far does an aeroplane travel if it accelerates from 400 kmh^{-1} at a rate of 40 kmh^{-2} for 1 hour?

$$u = 400, a = 40, t = 1$$

$$s = ut + \frac{at^2}{2} = (400 \times 1) + \frac{40 \times 1^2}{2} = 420 \text{ km}$$

Forces and Power

1. I hit a ball of mass 5g with a cue on a billiards table with a force of 20N. If friction opposes me with a force of 14.2N, what is the resultant acceleration of the ball away from the cue?

$$\text{Resultant force} = 20 - 14.2 = 5.8\text{N}$$

$$5.8 = (5 \times 10^{-3})a$$

$$a = 1160\text{ms}^{-2}$$

2. A 10g ball rolls down a 1.2m slope, and leaves it with a velocity of 4ms^{-1} . How much work is done by friction?

$$\text{g.p.e. at top of slope} = 10 \times 10^{-3} \times 9.81 \times 1.2 = 0.117\text{J}$$

$$\text{k.e. at bottom of slope} = 0.5 \times 10 \times 10^{-3} \times 4^2 = 0.08\text{J}$$

Assuming the remainder of the g.p.e. becomes heat / sound due to friction:

$$\text{work done by friction} = 0.117 - 0.08 = 0.037\text{J.}$$

3. An electric train is powered on a 30kV power supply, where the current is 100A. The train is travelling at 90 kmh^{-1} . What is the net force exerted on it in a forwards direction?

$$IV = Fv$$

$$90\text{ kmh}^{-1} = 0.025\text{ kms}^{-1} = 25\text{ ms}^{-1}$$

$$100 \times 30 \times 10^3 = 25F$$

$$F = 120000\text{N}$$

Exponential Relationships

1. Simplify Newton's Law of Cooling for the case when I place a warm object in a large tank of water which is on the point of freezing. Measure temperature in $^{\circ}\text{C}$.

$$\text{Newton's Law of Cooling states that } T_t = T_{env} + (T_0 - T_{env})e^{-rt}$$

The freezing point of water is 0°C , so, if we measure T in $^{\circ}\text{C}$, $T_{env} = 0$:

$$T_t = 0 + (T_0 - 0)e^{-rt}$$

$$T_t = T_0e^{-rt}$$

2. What will the temperature of an object at 40°C be after 30 seconds? (Take $r=10^{-3}\text{ s}^{-1}$.)

$$T_t = T_0e^{-rt} = 40 \times e^{-10^{-3} \times 30} = 38.8^{\circ}\text{C}$$

3. A body is found in a library (as per. Agatha Christie) at 8am. The temperature of the library is kept at a constant temperature of 20°C for 10 minutes. During these 10 minutes, the body cools from 25°C to 24°C . The body temperature of a healthy human being is 36.8°C . At what time was the person murdered?

First, we must calculate r:

$$24 = 20 + (25 - 20)e^{-10r}$$

$$4 = 5e^{-10r}$$

$$e^{-10r} = 0.8$$

$$-10r = \ln 0.8$$

$$r = \frac{\ln 0.8}{-10} = 0.0223\text{ minute}^{-1}$$

Then, calculate t - this is the time between the murder and 8am:

$$25 = 20 + (36.8 - 20)e^{-0.0223t}$$

$$5 = 16.8e^{-0.0223t}$$

$$e^{-0.0223t} = \frac{5}{16.8}$$

$$-0.0223t = \ln \frac{5}{16.8}$$

$$t = \frac{\ln \frac{5}{16.8}}{-0.0223} = 54 \text{ minutes}$$

Therefore, the murder occurred at 7:06am.

4. Suppose for a moment that the number of pages on Wikibooks p can be modelled as an exponential relationship. Let the number of pages required on average to attract an editor be a , and the average number of new pages created by an editor each year be z . Derive an equation expressing p in terms of the time in years since Wikibooks was created t .

Let n be the number of editors.

$$n = ap$$

$$\frac{dp}{dt} = nz = zap$$

$$dp = zapdt$$

$$\int \frac{1}{p} dp = \int zadt$$

$$\ln p = zat + c \text{ (where } c \text{ is the constant of integration)}$$

$$p = e^{zat+c} = e^{zat}e^c = ke^{zat} \text{ (where } k \text{ is a constant - } k = e^c)$$

There must have been a first page, which marked the point where $t = 0$, so:

$$1 = ke^{za \times 0} = ke^0 = k$$

Therefore:

$$p = e^{zat}$$

5. Wikibooks was created in mid-2003. How many pages should there have been 6 years later? (Take $a = 20$, $z = 10 \text{ yr}^{-1}$.)

$$p = e^{zat} = e^{10 \times 20 \times 6} = (e^{120})^{10} = (1.304 \times 10^{52})^{10} = 1.423 \times 10^{521}$$

That number is crazily large. In fact, it is so crazily large that you have to do the rearrangements above just to get a standard calculator to cope with it. This shows what 'exponential growth' really means.

6. The actual number of pages in Wikibooks in mid-2009 was 35,148. What are the problems with this model? What problems may develop, say, by 2103?

There are two key problems with this model:

- We have estimated the values of the constants. These should have been determined statistically.
- We have assumed that the constants are constant. In reality, as the amount of content on Wikibooks increases, more people think "Wikibooks already contains this content, so I am not going to add anything." This means that both z and a change with time. Our exponential model only applies over small periods of time. Each of these small periods of time has different values for the constants.

In the future, such as 2103, the constants will have changed so radically as to be useless. Question 5 shows how much they change over just 6 years - how much more must they change over a whole century!

Capacitors

1. A 2 mF capacitor is connected to a 10V DC power supply. How much charge can be stored by the capacitor?

$$Q = CV = 2 \times 10^{-3} \times 10 = 0.02 \text{ C}$$

Note that this is the maximum charge - since capacitors charge, as well as discharge, exponentially, we would have to leave the capacitor charging for an infinitely long period of time to charge it to capacity.

2. What is the highest possible energy stored by this capacitor?

$$E = \frac{1}{2}QV = 0.5 \times 0.02 \times 10 = 0.1 \text{ J}$$

3. The capacitor is placed in series with a 5Ω resistor and charged to capacity. How long would it take for the charge in the capacitor to be reduced to 1 mC?

$$Q = Q_0 e^{-\frac{t}{RC}}$$

$$Q_0 = Q e^{\frac{t}{RC}}$$

$$e^{\frac{t}{RC}} = \frac{Q_0}{Q}$$

$$\frac{t}{RC} = \ln \frac{Q_0}{Q}$$

$$t = RC \ln \frac{Q_0}{Q} = 5 \times 0.002 \times \ln \frac{0.02}{0.001} = 0.0300 \text{ s}$$

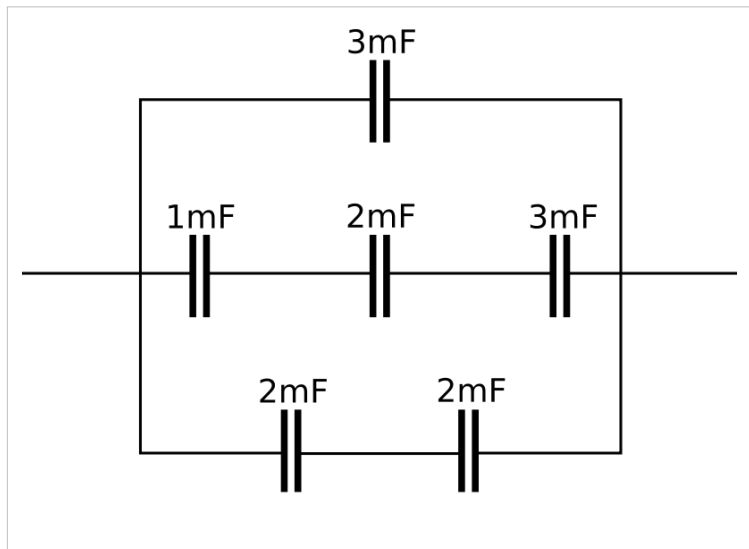
4. After this time has elapsed, how much energy is stored in the capacitor?

$$E = \frac{1}{2}QV = \frac{1}{2}Q_0V_0 e^{-\frac{2t}{RC}} = 0.5 \times 0.02 \times e^{-\frac{2 \times 0.03}{5 \times 0.002}} = 25 \mu\text{J}$$

5. What is the capacitance of the equivalent capacitor to the following network of capacitors?

First, ignore the fact that the capacitances are in mF, not F - we will not be using any other units, so if we put in mF, we will get out mF.

Then work out the equivalent capacitance of the second row:



$$\frac{1}{\Sigma C_2} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$\Sigma C_2 = \frac{6}{11} \text{ mF}$$

Then work out the equivalent capacitance of the third row:

$$\frac{1}{\Sigma C_3} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\Sigma C_3 = 1 \text{ mF}$$

The equivalent capacitance of the first row is easy, since it contains just 1 capacitor: 3mF. So, the total equivalent capacitance is:

$$\Sigma C = 3 + \frac{6}{11} + 1 = \frac{50}{11} \approx 4.55 \text{ mF}$$

Radioactive Decay

$$1 \text{ mole} = 6.02 \times 10^{23} \text{ atoms}$$

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

1. Americium-241 has a decay constant of $5.07 \times 10^{-11} \text{ s}^{-1}$. What is the activity of 1 mole of americium-241?

$$A = \lambda N = 5.07 \times 10^{-11} \times 6.02 \times 10^{23} = 3.05 \times 10^{13} \text{ Bq}$$

This is why we only need very small samples for use in, for example, smoke detectors. In fact, safety considerations necessitate a small sample - otherwise we would all have cancer!

2. How many g of lead-212 ($\lambda = 18.2 \mu\text{s}^{-1}$) are required to create an activity of $0.8 \times 10^{18} \text{ Bq}$?

$$N = \frac{A}{\lambda} = \frac{0.8 \times 10^{18}}{18.2 \times 10^{-6}} = 4.40 \times 10^{22} \text{ nuclei} = 4.40 \times 10^{22} \times 212 \times 1.66 \times 10^{-27} = 15.5 \text{ g}$$

3. How long does it take for 2kg of lead-212 to decay to 1.5kg of lead-212?

Mass is proportional to the number of atoms, so:

$$m = m_0 e^{-\lambda t}$$

$$\frac{m}{m_0} = e^{-\lambda t}$$

$$m_0$$

$$\frac{m_0}{m} = e^{\lambda t}$$

$$\lambda t = \ln \frac{m_0}{m}$$

$$t = \frac{\ln \frac{m_0}{m}}{\lambda} = \frac{\ln \frac{2}{1.5}}{18.2 \times 10^{-6}} = 15807 \text{ s} = 4.39 \text{ hours}$$

4. Where does the missing 0.5kg go?

It becomes another isotope - in this case, mercury-208. Some also becomes alpha particles.

5. Some americium-241 has an activity of 3kBq. What is its activity after 10 years?

$$A = A_0 e^{-\lambda t} = 3000 \times e^{-5.07 \times 10^{-11} \times 10 \times 365.24 \times 60 \times 60} = 2.998 \text{ kBq}$$

6. This model of radioactive decay is similar to taking some dice, rolling them once per. second, and removing the dice which roll a one or a two. What is the decay constant of the dice?

The decay constant is the probability of removing a die - $\lambda = 1/3$.

7. If you started out with 10 dice, how many dice would you have left after 10s? What is the problem with this model of radioactive decay?

$$N = N_0 e^{-\lambda t} = 10 \times e^{-\frac{1}{3} \times 10} = 0.357 \text{ dice}$$

Obviously, you can't have 0.357 dice. The problem with this model of radioactive decay is that, once you have sufficiently few nuclei, the decay ceases to be continuous. As time passes, the pattern becomes relatively more random. The model also says that the number of nuclei will always decrease. In reality, since there can only be an

integer number of nuclei, there will eventually come a point when there are no nuclei left.

Half-lives

1. Radon-222 has a decay constant of $2.1\mu\text{s}^{-1}$. What is its half-life?

$$t_{\frac{1}{2}} = \frac{\ln 2}{2.1 \times 10^{-6}} = 330070 \text{ s} = 3.82 \text{ days}$$

2. Uranium-238 has a half-life of 4.5 billion years. How long will it take for a 5 gram sample of U-238 to decay to contain 1.25 grams of U-238?

2 half-lives, since 1.25 is a quarter of 5. $2 \times 4.5 = 9$ billion years.

3. How long will it be until it contains 0.5 grams of U-238?

First calculate the decay constant:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{\ln 2}{4.5 \times 10^9} = 1.54 \times 10^{-10} \text{ yr}^{-1}$$

$$0.5 = 5e^{-1.54 \times 10^{-10}t}$$

$$0.1 = e^{-1.54 \times 10^{-10}t}$$

$$\ln 0.1 = -1.54 \times 10^{-10}t$$

$$t = \frac{\ln 0.1}{-1.54 \times 10^{-10}} = 14.9 \text{ Gyr}$$

4. Tritium, a radioisotope of Hydrogen, decays into Helium-3. After 1 year, 94.5% is left. What is the half-life of tritium (H-3)?

$$0.945 = e^{-\lambda \times 1} \text{ (if } \lambda \text{ is measured in yr}^{-1}\text{)}$$

$$\lambda = -\ln 0.945 = 0.0566 \text{ yr}^{-1} = \frac{\ln 2}{t_{\frac{1}{2}}}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{0.0566} = 12.3 \text{ yr}$$

5. A large capacitor has capacitance 0.5F. It is placed in series with a 5Ω resistor and contains 5C of charge. What is its time constant?

$$\tau = RC = 5 \times 0.5 = 2.5 \text{ s}$$

6. How long will it take for the charge in the capacitor to reach 0.677C? ($0.677 = \frac{5}{e^2}$)

$$2 \times \tau = 5\text{s}$$

Gravitational Force

1. Jupiter orbits the Sun at a radius of around 7.8×10^{11} m. The mass of Jupiter is 1.9×10^{27} kg, and the mass of the Sun is 2.0×10^{30} kg. What is the gravitational force acting on Jupiter? What is the gravitational force acting on the Sun?

$$F_{grav} = \frac{-GMm}{r^2} = \frac{-6.67 \times 10^{-11} \times 2 \times 10^{30} \times 1.9 \times 10^{27}}{(7.8 \times 10^{11})^2} = -4.17 \times 10^{23} \text{ N}$$

2. The force exerted by the Sun on an object at a certain distance is 10^6 N. The object travels half the distance to the Sun. What is the force exerted by the Sun on the object now?

$$\frac{1}{\left(\frac{1}{2}\right)^2} = 4$$

So, the new force is 4 MN.

3. How much gravitational force do two 1kg weights 5cm apart exert on each other?

$$F_{grav} = \frac{-GMm}{r^2} = \frac{-6.67 \times 10^{-11} \times 1 \times 1}{0.05^2} = -2.67 \times 10^{-8} \text{ N}$$

In other words, ordinary objects exert negligible gravitational force.

4. The radius of the Earth is 6360km, and its mass is 5.97×10^{24} kg. What is the difference between the gravitational force on 1kg at the top of your body, and on 1kg at your head, and 1kg at your feet? (Assume that you are 2m tall.)

$$\Delta F_{grav} = GMm \left(\frac{1}{6360000^2} - \frac{1}{6360002^2} \right) = (1.55 \times 10^{-20}) GMm = 1.55 \times 10^{-20} \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1$$

This is why it is acceptable to approximate the acceleration due to gravity as constant over small distances.

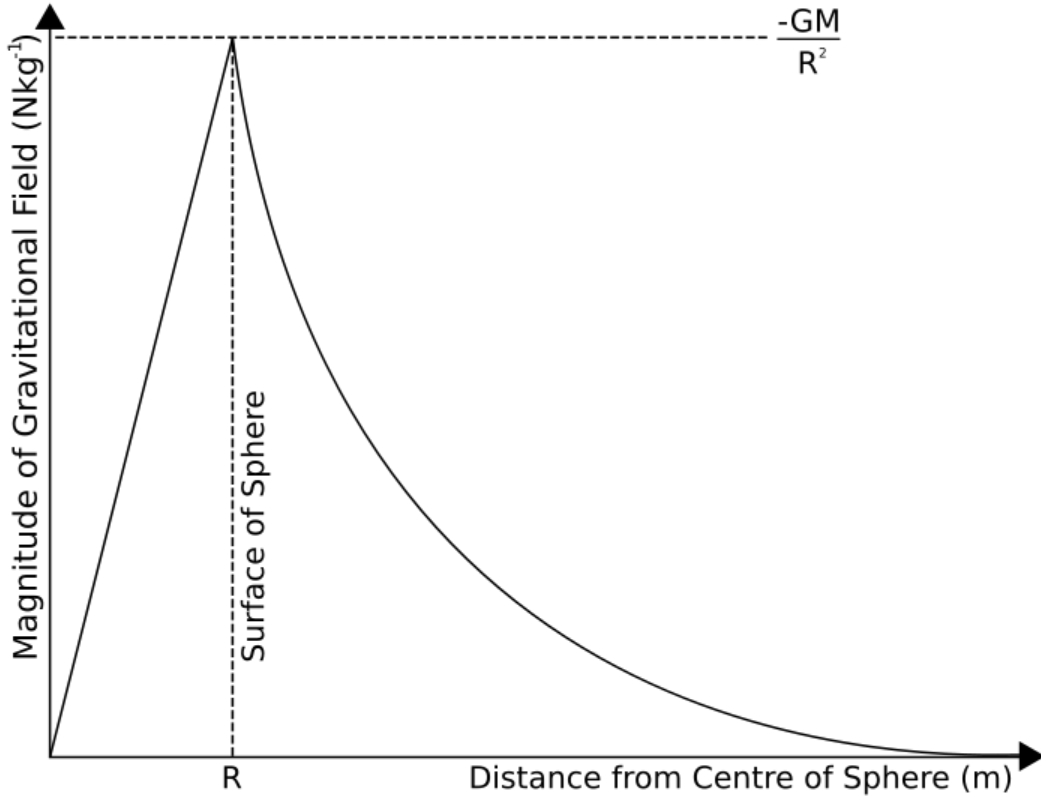
Gravitational Field

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

1. A 15kg object has a weight of 8000N. What is the gravitational field strength at this point?

$$g = \frac{F_{grav}}{m} = \frac{8000}{15} = 533 \text{ Nkg}^{-1}$$

2. Draw a graph of gravitational field strength against distance.



Since gravitational field strength is proportional to gravitational force, the two graphs look very similar.

3. What is the gravitational field strength of the Sun (mass 2×10^{30} kg) on the Earth (mass 6×10^{24} kg, mean orbital radius 15×10^{10} m)?

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{(15 \times 10^{10})^2} = 5.93 \text{ mNkg}^{-1}$$

4. What is the difference in the acceleration due to gravity over a vertical distance d?

$$\Delta g = \frac{GM}{(r+d)^2} - \frac{GM}{r^2} = GM \left(\frac{1}{(r+d)^2} - \frac{1}{r^2} \right) = \frac{GM(r^2 - (r+d)^2)}{r^2(r+d)^2} = \frac{GM(r^2 - r^2 - 2rd - d^2)}{r^2(r+d)^2} = \frac{-GM(2rd + d^2)}{r^2(r+d)^2}$$

5. How far would one have to travel upwards from the Earth's surface to notice a 1 Nkg^{-1} difference in gravitational field? (The Earth has a radius of 6400 km.)

$$1 = \frac{GMd(2r+d)}{r^2(r+d)^2}$$

$$GMd(2r+d) = r^2(r+d)^2$$

$$2GMrd + GMd^2 = r^2(r^2 + 2rd + d^2) = r^4 + 2r^3d + r^2d^2$$

$$d^2(r^2 - GM) + dr(2r^2 - 2GM) + r^4 = 0$$

Using the quadratic formula:

$$d = \frac{2GMr - 2r^3 \pm \sqrt{r^2(2r^2 - 2GM)^2 - 4r^4(r^2 - GM)}}{2(r^2 - GM)} = \frac{2GMr - 2r^3 \pm \sqrt{4r^6 - 8GMr^4 + 4G^2M^2r^2 - 4r^4 + 4GMr^2}}{2(r^2 - GM)}$$

$$= \frac{2GMr - 2r^3 \pm r\sqrt{4r^4 - 8GMr^2 + 4G^2M^2 - 4r^4 + 4GMr^2}}{2(r^2 - GM)}$$

Which is very horrible. If you plug in the numbers, you get:

$$d = \frac{4.598272 \times 10^{21} \pm 4.853341552 \times 10^{21}}{-7.1848 \times 10^{14}}$$

Since we want a negative d (because of the minus sign we ignored right at the beginning):

$$d = \frac{4.598272 \times 10^{21} + 4.853341552 \times 10^{21}}{-7.1848 \times 10^{14}} = -13155 \text{ km}$$

Although, I have to confess, there was so much scope for error that it is almost certain that this is the wrong answer.

If you fancy wading through and checking, feel free! --Sjlegg (talk) 14:18, 23 April 2009 (UTC)

Gravitational Potential Energy

1. A ball rolls down a 3m-high smooth ramp. What speed does it have at the bottom?

$$mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 3} = 7.67 \text{ ms}^{-1}$$

2. In an otherwise empty universe, two planets of mass 10^{25} kg are 10^{12} m apart. What are their speeds when they collide?

$$\frac{GMm}{0.5r} = \frac{1}{2}mv^2$$

$$\frac{GM}{0.5r} = \frac{1}{2}v^2$$

$$\frac{4GM}{r} = v^2$$

$$v = 2\sqrt{\frac{GM}{r}} = 2\sqrt{\frac{6.67 \times 10^{-11} \times 10^{25}}{10^{12}}} = 52 \text{ ms}^{-1}$$

(Not too sure about this one. Please check.)

3. What is the least work a 2000kg car must do to drive up a 100m hill?

$$mgh = 2000 \times 9.81 \times 100 = 1.962 \text{ MJ}$$

4. How does the speed of a planet in an elliptical orbit change as it nears its star?

As it nears the star, it loses gravitational potential energy, and so gains kinetic energy, so its speed increases.

Gravitational Potential

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$g = 9.81 \text{ ms}^{-2}$$

1. What is the gravitational potential at the Earth's surface? (mass of Earth = 5.97×10^{24} kg, radius of Earth = 6371 km)

$$V_{grav} = \frac{-GM}{r} = \frac{-6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6371000} = 62.5 \text{ MJkg}^{-1}$$

2. Taking the Earth's surface as $V_{grav} = 0$, what is the gravitational potential 2m above the ground?

$$V_{grav} \approx g\Delta h = 9.81 \times 2 = 19.62 \text{ Jkg}^{-1}$$

3. A 0.2kg firework reaches a gravitational potential relative to the ground of 500 Jkg^{-1} . If the firework is 30% efficient, how much energy was expended to get there?

$$V_{grav} = \frac{E_{grav}}{m}$$

$$E_{grav} = mV_{grav} = 0.2 \times 500 = 100 \text{ J}$$

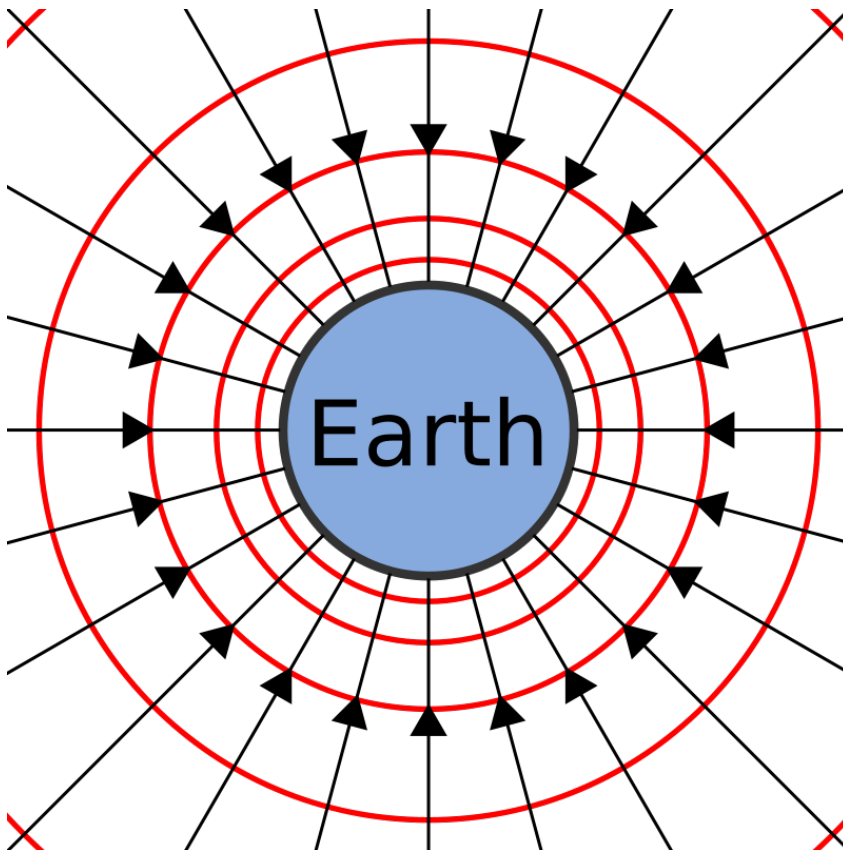
However, this is only 30% of the energy expended, so:

$$E_{expended} = \frac{100}{0.3} \approx 333 \text{ J}$$

4. Express gravitational potential in terms of gravitational force.

$$V_{grav} = \frac{\int F_{grav} dr}{m}$$

5. Draw the equipotentials and field lines surrounding the Earth.



Simple Harmonic Motion

1. A 10N weight extends a spring by 5cm. Another 10N weight is added, and the spring extends another 5cm. What is the spring constant of the spring?

$$\Delta F = k\Delta x$$

$$10 = 0.05k$$

$$\text{So } k = \frac{10}{0.05} = 200 \text{ Nm}^{-1}$$

2. The spring is taken into outer space, and is stretched 10cm with the two weights attached. What is the time period of its oscillation?

First calculate the mass of the two weights:

$$20 = 9.81m$$

$$m = 2.04 \text{ kg}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2.04}{200}} = 0.634 \text{ s}$$

3. What force is acting on the spring after 1 second? In what direction?

We are starting the oscillation at $t=0$ with a displacement. This displacement is the amplitude of the oscillation, and we need $f(\omega t)$ to be positive at $t=0$. So, we use an equation for the displacement with a cosine in it. We have already derived the acceleration in this case:

$$a = -\omega^2 \cos \omega t$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.34} = 0.99 \text{ rad s}^{-1}$$

So:

$$a = -0.99^2 \cos 0.99 \times t = -0.538 \text{ ms}^{-2}$$

The minus sign means that the acceleration is in the opposite direction to the initial displacement.

4. A pendulum oscillates with a frequency of 0.5 Hz. What is the length of the pendulum?

$$\omega = 2\pi f$$

$$\sqrt{\frac{g}{l}} = 2 \times \pi \times 0.5$$

$$\sqrt{\frac{9.81}{l}} = \pi$$

$$\frac{9.81}{l} = \pi^2$$

$$l = \frac{9.81}{\pi^2} = 0.994 \text{ m}$$

5. The following graph shows the displacement of a simple harmonic oscillator. Draw graphs of its velocity, momentum, acceleration and the force acting on it.

x is a sine wave, so:

$$x = A \sin \omega t$$

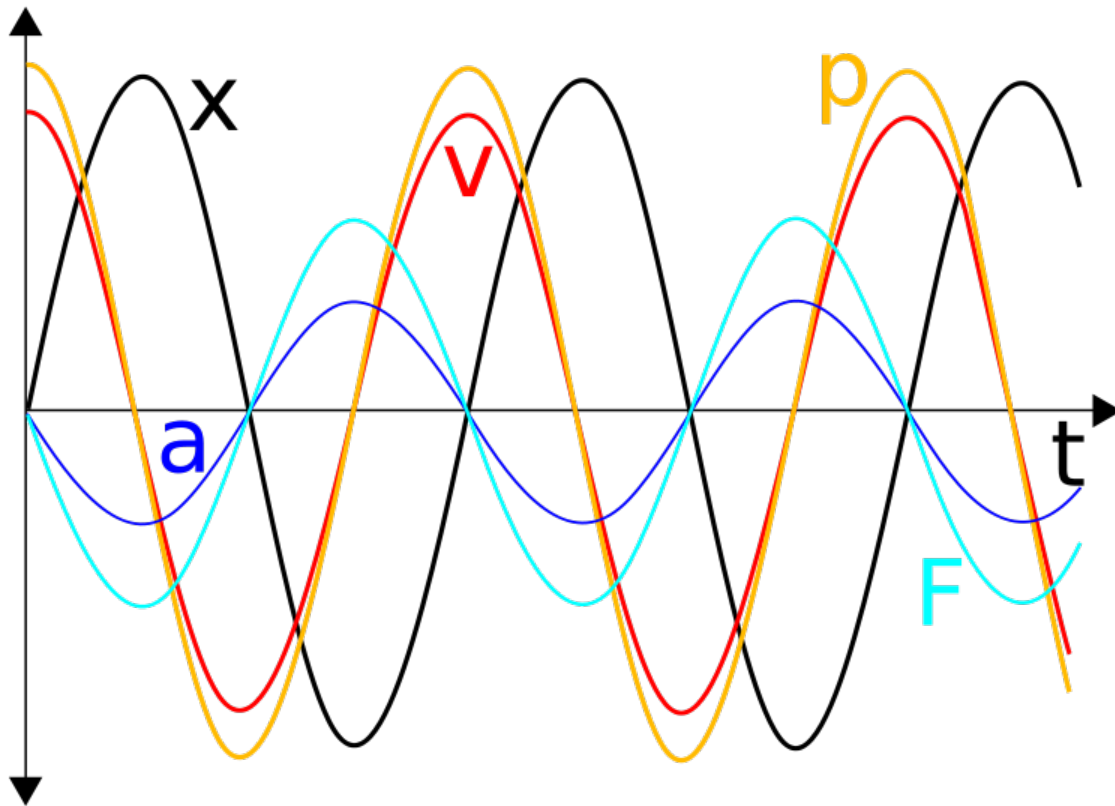
$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2 \sin \omega t$$

$$p = mv = mA\omega \cos \omega t$$

$$F = ma = -mA\omega^2 \sin \omega t$$

Since you haven't been given any details, the amplitudes of the waves don't matter. The phase differences, however, do.



6. A pendulum can only be modelled as a simple harmonic oscillator if the angle over which it oscillates is small. Why is this?

Simple harmonic oscillators work because the force acts in the opposite direction to the displacement. As the pendulum moves away from the area immediately below the peg it is hanging on, the force no longer acts in the opposite direction to the displacement.

Energy in Simple Harmonic Motion

1. A 10g mass causes a spring to extend 5cm. How much energy is stored by the spring?

$$k = \frac{\Delta F}{\Delta x} = \frac{mg}{\Delta x} = \frac{0.01 \times 9.81}{0.05} = 1.962 \text{ Nm}^{-1}$$

$$E = \frac{1}{2}kx^2 = 0.5 \times 1.962 \times 0.05^2 = 2.45 \text{ mJ}$$

2. A 500g mass on a spring ($k=100$) is extended by 0.2m, and begins to oscillate in an otherwise empty universe. What is the maximum velocity which it reaches?

$$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kx_{max}^2$$

$$v_{max}^2 = \frac{kx_{max}^2}{m}$$

$$v_{max} = x_{max} \sqrt{\frac{k}{m}} = 0.2 \times \sqrt{\frac{100}{0.5}} = 2.83 \text{ ms}^{-1}$$

3. Another 500g mass on another spring in another otherwise empty universe is extended by 0.5m, and begins to oscillate. If it reaches a maximum velocity of 15ms^{-1} , what is the spring constant of the spring?

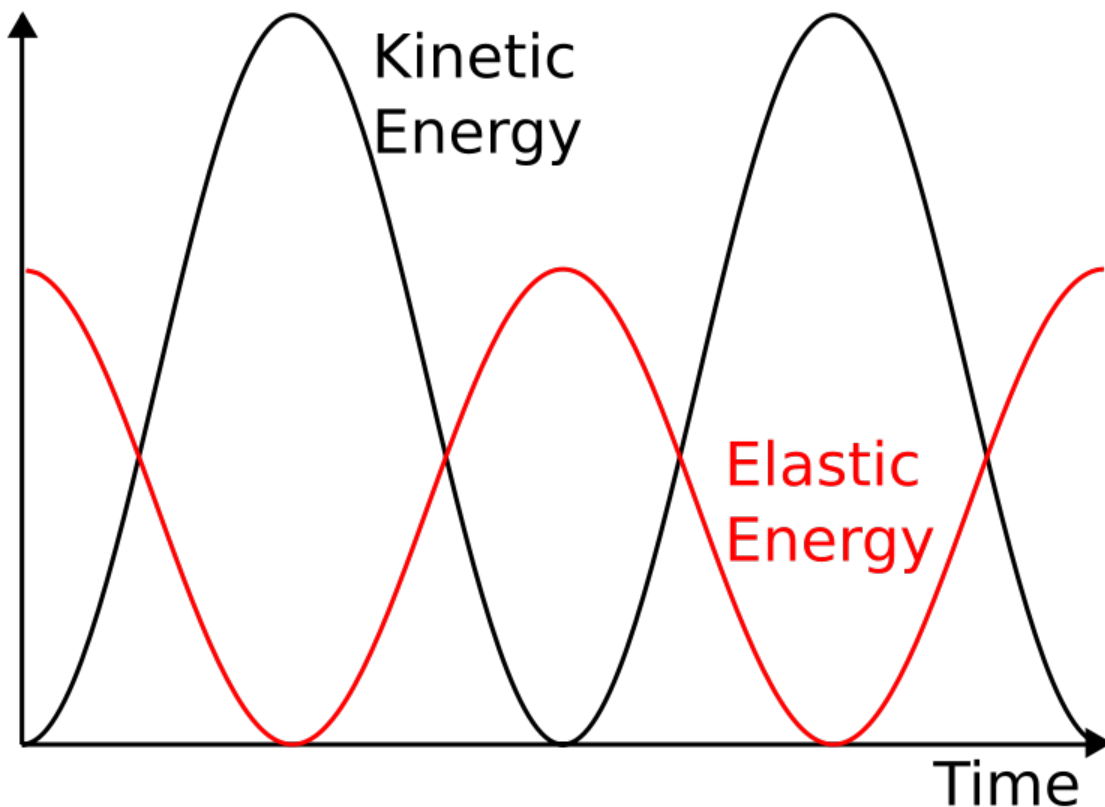
$$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kx_{max}^2$$

$$k = \frac{mv_{max}^2}{x_{max}^2} = \frac{0.5 \times 15^2}{0.5^2} = 450 \text{ Nm}^{-1}$$

4. Draw graphs of the kinetic and elastic energies of a mass on a spring (ignoring gravity).

$$E_e \propto \cos^2 \omega t$$

$$E_k \propto \sin^2 \omega t$$



5. Use the trigonometric formulae for x and v to derive an equation for the total energy stored by an oscillating mass on a spring, ignoring gravity and air resistance, which is constant with respect to time.

$$x = A \cos \omega t$$

$$v = -A\omega \sin \omega t$$

Substitute these into the equation for the total energy:

$$\Sigma E = \frac{1}{2}(kx^2 + mv^2) = \frac{1}{2}(k(A \cos \omega t)^2 + m(-A\omega \sin \omega t)^2) = \frac{1}{2}(kA^2 \cos^2 \omega t + mA^2\omega^2 \sin^2 \omega t) = \frac{A^2}{2}(k \cos^2 \omega t + m\omega^2 \sin^2 \omega t)$$

We know that:

$$\omega = \sqrt{\frac{k}{m}}$$

Therefore:

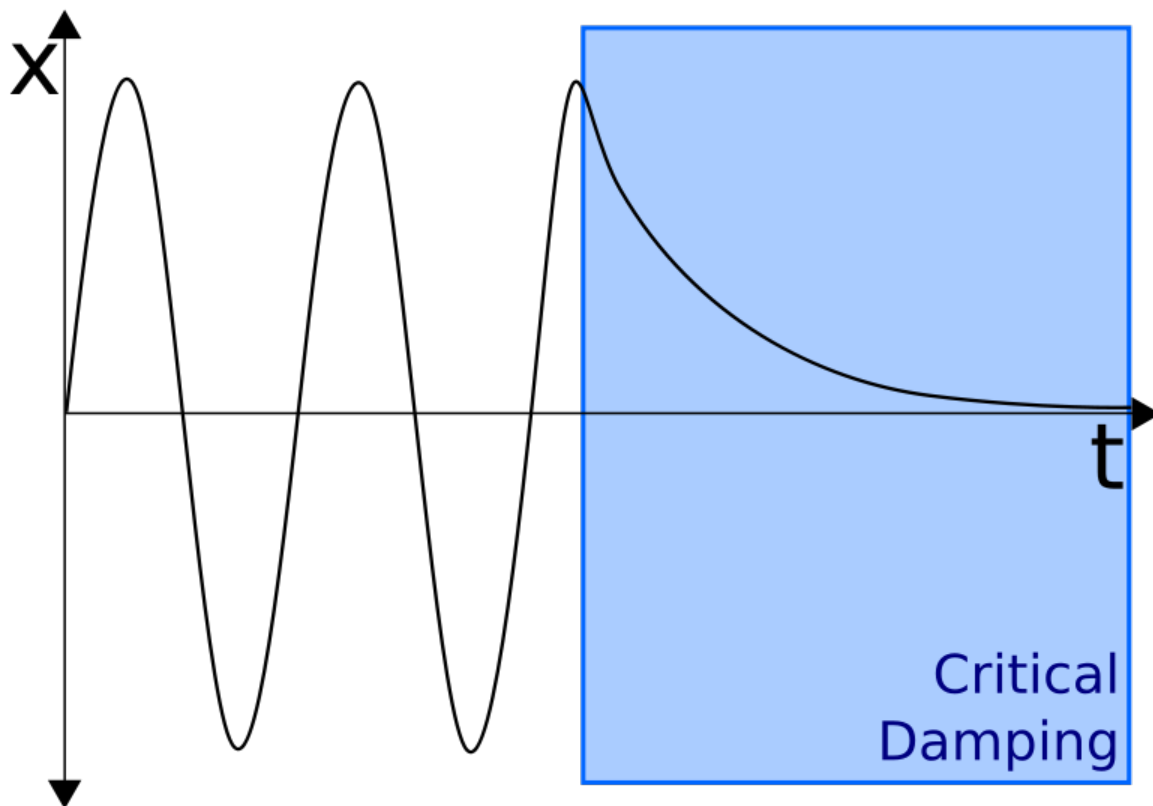
$$\omega^2 = \frac{k}{m}$$

By substitution:

$$\Sigma E = \frac{A^2}{2}(k \cos^2 \omega t + \frac{mk}{m} \sin^2 \omega t) = \frac{A^2}{2}(k \cos^2 \omega t + k \sin^2 \omega t) = \frac{kA^2}{2}(\cos^2 \omega t + \sin^2 \omega t) = \frac{kA^2}{2}$$

Damping

1. Draw a graph of displacement for a critically damped oscillation.



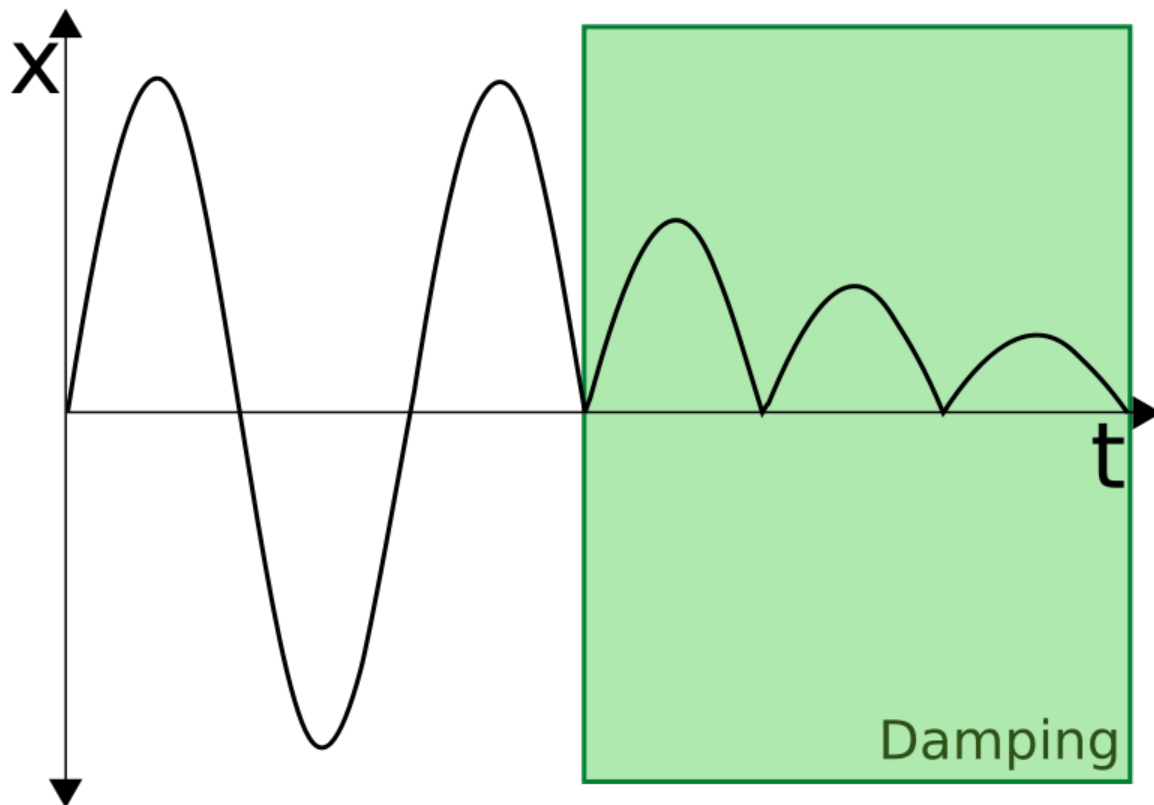
2. How would you critically damp an oscillating pendulum?

Grab the weight, move it to its equilibrium position, and stop it moving.

3. How would you damp an oscillating pendulum using only a weighted polystyrene block?

Put the block in the path of the pendulum, which will bounce off the weight, losing a bit of energy each oscillation.

4. What would the displacement graph look like for this oscillation, before and after damping began?



5. The graph above is an exponentially damped oscillation. If the displacement of the undamped oscillation is given by $\sin \omega t$, what is an approximate equation for the damped oscillation, in terms of a constant k which describes the degree to which the oscillation is damped?

$$x = e^{-k\omega t} \sin \omega t$$

If k doubles, the e^{-kt} will be squashed to half its size along the t -axis, so as k increases, the rate of damping increases.

Conservation of Momentum

1. A ball of mass 0.5kg collides with a stationary ball of 0.6kg at a velocity of 3ms^{-1} . If the stationary ball moves off at a speed of 2ms^{-1} , what is the new velocity of the first ball?

$$0.5 \times 3 = 0.5v + (0.6 \times 2)$$

$$1.5 = 0.5v + 1.2$$

$$0.5v = 0.3$$

$$v = 0.6 \text{ ms}^{-1}$$

2. Two balls are moving in opposite directions with velocities 5ms^{-1} and 10ms^{-1} . They collide, and move off in opposite directions with new velocities of 7.5ms^{-1} each. If the mass of the first ball was 1.25kg, what is the mass of the second ball?

$$(5 \times 1.25) + (-10m) = (-7.5 \times 1.25) + (7.5m)$$

$$6.25 - 10m = 7.5m - 9.375$$

$$17.5m = 15.625$$

$$m = 0.893 \text{ kg}$$

3. A totally elastic collision occurs between two balls of equal mass. One of the balls is stationary. What happens?

Since momentum must be conserved:

$$mu = mv_1 + mv_2$$

$$u = v_1 + v_2(1)$$

Since kinetic energy must be conserved:

$$\frac{mu^2}{2} = \frac{mv_1^2}{2} + \frac{mv_2^2}{2}$$

$$u^2 = v_1^2 + v_2^2$$

Substitute in the value of u from (1):

$$(v_1 + v_2)^2 = v_1^2 + v_2^2$$

$$v_1^2 + 2v_1v_2 + v_2^2 = v_1^2 + v_2^2$$

$$2v_1v_2 = 0$$

Therefore, either v_1 or v_2 is 0. Using equation (1), if v_1 is zero, then $v_2 = u$, and vice versa. However, v_1 cannot be the same as u, as this would mean that the first ball had to move through the second ball! So, the only physical solution is that the first ball stops, and the second ball continues moving with the first ball's original velocity.

4. A particle explodes to become two particles with masses 1kg and 2kg. The 1kg particle moves with velocity 45ms^{-1} . With what velocity does the other particle move?

$$0 = (1 \times 45) + 2v$$

$$2v = -45$$

$$v = -22.5 \text{ ms}^{-1}$$

ie. in the opposite direction to the motion of the 1kg particle.

5. A 3kg ball moving at 3ms^{-1} collides with a 5kg ball moving at -5ms^{-1} . The collision is perfectly elastic. What are the new velocities of the balls?

Since momentum must be conserved:

$$(3 \times 3) + (5 \times -5) = 3v_1 + 5v_2 = -16 \text{ kgms}^{-1}(1)$$

Since kinetic energy must be conserved:

$$\frac{3 \times 3^2}{2} + \frac{5 \times (-5)^2}{2} = \frac{3 \times v_1^2}{2} + \frac{5 \times v_2^2}{2}$$

$$3v_1^2 + 5v_2^2 = 152$$

From (1):

$$v_2 = \frac{-16 - 3v_1}{5}$$

$$3v_1^2 + 5 \left(\frac{-16 - 3v_1}{5} \right)^2 = 152$$

$$3v_1^2 + 5 \left(\frac{9v_1^2 + 96v_1 + 256}{25} \right) = 152$$

$$3v_1^2 + \frac{9v_1^2 + 96v_1 + 256}{5} = 152$$

$$15v_1^2 + 9v_1^2 + 96v_1 + 256 = 760$$

$$24v_1^2 + 96v_1 - 504 = 0$$

$$v_1^2 + 4v_1 - 21 = 0$$

$$(v_1 + 7)(v_1 - 3) = 0$$

So, v_1 is either -7 or 3 ms^{-1} .

If $v_1 = -7 \text{ ms}^{-1}$, by (1):

$$-21 + 5v_2 = -16$$

$$5v_2 = 5$$

$$v_2 = 1 \text{ ms}^{-1}$$

If $v_1 = 3 \text{ ms}^{-1}$:

$$9 + 5v_2 = -16$$

$$5v_2 = -25$$

$$v_2 = -5 \text{ ms}^{-1}$$

This last solution is non-physical since it requires the balls to move through each other. So, $v_1 = -7 \text{ ms}^{-1}$ and $v_2 = 1 \text{ ms}^{-1}$

6. A ball collides with a wall, and rebounds at the same velocity. Why doesn't the wall move?

Let the mass of the wall be M , and the mass of the ball be m :

$$mu = Mv_{wall} - mu$$

$$2mu = Mv_{wall}$$

$$v_{wall} = \frac{2mu}{M}$$

The mass of the wall is large. As M tends to infinity, therefore, v_{wall} tends to 0.

Forces and Impulse in Collisions

1. Escape velocity from the Earth is 11.2kms^{-1} . How much impulse must be exerted on a 47000kg payload to get it to travel away from the Earth?

$$I = m(v - u) = 47000(11200 - 0) = 526.4 \text{ MNs}$$

2. Two billiard balls, of mass 10g , collide. One is moving at 5ms^{-1} , and the other at 2ms^{-1} . After the collision, the first billiard ball is moving backwards at 4ms^{-1} . The collision takes 1 ms . What force was exerted on this ball?

$$I = m(v - u) = 0.01(5 - (-4)) = 0.09 \text{ Ns}$$

$$F = \frac{I}{\Delta t} = \frac{0.09}{0.001} = 90 \text{ N}$$

3. What impulse and force were exerted on the second ball?

The impulse was -0.09Ns and the force was -90N .

4. A 60kg spacewalker uses a jet of gas to exert an impulse of 10Ns . How many times would he have to do this to reach a speed of 1 ms^{-1} from stationary?

$$\Delta v = \frac{I}{m} = \frac{10}{60} = \frac{1}{6} \text{ ms}^{-1}$$

So, the spacewalker would have to do this 6 times to reach a speed of 1ms^{-1} .

5. A 5kg bowling ball collides with a stationary tennis ball of mass 0.1kg at 3ms^{-1} , slowing to 2.5ms^{-1} . It exerts a force of 100N on the ball. How long did the collision take?

$$F = \frac{m(v - u)}{\Delta t}$$

$$\Delta t = \frac{m(v - u)}{F} = \frac{5(3 - 2.5)}{100} = 0.025 \text{ s}$$

Rockets, Hoses and Machine Guns

1. A machine gun fires 300 5g bullets per. minute at 800ms^{-1} . What force is exerted on the gun?

$$F = v \frac{dm}{dt} = 800 \times \frac{300 \times 0.005}{60} = 20 \text{ N}$$

2. 1 litre of water is pumped out of a tank in 5 seconds through a hose. If a 2N force is exerted on the tank, at what speed does the water leave the hose?

The flow rate is about 0.2kgs^{-1} .

$$2 = 0.2v$$

$$v = \frac{2}{0.2} = 10 \text{ ms}^{-1}$$

3. If the hose were connected to the mains, what problems would there be with the above formula?

The force would not be exerted on the tank, but would instead be referred back through the pipe - it is not gaining velocity at the nozzle.

4. The thrust of the first stage of a Saturn V rocket is 34 MN, using 131000kg of solid fuel in 168 seconds. At what velocity does the fuel leave the tank?

$$34 \times 10^6 = v \frac{131000}{168}$$

$$v = \frac{34 \times 10^6 \times 168}{131000} = 43600 \text{ ms}^{-1}$$

5. Escape velocity from the Earth is 11kms^{-1} . What is the velocity of the rocket after the first stage is used up, if the total mass of the rocket is $3 \times 10^6 \text{ kg}$? How does this compare to escape velocity?

$$ma = 34 \times 10^6 - mg$$

$$a = \frac{34 \times 10^6}{3 \times 10^6} - 9.81 = 1.52 \text{ ms}^{-2}$$

$$v = at = 168 \times 1.52 = 255.92 \text{ ms}^{-1} \approx 0.256 \text{ kms}^{-1}$$

This is 0.002% of escape velocity, hence the need for the other stages of the rocket.

Circular Motion

1. A tennis ball of mass 10g is attached to the end of a 0.75m string and is swung in a circle around someone's head at a frequency of 1.5Hz. What is the tension in the string?

$$\omega = 2\pi f = 2\pi \times 1.5 = 3\pi \text{ rad s}^{-1}$$

$$F = T = m\omega^2 r = 0.01 \times (3\pi)^2 \times 0.75 = 0.0675\pi^2 = 0.666 \text{ N}$$

2. A planet orbits a star in a circle. Its year is 100 days, and the distance from the star to the planet is 70 Gm from the star. What is the mass of the star?

$$100 \text{ days} = 100 \times 365.24 \times 24 \times 60 \times 60 = 3155673600 \text{ s}$$

$$f = \frac{1}{T} = \frac{1}{3155673600} = 3.17 \times 10^{-10} \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 3.17 \times 10^{-10} = 1.99 \text{ nrad s}^{-1}$$

$$\frac{GM}{r^2} = \omega^2 r$$

$$M = \frac{\omega^2 r^3}{G} = \frac{(1.99 \times 10^{-9})^2 \times (70 \times 10^9)^3}{6.67 \times 10^{-11}} = 2.04 \times 10^{25} \text{ kg}$$

3. A 2000kg car turns a corner, which is the arc of a circle, at 20kmh⁻¹. The centripetal force due to friction is 1.5 times the weight of the car. What is the radius of the corner?

$$20 \text{ kmh}^{-1} = 20000 / 3600 = 5.56 \text{ ms}^{-1}$$

$$W = 2000 \times 9.81 = 19620 \text{ N}$$

$$F_r = 1.5 \times 19620 = 29430 \text{ N}$$

$$29430 = \frac{mv^2}{r} = \frac{2000 \times 5.56^2}{r} = \frac{61728}{r}$$

$$r = \frac{61728}{29430} = 2.1 \text{ m}$$

This is a bit unrealistic, I know...

4. Using the formulae for centripetal acceleration and gravitational field strength, and the definition of angular velocity, derive an equation linking the orbital period of a planet to the radius of its orbit.

$$\omega^2 r = \frac{GM_{star}}{r^2}$$

$$\omega^2 r^3 = GM_{star}$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{4\pi^2 r^3}{T^2} = GM_{star}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_{star}}$$

So, orbital period squared is proportional to radius of orbit cubed. Incidentally, this is Kepler's Third Law in the special case of a circular orbit (a circle is a type of ellipse).

Radar and Triangulation

1. A radar pulse takes 8 minutes to travel to Venus and back. How far away is Venus at this time?

$$2d = ct = 3 \times 10^8 \times 8 \times 60 = 1.44 \times 10^{11} \text{ m}$$

$$d = 1.44 \times 10^{11} \times 0.5 = 7.2 \times 10^{10} \text{ m}$$

2. Why can't a radar pulse be used to measure the distance to the Sun?

It would be impossible to pick up the reflected signal due to all the other signals coming from the Sun. Also, the signal would almost certainly be absorbed anyway.

3. Radar is used to measure the velocity of a spacecraft travelling between the Earth and the Moon. Use the following data to measure this velocity:

Pulse	Transmission Time	Reception Time
1	3:26:45.31213	3:26:45.51213
2	3:26:46.32742	3:26:46.52785

First, calculate the distance of the spacecraft from the Earth at each time:

$$d_1 = \frac{c\Delta t_1}{2} = \frac{3.0 \times 10^8 \times (45.51213 - 45.31213)}{2} = 30,000 \text{ km}$$

$$d_2 = \frac{c\Delta t_2}{2} = \frac{3.0 \times 10^8 \times (46.52785 - 46.32742)}{2} = 30,064.5 \text{ km}$$

Next, calculate the distance the spacecraft has travelled between the two pulses:

$$\Delta d = d_2 - d_1 = 30,000 - 30,064.5 = 64.5 \text{ km}$$

Now, calculate the time elapsed between the transmission of the two pulses:

$$\Delta t = t_\beta - t_\alpha = 46.32742 - 45.31213 = 1.01529 \text{ s}$$

Finally, divide the distance the spacecraft has travelled between the two pulses by the time between the transmission of the two pulses, to give the average velocity of the spacecraft in that interval of time:

$$v = \frac{\Delta d}{\Delta t} = \frac{64.5}{1.01529} \approx 63.5 \text{ kms}^{-1}$$

4. The angles between the horizontal and a star are measured at midnight on January 1 as 89.99980° and at midnight on June 1 as 89.99982° . How far away is the star?

$$d = \frac{2r \tan a \tan b}{\tan a + \tan b} = \frac{2 \times 150 \times 10^9 \times \tan 89.9998 \tan 89.99982}{\tan 89.9998 + \tan 89.99982} = 4.52 \times 10^{13} \text{ km}$$

5. Why can't triangulation be used to measure the distance to another galaxy?

The difference between the two angles becomes so tiny that we don't have good enough equipment to measure it.

Large Units

1. What is one parsec in m?

$$206265 \text{ AU} \times 150 \times 10^9 = 3.09 \times 10^{16} \text{ m}$$

2. Convert 3 light days into km.

$$c = 3 \times 10^5 \text{ kms}^{-1}$$

$$3 \times 3 \times 10^5 \times 24 \times 60 \times 60 = 7.78 \times 10^{10} \text{ km}$$

3. Convert 5.5 parsecs into light years.

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

$$5.5 \text{ pc} = 5.5 \times 3.09 \times 10^{16} \text{ m} = 1.70 \times 10^{17} \text{ m}$$

Divide by 1 ly in metres to get a final answer of approximately 18ly.

4. The difference in angle of a star on the perpendicular to the plane of the Earth's orbit which passes through the Sun when viewed from either side of the Earth's orbit is 0.1° . How far away is the star in parsecs?





The angle between the perpendicular and the line which goes through the Earth is $0.05^\circ = 180''$. So, the star is 180pc away.

Orbits

1. The semi-major axis of an elliptical orbit can be approximated reasonably accurately by the mean distance of the planet for the Sun. How would you test, using the data in the table above, that the inner planets of the Solar System obey Kepler's Third Law?

Divide T^2 by R^3 and for each planet and see if this value is roughly constant.

2. Perform this test. Does Kepler's Third Law hold?

Planet	Mercury	Venus	Earth	Mars
Picture				
Mean distance from Sun (km)	57,909,175	108,208,930	149,597,890	227,936,640
Orbital period (years)	0.2408467	0.61519726	1.0000174	1.8808476
$\frac{T^2}{R^3}$ (year ² km ⁻³ x 10 ⁻²⁵)	2.987027815	4.85540079	2.986972006	1.588221903

So, Kepler's Third Law does hold for the inner planets, using this rough approximation for the semi-major axis.

3. If $T^2 \propto R^3$, express a constant C in terms of T and R.

$$C = \frac{T^2}{R^3}$$

Or, alternatively:

$$C = \frac{R^3}{T^2}$$

We will be using the former to answer the next two questions, but you should be able to get the same answers using the latter.

4. Io, one of Jupiter's moons, has a mean orbital radius of 421600km, and a year of 1.77 Earth days. What is the value of C for Jupiter's moons?

$$C = \frac{T^2}{R^3} = \frac{(1.77 \times 24 \times 60 \times 60)^2}{(421600000)^3} = 3.12 \times 10^{-16} \text{ s}^2 \text{m}^{-3}$$

5. Ganymede, another of Jupiter's moons, has a mean orbital radius of 1070400km. How long is its year?

$$3.12 \times 10^{-16} = \frac{T^2}{1070400000^3}$$

$$T = \sqrt{3.12 \times 10^{-16} \times 1070400000^3} = 618665 \text{ s} = 7.16 \text{ days}$$

Which isn't that accurate, due to the approximations that we used.

Doppler Effect

1. M31 (the Andromeda galaxy) is approaching us at about 120kms⁻¹. What is its red-shift?

$$z = \frac{v_s}{c} = \frac{-120000}{300000000} = -0.4 \times 10^{-3}$$

The minus sign is important! Andromeda is blue-shifted!

2. Some light from M31 reaches us with a wavelength of 590nm. What is its wavelength, relative to M31?

$$-0.0004 = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1 = \frac{590 \times 10^{-9}}{\lambda_0} - 1$$

$$0.9996 = \frac{590 \times 10^{-9}}{\lambda_0}$$

$$\lambda_0 = \frac{590 \times 10^{-9}}{0.9996} = 590.23 \text{ nm}$$

3. Some light has a wavelength, relative to M31, of 480nm. What is its wavelength, relative to us?

$$0.9996 = \frac{\lambda}{\lambda_0} = \frac{\lambda}{480 \times 10^{-9}}$$

$$\lambda = 0.9996 \times 480 \times 10^{-9} = 479.808 \text{ nm}$$

4. A quasar emits electromagnetic radiation at a wavelength of 121.6nm. If, relative to us, this wavelength is red-shifted 0.2nm, what is the velocity of recession of the quasar?

$$\frac{v_s}{c} = \frac{\Delta\lambda}{\lambda_0}$$

$$\frac{v_s}{3 \times 10^8} = \frac{0.2}{121.6} = 0.00164$$

$$v_s = 3 \times 10^8 \times 0.00164 = 493 \text{ kms}^{-1}$$

However, this is about as high a velocity as we can use the classical Doppler effect for.

The Big Bang

1. What is the Hubble Constant in s^{-1} ?

$$H_0 = 70 \text{ kms}^{-1} \text{Mpc}^{-1} = 0.07 \text{ ms}^{-1} \text{pc}^{-1}$$

$$1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$$

$$H_0 = \frac{0.07}{3.09 \times 10^{16}} = 2.27 \times 10^{-18} \text{ s}^{-1}$$

2. How old is the universe?

$$\frac{1}{H_0} = 4.41 \times 10^{17} \text{ s} = 14.0 \text{ bn years}$$

3. What effect might gravity have had on this figure?

Gravity would slow the expansion of the universe, so the age of the universe would actually be lower than 14.0 billion years.

4. Polaris is 132pc away. What is its velocity of recession, according to Hubble's Law?

$$v = H_0 d = 70 \times 132 \times 10^{-6} = 9.24 \text{ ms}^{-1}$$

Heat and Energy

1. Carbon dioxide sublimates at 195°K. Roughly what energy per. particle does this correspond to?

$$E \approx kT = 1.38 \times 10^{-23} \times 195 \approx 10^{-21} \text{ J particle}^{-1}$$

2. A certain chemical reaction requires particles with mass of the order 10^{-26} to move, on average, at 10ms^{-1} . Roughly what temperature does this correspond to?

$$E_k = \frac{1}{2}mv^2 = 0.5 \times 10^{-26} \times 10 = 5 \times 10^{-26}$$

$$5 \times 10^{-26} = kT$$

$$T = \frac{5 \times 10^{-26}}{1.38 \times 10^{-23}} \approx 10^{-3} \text{°K}$$

3. The boiling point of water is 100°C. Roughly what energy per. particle does this correspond to?

$$100^\circ\text{C} = 373^\circ\text{K}$$

$$E \approx kT = 1.38 \times 10^{-23} \times 373 \approx 5 \times 10^{-21} \text{ J particle}^{-1}$$

4. Thermionic emission from copper requires around 5eV of energy per. particle. How hot will the wire be at this energy level?

$$5\text{eV} = 5 \times 1.6 \times 10^{-19} = 8 \times 10^{-19} \text{ J particle}^{-1}$$

$$T = \frac{8 \times 10^{-19}}{1.38 \times 10^{-23}} \approx 6 \times 10^4 \text{°K}$$

Specific Heat Capacity

1. How much energy would it take to heat 100kg of liquid water from 20°C to 36.8°C?

$$\Delta E = mc\Delta\theta = 100 \times 4180 \times 16.8 = 7.022 \text{ MJ}$$

2. How much energy would it take to heat a well-insulated room from 15°C to 21°C, if the room is a cube with side length 10m, and the density of the air is 1.2kgm⁻³?

$$V = 10^3 = 1000 \text{ m}^3$$

$$\rho = \frac{m}{V}$$

$$m = V\rho = 1000 \times 1.2 = 1200 \text{ kg}$$

$$\Delta E = mc\Delta\theta = 1200 \times 1010 \times 6 = 7.272 \text{ MJ}$$

3. A 10kg block of iron at 80°C is placed in the room above once it has reached 21°C. If the iron cools by 40°C, what is the new temperature of the room?

Energy must be conserved, so:

$$10 \times 450 \times 40 = 1200 \times 1010 \times \Delta\theta$$

$$\Delta\theta = \frac{180000}{1212000} = 0.149^\circ\text{C}$$

So, the new temperature of the room is 21.149°C.

Ideal Gases

1. I heat some argon from 250°K to 300°K. If the pressure of the gas at 250°K is 0.1 MPa, what is its pressure after heating?

The temperature has increased by 20%, so the pressure will also increase by 20% (using the pressure law). So, the new pressure is 0.1 MPa x 1.2 = 0.12 MPa.

2. The argon is in a 0.5m long cylindrical tank with radius 10cm. What volume does it occupy?

$$V = \pi r^2 l = \pi \times 0.1^2 \times 0.5 = 0.016 \text{ m}^3$$

3. The argon is then squeezed with a piston so that it only occupies 0.4m of the tank's length. What is its new pressure?

The volume occupied decreases by 20%, which is equivalent to multiplying by 0.8. So, by Boyle's Law, the pressure is multiplied by the reciprocal of 0.8 - 1.25. So, the pressure increases by 25% to 0.15 MPa.

4. What is its new temperature?

If we combine Charles' Law with the pressure law, we find that:

$$T \propto pV$$

Volume is multiplied by 0.8, but pressure is multiplied by 1.25, so there is no change in temperature - it remains at 300°K.

5. 25% of the argon is sucked out. What is its pressure now?

Using the amount law, if N decreases by 25%, the pressure must also decrease by 25% to 0.1125 MPa.

Kinetic Theory

1. Five molecules are moving at speeds of 1,5,6,8, and 36ms⁻¹. What is their mean square speed?

$$\bar{c}^2 = \frac{1^2 + 5^2 + 6^2 + 8^2 + 36^2}{5} = 284.4 \text{ m}^2\text{s}^{-2}$$

2. What is the mass of one molecule of N₂ (atomic mass 14, 1u = 1.66 x 10⁻²³kg)?

$$2 \times 14 \times 1.66 \times 10^{-23} = 4.648 \times 10^{-22} \text{ kg}$$

3. Atmospheric pressure is 101,325 Pa. If one mole of Nitrogen takes up 2.3 m³ at about 10°C, what is the mean square speed of the molecules in the air outside, assuming that the atmosphere is 100% nitrogen (in reality, it is only 78%)?

$$pV = \frac{1}{3}Nmc^2$$

$$101235 \times 2.3 = \frac{1}{3} \times 6.02 \times 10^{23} \times 6.648 \times 10^{-22} \bar{c}^2$$

$$\bar{c}^2 = \frac{3 \times 101235 \times 2.3}{6.02 \times 10^{23} \times 6.648 \times 10^{-22}} = 1745 \text{ m}^2\text{s}^{-2}$$

4. What is the average speed of a nitrogen molecule under the above conditions?

$$\bar{c} = \sqrt{\bar{c}^2} = \sqrt{1745} = 41.8 \text{ ms}^{-1}$$

5. The particles in question 1 are duplicated 3000 times. If they have a completely unrealistic mass of 1g, what is their pressure when they are crammed into a cube with side length 0.5m?

$$p = \frac{Nmc^2}{3V} = \frac{5 \times 3000 \times 10^{-3} \times 284.4}{3 \times 0.5^3} = 11376 \text{ Pa}$$

Boltzmann Factor

$$1u = 1.66 \times 10^{-27} \text{ kg}$$

$$g = 9.81 \text{ ms}^{-2}$$

1. A nitrogen molecule has a molecular mass of 28u. If the Earth's atmosphere is 100% nitrous, with a temperature of 18°C, what proportion of nitrogen molecules reach a height of 2km?

$$\epsilon = mgh = 28 \times 1.66 \times 10^{-27} \times 9.81 \times 2000 = 9.12 \times 10^{-22} \text{ J}$$

$$\frac{n}{n_0} = e^{\frac{-9.12 \times 10^{-22}}{1.38 \times 10^{-23} \times 291}} = 0.797$$

So, 79.7% reach a height of at least 2km.

2. What proportion of the molecules in a box of hydrogen (molecular mass 2u) at 0°C have a velocity greater than 5ms⁻¹?

$$\epsilon = \frac{1}{2}mv^2 = 0.5 \times 2 \times 1.66 \times 10^{-27} \times 5^2 = 4.15 \times 10^{-26} \text{ J}$$

$$\frac{n}{n_0} = e^{\frac{-4.15 \times 10^{-26}}{1.38 \times 10^{-23} \times 273}} = 0.99999$$

(Practically all of them.)

3. What is the temperature of the hydrogen if half of the hydrogen is moving at at least 10ms⁻¹?

$$\epsilon = \frac{1}{2}mv^2 = 0.5 \times 2 \times 1.66 \times 10^{-27} \times 10^2 = 1.66 \times 10^{-25} \text{ J}$$

$$0.5 = e^{\frac{-\epsilon}{kT}}$$

$$\ln 0.5 = -\ln 2 = \frac{-\epsilon}{kT}$$

$$T = \frac{\epsilon}{k \ln 2} = \frac{1.66 \times 10^{-25}}{1.38 \times 10^{-23} \times \ln 2} = 0.0174 \text{ K}$$

(almost absolute zero)

4. Some ionised hydrogen (charge -1.6 x 10⁻¹⁹ C) is placed in an uniform electric field. The potential difference between the two plates is 20V, and they are 1m apart. What proportion of the molecules are at least 0.5m from the positive plate (ignoring gravity) at 350°K?

$$\epsilon = 0.5 \times 20 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-18} \text{ J}$$

$$\frac{n}{n_0} = e^{\frac{-1.6 \times 10^{-18}}{1.38 \times 10^{-23} \times 350}} \approx 0$$

They would all fly onto the positive plate.

Magnetic Flux

1. A circular steel core has a cross-sectional area of 9 cm^2 , and a length of 0.5 m . If the permeability of steel is $875 \mu\text{NA}^{-2}$, what is the permeance of the core?

$$\Lambda = \frac{\mu A}{L} = \frac{875 \times 10^{-6} \times 9 \times 10^{-4}}{0.5} = 1.58 \mu\text{WbA}^{-1}$$

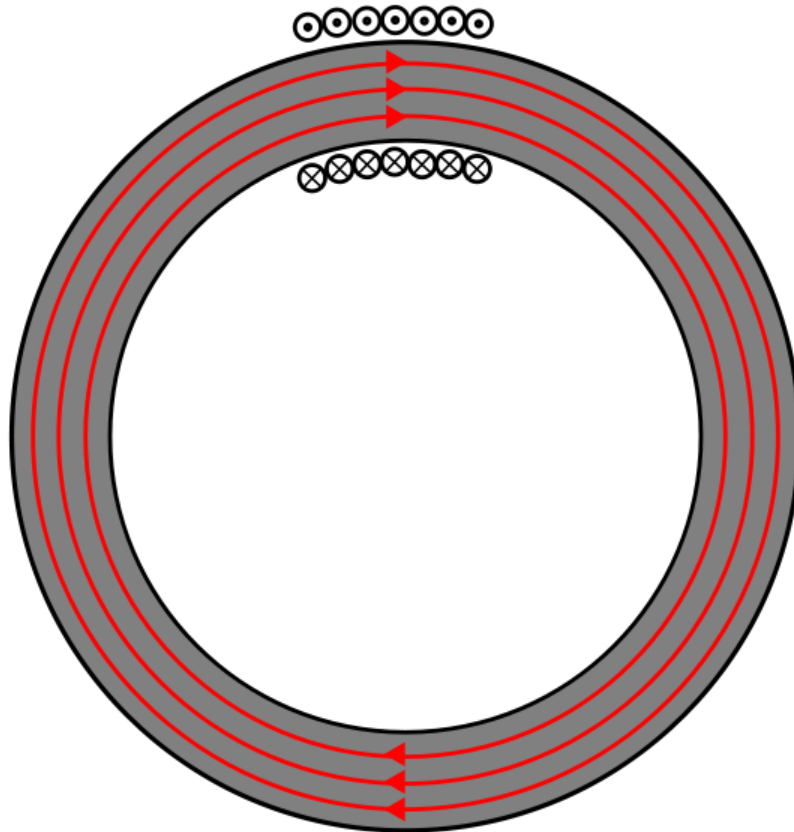
2. A coil of insulated wire is wrapped 60 times around the top of the core, and a 9A direct current is put through the coil. How much flux is induced?

$$\Phi = \Lambda NI = 1.58 \times 10^{-6} \times 60 \times 9 = 0.851 \text{ mWb}$$

3. Assuming that all the flux goes through the core, what is the flux density at any point in the core?

$$B = \frac{\Phi}{A} = \frac{0.851 \times 10^{-3}}{9 \times 10^{-4}} = 0.945 \text{ Wbm}^{-2}$$

4. Draw a diagram showing the lines of flux within the core.



Induction

1. What is the flux linkage of a 30cm coil of 0.5mm thick wire with a flux perpendicular to it of 10Wb?

$$N = \frac{30 \times 10^{-2}}{0.5 \times 10^{-3}} = 600 \text{ coils}$$

$$N\Phi = 600 \times 10 = 6000 \text{ Wb}$$

2. If the above coil is crushed steadily over a period of 2s, what emf is maintained?

$$|\epsilon| = \frac{dN\Phi}{dt} = \frac{6000}{2} = 3000 \text{ V}$$

This principle of removing the flux linkage rapidly (although not usually by destroying one's equipment) is used to create very high voltages for short periods of time.

3. The flux in a flux circuit varies according to the equation $\Phi = \sin \omega t$. What is the equation for the emf induced?

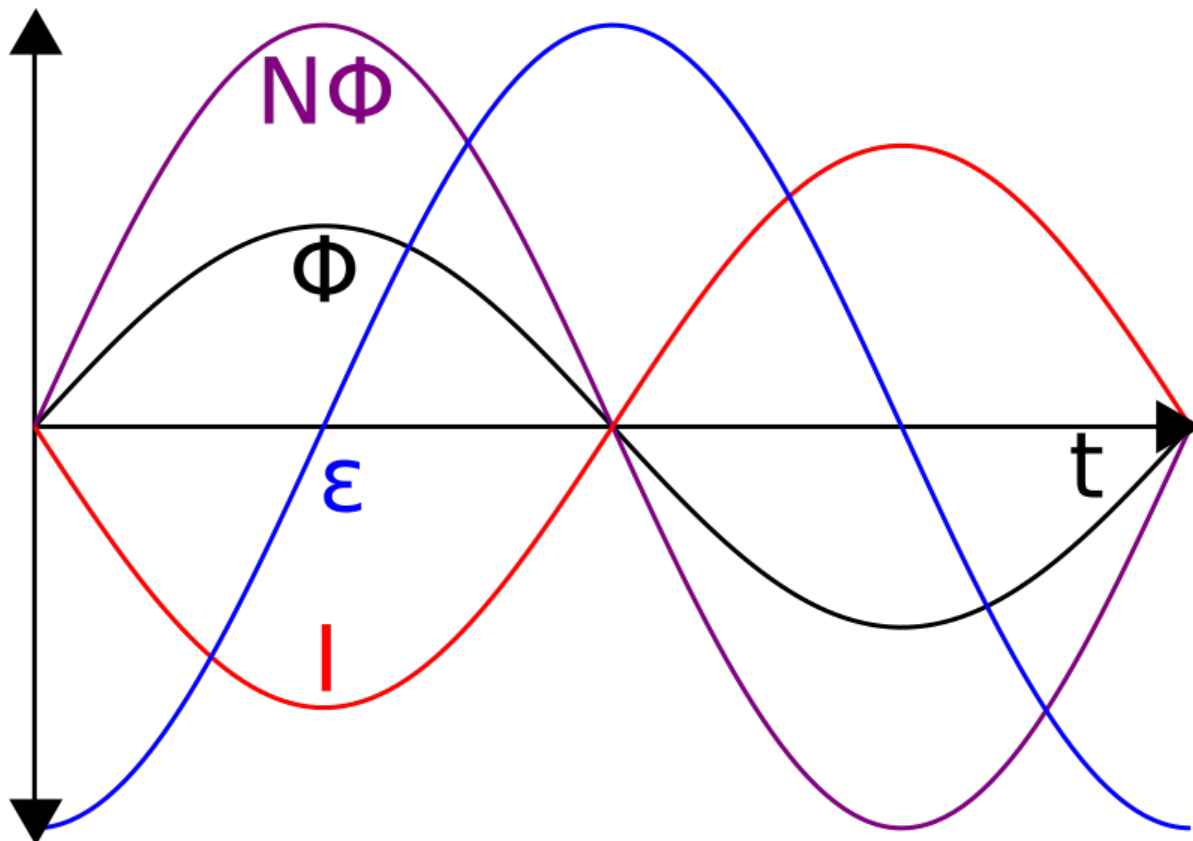
$$\epsilon = -\frac{dN\Phi}{dt} = -N \frac{d}{dt} \sin \omega t = -N\omega \cos \omega t$$

4. Using a constant k, what is the equation for a current which could induce the flux in the flux circuit above?

$$I = -k \sin \omega t$$

5. Draw a graph of the flux, flux linkage, emf and current as deduced in the previous two questions.

In order to keep the scales sane, I used a 2-coil ($N=2$) coil to plot the following graph:



Remember that the current here is inducing the flux (and flux linkage), which is in turn inducing the emf.

Magnetic Force

1. What force is exerted by a 1T magnetic field on an electron (of charge $-1.6 \times 10^{-19}\text{C}$) moving at 5% of the speed of light ($3 \times 10^8 \text{ms}^{-1}$)?

$$F = Bqv = 1 \times -1.6 \times 10^{-19} \times 0.05 \times 3 \times 10^8 = 2.4 \times 10^{-12} \text{N}$$

This may seem tiny, but remember that the acceleration of the electron will be far greater, as it has such a small mass. The fact that the electron was moving at $0.05c$ means that we don't have to do any relativistic corrections.

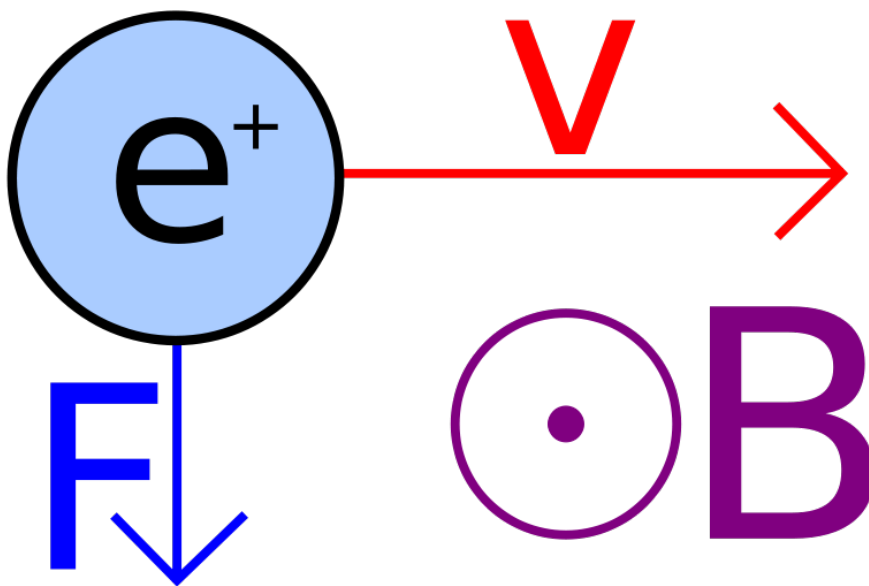
2. What force is exerted by a 5mT magnetic field on a 20cm wire with resistance $1\mu\Omega$ attached to a 9V battery?

$$V = IR$$

$$I = \frac{V}{R}$$

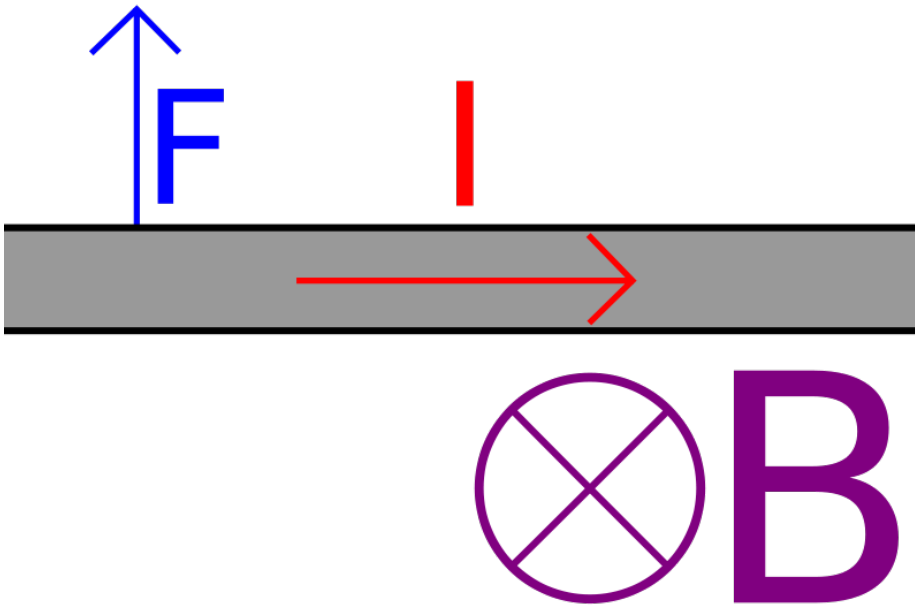
$$F = BIl = \frac{BVI}{R} = \frac{5 \times 10^{-3} \times 9 \times 0.2}{10^{-6}} = 9 \text{kN}$$

3. The following diagram shows a positive charge moving through a magnetic field. Draw an arrow representing the direction of the force on the charge.



Note that, if the charge had been negative, the force would be in the opposite direction (upwards).

4. The following diagram shows a wire in a magnetic field. Draw an arrow representing the direction of the force on the wire.



Transformers

1. A step-down transformer has 300 coils on one coil, and 50 coils on the other. If 30 kV AC is put in, what voltage comes out?

$$\frac{300}{50} = \frac{30000}{V_s} = 6$$

$$V_s = \frac{30000}{6} = 5 \text{ kV}$$

2. A step-up transformer has 200 coils on one coil, and 980 coils on the other. If 25 kV AC comes out, what voltage was put in?

$$\frac{200}{980} = \frac{V_p}{25000}$$

$$V_p = \frac{25000 \times 200}{980} \approx 5.1 \text{ kV}$$

3. An ideal transformer transforms a 50A current into a 1A current. It has 40 coils on the primary coil. How many coils are in the secondary coil?

$$\frac{50}{1} = \frac{N_s}{40} = 50$$

$$N_s = 40 \times 50 = 2000 \text{ coils}$$

4. Transformers tend to vibrate. Why is this? What effect does this have on the efficiency of the transformer?

There are wires with current in them going perpendicularly through a magnetic field. According to $F = BIl$, a force is exerted on the wires. This means that some of the energy is converted into kinetic energy, reducing the efficiency of the transformer.

5. Air does have some permeability. What effect does this have on the efficiency of the transformer? Why?

Some of the flux passes through the air instead of the core, and so does not pass through the secondary coil, and is not used to create AC. This means that some of the energy put in oscillates in and out of the magnetic field, reducing the efficiency of the transformer.

Motors

1. How could you adapt the simple DC motor to use AC?

Connect the coil in the stator in series with the coil in the rotor via a slip-ring commutator. This must be in the correct direction to ensure that the fields created by the two parts of the coil are constantly repelling each other, rather than constantly attracting each other.

2. Why does a three-phase motor have a constant angular velocity?

The vectorial sum of the three currents is constant, so the vectorial sum of the forces on the rotor due to each of the three currents is constant.

3. What is the difference between a split-ring and a slip-ring commutator?

A split-ring commutator makes the current change direction every half-rotation, whereas a slip-ring commutator merely maintains a connection between the moving rotor and the stationary stator.

4. How could the angular velocity of a three-phase motor be increased?

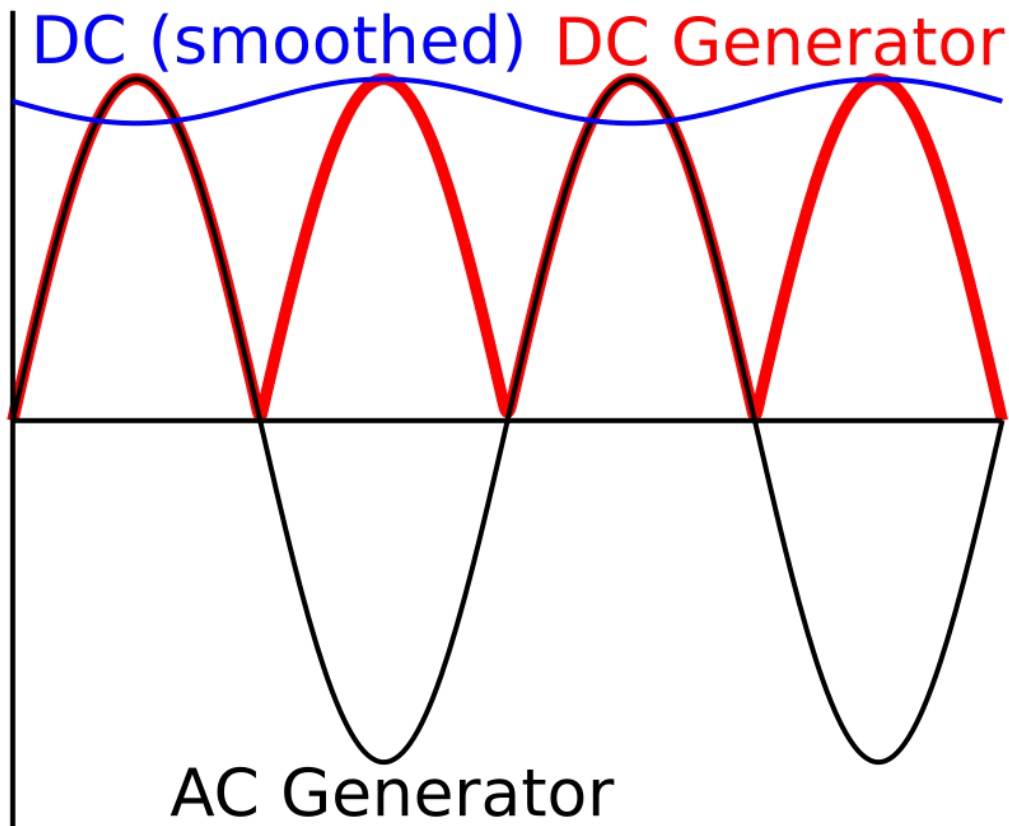
Increase the frequency of the three-phase power.

5. A squirrel-cage motor relies on eddy currents running along the rotor to function. However, if eddy currents run across the rotor, then the force on the rotor is reduced. How may these eddy currents be reduced without reducing the desired eddy currents?

Laminate the plates which connect the rods to each other so that currents are restricted from running in the wrong direction.

Generators

1. Draw diagrams of an alternating current, the 'direct current' produced by a DC generator, and this current once it has been smoothed with a capacitor.



2. What is the phase difference (in radians) between the voltages produced by a three-phase generator?

$$\frac{2\pi}{3}$$

3. According to Faraday's law, what three things will increase the amplitude of the emf created by a generator?

$$\epsilon = - \frac{dN\phi}{dt}$$

So, more coils, a stronger magnetic field or a higher frequency of rotation will increase the amplitude of the emf.

4. If an albatross touched two power cables carrying AC in phase, what would happen?

Nothing - there is no potential difference between the two cables.

5. What would happen if the two cables carried three-phase power?

Unfortunately, dead albatross - there is potential difference between the two cables.

Electric Force

$$e = 1.6 \times 10^{-19} \text{C}$$

1. A positron (charge +e) is 1 μm from a lithium nucleus (charge +3e). What is the magnitude of the force acting on each of the particles? In what direction is it acting?

$$F_{\text{electric}} = \frac{kQq}{r^2} = \frac{8.99 \times 10^9 \times 1 \times 3 \times (1.6 \times 10^{-19})^2}{(10^{-6})^2} = 6.90 \times 10^{-16} \text{ N}$$

This may seem tiny, but the mass of an electron is also tiny, so the acceleration is considerable.

2. An electron is 1mm from the positively charged plate in an uniform electric field. The potential difference between the plates is 20V, and the plates are 10cm apart. What force is acting on the electron? In what direction?

The 1mm is a red herring.

$$F_{\text{electric}} = \frac{qV}{d} = \frac{1.6 \times 10^{-19} \times 20}{0.1} = 3.2 \times 10^{-17} \text{ N}$$

3. The acceleration due to gravity around a point mass is constant, irrespective of the mass of the objects it is acting on. The acceleration due to electricity around a point charge is not. Use Newton's Second Law ($F=ma$) to explain this.

$$a = \frac{F}{m}$$

$$a_{\text{grav}} = \frac{GMm}{mr^2} = \frac{GM}{r^2}$$

$$a_{\text{electric}} = \frac{kQq}{mr^2}$$

The m cancels for gravitational acceleration, but not for electrical acceleration, since charge does not provide a resistance to a force.

4. An insulator contains charged particles, even though the overall charge on the insulator is 0. Why is the insulator attracted by a nearby charge?

The electrons in the atoms can move within certain bounds. Being in an electric field means that they spend more time closer to a positive charge, and so the parts of the insulator which are attracted to the charge end up being closer to the charge than those which are repelled. This means that the attractive force on the insulator is greater than the repulsive force. This causes the insulator as a whole to be attracted to the charge if it is positive. If it is negative, the reverse happens, and the insulator is repelled.

5. Where in the charged conducting plates which create an uniform electric field would you expect to find the charge located? Why?

The charges are free to move within a conductor. The opposite charges in each plate are attracted to each other and try to move as close to each other as possible. So, they end up on the inside edge of the plates.

Electric Field

1. Two metal plates are connected to a 9V battery with negligible internal resistance. If the plates are 10cm apart, what is the electric field at either of the plates?

$$E_{electric} = \frac{V_{electric}}{d} = \frac{9}{0.1} = 90 \text{ Vm}^{-1}$$

2. What is the electric field at the midpoint between the plates?

The whole point of a *uniform* electric field is that the field does not change anywhere between the plates - at the midpoint, as anywhere, it is 90 NC^{-1} .

3. The charge on an electron is $-1.6 \times 10^{-19} \text{ C}$. What is the electric field $1\mu\text{m}$ from a hydrogen nucleus?

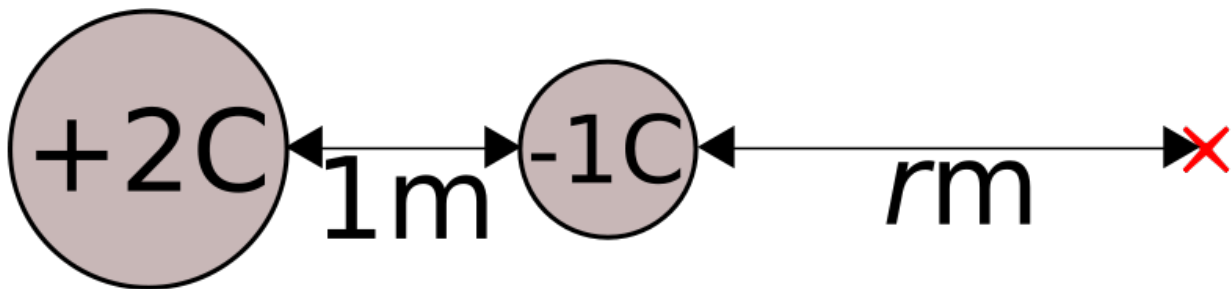
$$E_{electric} = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (10^{-6})^2} = 1440 \text{ Vm}^{-1}$$

4. What is the direction of this field?

The hydrogen nucleus has a positive charge, so the electric field goes away from the nucleus (by convention).

5. A 2C charge is placed 1m from a -1C charge. At what point will the electric field be 0?

Define the distance r as shown:



$$\frac{2}{4\pi\epsilon_0(1+r)^2} = \frac{1}{4\pi\epsilon_0 r^2}$$

$$\frac{2}{(1+r)^2} = \frac{1}{r^2}$$

$$\frac{(1+r)^2}{2} = r^2$$

$$(1+r)^2 = 2r^2$$

$$1+r = r\sqrt{2}$$

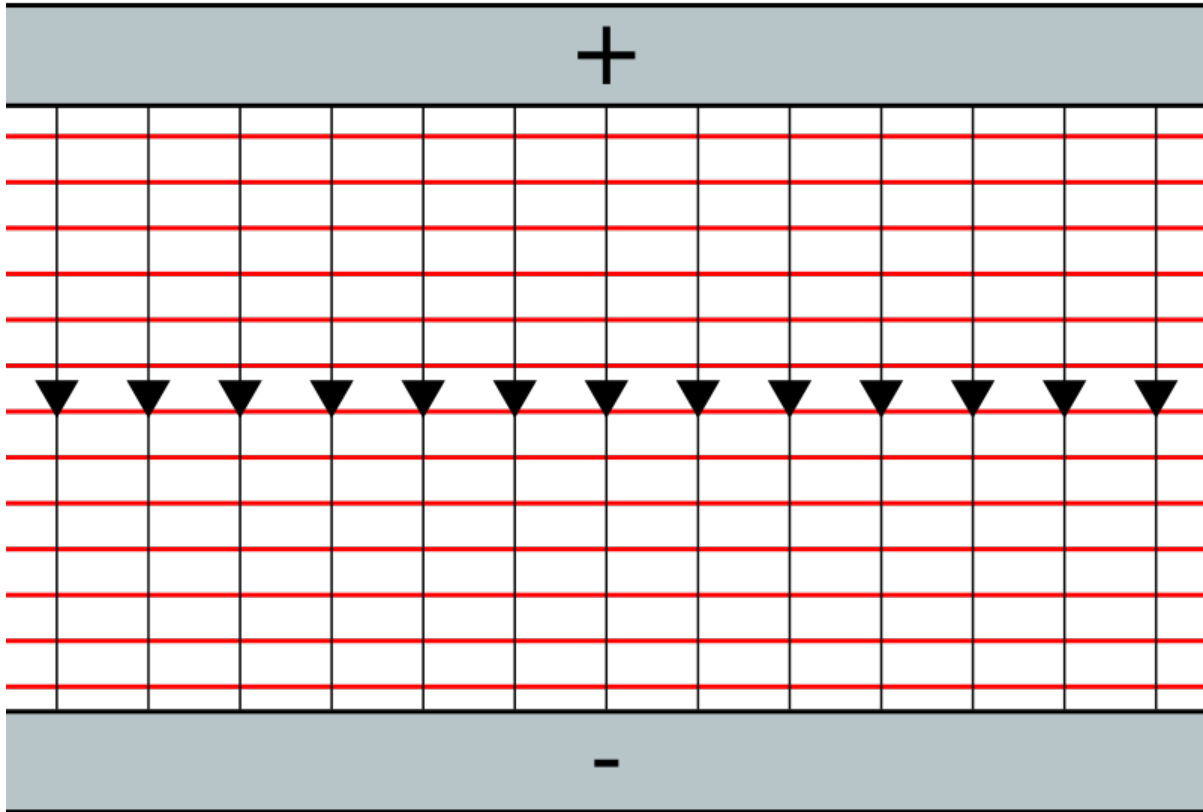
$$r(\sqrt{2}-1) = 1$$

$$r = \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)} = \sqrt{2}+1 \approx 2.41 \text{ m}$$

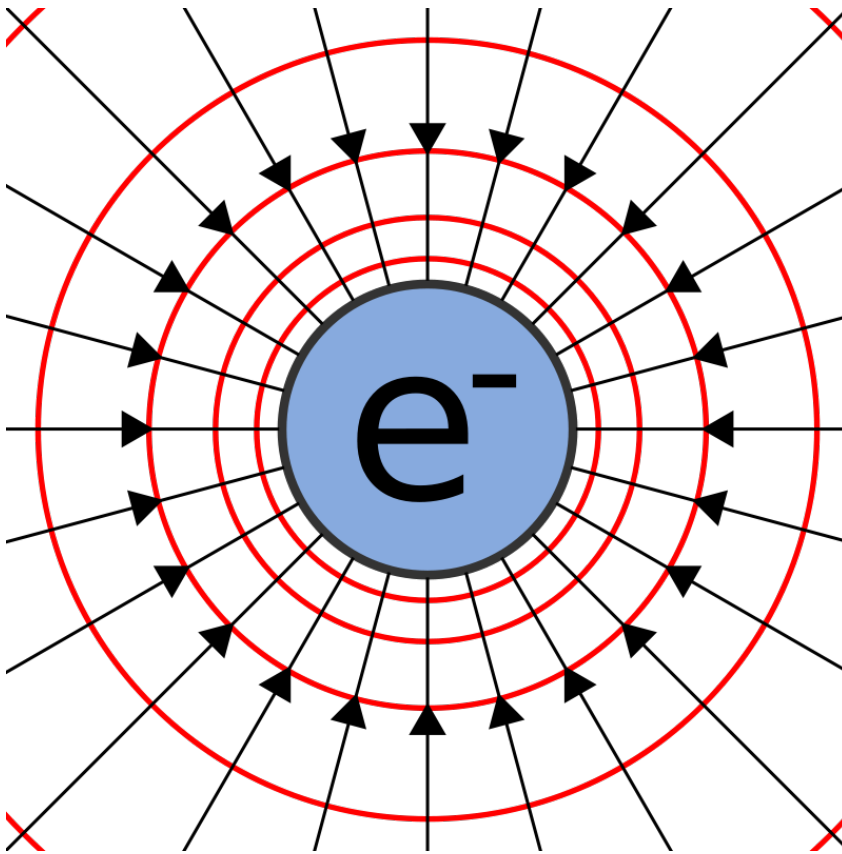
Electric Potential

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$$

1. Draw a diagram of an uniform electric field between two plates, showing the field lines and the equipotentials.



2. Do the same for the electric field around a point charge.



3. The potential difference between two plates is 100V. What is the potential difference between a point halfway between the plates and one of the plates?

Half of 100 is 50V.

4. What is the electric potential at a point 0.2m from an alpha particle (charge on an electron = $-1.6 \times 10^{-19}\text{C}$)?

The charge on an alpha particle is $3.2 \times 10^{-19}\text{C}$.

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{3.2 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times 0.2} = 1.44 \times 10^{-8} \text{ V}$$

5. What is the electric potential energy of an electron at the negative electrode of an electron gun if the potential difference between the electrodes is 10V?

$$U = Vq = 10 \times -1.6 \times 10^{-19} = -1.6 \times 10^{-18} \text{ J} = -10 \text{ eV}$$

Electric Potential Energy

$$k = 8.99 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

1. Convert 5×10^{-13} J to MeV.

$$\frac{5 \times 10^{-13}}{1.6 \times 10^{-19}} = 3125000 \text{ eV} = 3.125 \text{ MeV}$$

2. Convert 0.9 GeV to J.

$$0.9 \times 10^9 \times 1.6 \times 10^{-19} = 1.44 \times 10^{-10} \text{ J}$$

3. What is the potential energy of an electron at the negatively charged plate of an uniform electric field when the potential difference between the two plates is 100V?

$$\epsilon_{elec} = V_{elec}q = 100 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-17} \text{ J} = 0.1 \text{ keV}$$

4. What is the potential energy of a 2C charge 2cm from a 0.5C charge?

$$\epsilon_{elec} = \frac{kQq}{r} = \frac{8.99 \times 10^9 \times 2 \times 0.5}{0.02} = 4.5 \times 10^{11} \text{ J}$$

Remember, coulombs are big!

5. What is represented by the gradient of a graph of electric potential energy against distance from some charge?

Force exerted on the charge with that energy.

The Standard Model

1. The third generation top quark was the last quark in the Standard Model to have its existence proven experimentally (in 1995). It is also the most massive of the quarks. Why was it so difficult to observe a top quark?

Creating massive particles requires a lot of energy. It took a long time to develop a particle accelerator which accelerated other particles to the energy required to create top quarks.

2. What observable phenomena does the Standard Model not explain?

Gravity, as well as mass, and the large number of constants in the Standard Model.

3. How much more massive is an up quark than an electron?

$$\frac{2.4}{0.511} = 4.7 \text{ times}$$

4. How many fermions are there in the Standard Model?

$12 + 12 = 24$ (remember the antiparticles)

5. The antiparticle of the electron (e^-) is the positron. What is the charge and rest mass of a positron?

Charge: $+1.6 \times 10^{-19}\text{C}$

Mass: $0.511 \text{ MeV} = 8.176 \times 10^{-14} \text{ kg}$

Quarks

1. The Δ^{++} baryon is made up of up quarks. What is its total charge?

Baryons are made of three quarks. $3 \times +\frac{2}{3} = +2e$

2. The Δ^- baryon has a total charge of $-1e$. Given that it is made up of only one type of first generation quark, what is this quark?

Three identical quarks must have a total charge of $-1e$, so one of these quarks has a charge of $-\frac{1}{3}e$. This is a down quark.

3. What is an antiproton made of? What is its charge?

$\bar{u}\bar{u}\bar{d}$, total charge $-1e$.

4. A K^+ meson is made of an up quark and an anti strange quark. What is its total charge?

$\frac{2}{3} + \frac{1}{3} = +1e$

5. Lambda (Λ) baryons are made up of an up quark, a down quark, and another quark (not an antiquark). The Λ^0 is neutral, and contains a second generation quark. What is this quark?

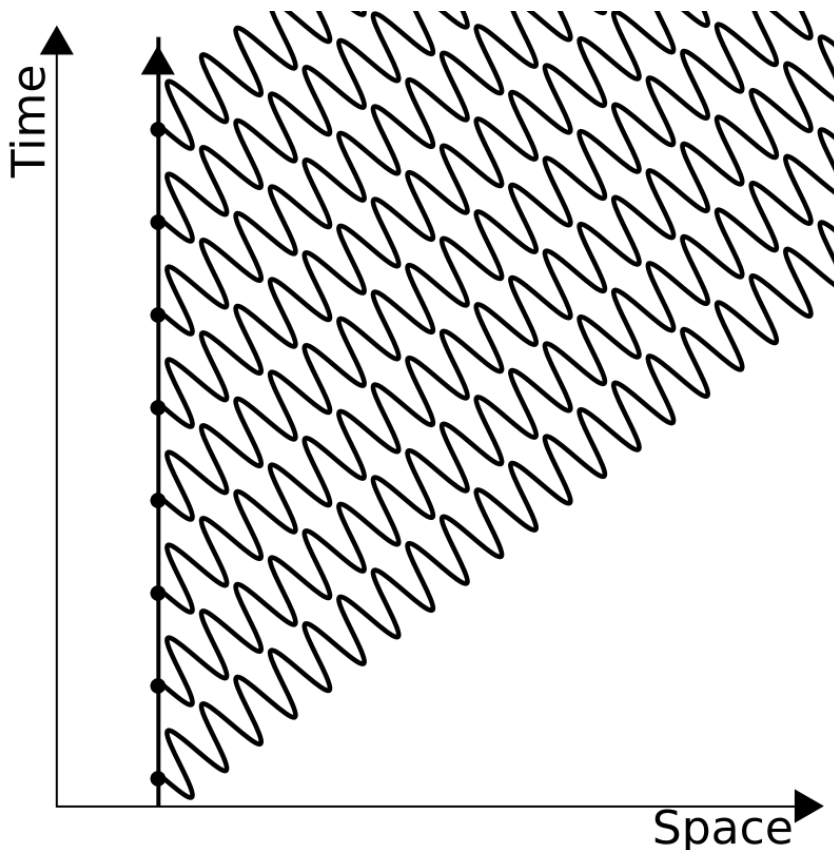
$0 = +\frac{2}{3} - \frac{1}{3} + q$

$q = -\frac{1}{3}$

The second generation quark with a charge of $-\frac{1}{3}e$ is the strange quark.

Bosons

1. A stationary light source emits single photons at regular intervals. Draw a Feynman diagram to represent this.



2. Write two equations (including a W^+ boson) which describe positron emission.

$$p \rightarrow n + W^+$$

$$W^+ \rightarrow \nu + e^+$$

3. What is the charge on a W^- boson?

$-1.6 \times 10^{-19} \text{C}$, since it has to balance the charge on a proton.

4. Read Richard Feynman's excellent book, "QED - the Strange Theory of Light and Matter", ISBN 978-0-140-12505-4.

Don't look here. I can't read a book for you. Visit your library, bookshop, or whatever.

Leptons

1. An electron is produced by a nuclear reaction, but an electron-antineutrino is not produced. What other particle is produced?

A positron, since this is the only other first-generation particle with a lepton number of -1.

2. Why do electrons not make up part of the nucleus?

Electrons do not interact with the strong *nuclear* force.

3. Why did it take until the 1950s to detect the first antineutrino?

They are tiny, chargeless, and almost massless.

4. Complete the following equation for the emission of a beta particle from a nucleus:

$${}^1_0n \rightarrow {}^1_1p + {}^0_{-1}e^- + {}^0_0\bar{\nu}_e$$

5. Complete the following equation for the emission of an antielectron from a nucleus:

$${}^1_1p \rightarrow {}^1_0n + {}^0_1e^+ + {}^0_0\nu_e$$

6. Complete the following equation for the capture of an electron by a nucleus:

$${}^1_1p + {}^0_{-1}e^- \rightarrow {}^1_0n + {}^0_0\nu_e$$

Millikan's Experiment

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$g = 9.81 \text{ ms}^{-2}$$

1. Rearrange the formula above in terms of q.

$$\frac{qV}{d} = mg$$

$$qV = mgd$$

$$q = \frac{mgd}{V}$$

2. The mass of an oil drop cannot be measured easily. Express the mass of an oil drop in terms of its radius r and its density ρ , and, by substitution, find a more useful formula for q.

$$\rho = \frac{m}{\text{Volume}}$$

$$m = \text{Volume} \times \rho$$

Assuming that the oil drop is spherical:

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$m = \frac{4}{3}\rho\pi r^3$$

$$q = \frac{4\pi\rho gdr^3}{3V}$$

3. An oil droplet of density 885kgm^{-3} and radius $1\mu\text{m}$ is held stationary in between two plates which are 10cm apart. At what potential differences between the plates is this possible?

q must be a multiple of e, the charge on an electron:

$$q = ne, \text{ where } n \text{ is an integer.}$$

$$ne = \frac{4\pi\rho gdr^3}{3V}$$

$$V = \frac{4\pi\rho gdr^3}{3ne} = \frac{4\pi \times 885 \times 9.81 \times 0.1 \times (10^{-6})^3}{3 \times 1.6 \times 10^{-19} \times n} = \frac{25.7}{n}$$

Then, to find some actual values, take $n = 1, 2, 3 \dots$, so $V = 25.7\text{V}, 12.8\text{V}, 8.6\text{V} \dots$

4. If the X-rays used to ionise the oil are of wavelength 1nm, how much energy do they give to the electrons? Why does this mean that the oil drops are ionised?

$$c = f\lambda$$

$$f = \frac{c}{\lambda}$$

$$E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-9}} = 1.989 \times 10^{-16} \text{ J} = 1.24 \text{ keV}$$

This provides the energy required for an electron to escape from its energy level, becoming unbound.

5. In reality, the oil drops are moving when they enter the uniform electric field. How can this be compensated for?

Either:

- Use a slightly stronger potential difference to slow the oil drop down, and then reduce the potential difference to keep the oil drop stationary.
- Measure the voltage required to keep the oil drop moving at a constant (terminal) velocity, using more sophisticated equipment.

Pair Production and Annihilation

$$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$$

1. The mass of an electron is 9.11×10^{-31} kg. What is the minimum amount of energy a photon must have to create an electron?

$$E = 2mc^2 = 2 \times 9.11 \times 10^{-31} \times (3 \times 10^8)^2 = 1.64 \times 10^{-13} \text{ J} = 1.02 \text{ MeV}$$

2. A 1.1 MeV electron annihilates with a 1.1 MeV positron. What is the total energy of the photon produced?

$$2.2 \text{ MeV} = 3.52 \times 10^{-13} \text{ J}$$

3. What is its frequency?

$$f = \frac{E}{h} = \frac{3.52 \times 10^{-13}}{6.63 \times 10^{-34}} = 5.31 \times 10^{20} \text{ Hz}$$

4. What is its wavelength?

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5.31 \times 10^{20}} = 5.65 \times 10^{-13} \text{ m}$$

5. What classical physical conditions might cause a newly produced electron-positron pair to annihilate almost immediately?

Electrons and positrons have opposite charges, and so they attract each other. When they are created, they must have enough kinetic energy to escape each other's attraction. If this is not the case, they will annihilate.

Particle Accelerators

1. Use the formula for centripetal force to show that the radius of motion depends on the speed of the moving object.

$$F = \frac{mv^2}{r}$$

$$r = \frac{mv^2}{F}$$

So, if F is constant:

$$r \propto v^2$$

2. A cyclotron with a diameter of 1.5m is used to accelerate electrons (mass 9.11×10^{-31} kg). The maximum force exerted on an electron is 2.4×10^{-18} N. What is the maximum velocity of the electrons?

$$F = \frac{mv^2}{r}$$

$$v^2 = \frac{Fr}{m} = \frac{2.4 \times 10^{-18} \times 1.5}{9.11 \times 10^{-31}} = 3.95 \times 10^{12} \text{ m}^2\text{s}^{-2}$$

$$v = 1.99 \times 10^6 \text{ ms}^{-1}$$

3. What are the problems involved in constructing a large cyclotron?

Creating an uniform magnetic field over such a large area (for example, the land area enclosed by the LHC!) means that cyclotrons have been largely replaced in experimental particle physics by tubes, instead of flat cylinders.

4. Why don't particles stick to the electrodes when passing through them?

There are two reasons. Firstly, alternating current is sinusoidal, so there is no charge on the electrode when the particles are passing through them. Secondly, even if there were charge on the electrode, the net force on the particle would still be 0, since the force would be equal in every direction.

Cloud Chambers and Mass Spectrometers

Charge of electron = $-1.6 \times 10^{-19}\text{C}$

Mass of electron = $9.11 \times 10^{-31}\text{kg}$

$u = 1.66 \times 10^{-27}\text{kg}$

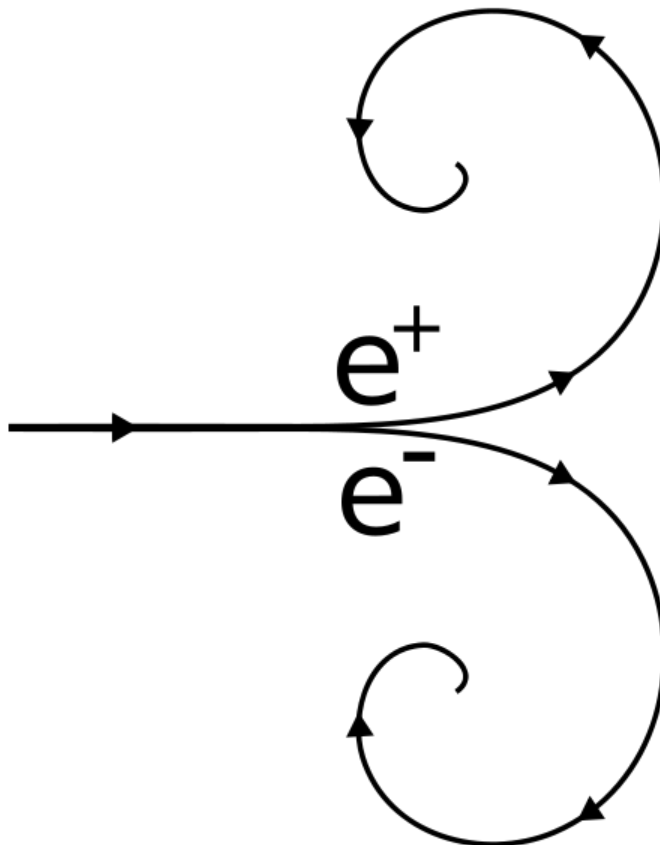
1. An electron enters a cloud chamber, passing into a 0.1T magnetic field. The initial curvature (the reciprocal of its radius) of its path is 100m^{-1} . At what speed was it moving when it entered the magnetic field?

$$r = \frac{mv}{qB}$$

$$v = \frac{qBr}{m} = \frac{1.6 \times 10^{-19} \times 0.1 \times 0.01}{9.11 \times 10^{-31}} = 1.76 \times 10^8 \text{ ms}^{-1} = 0.585c$$

This is too close to the speed of light to ignore special relativity, however. Which we just did.

2. The electron spirals inwards in a clockwise direction, as show in the diagram on the right. What would the path of a positron, moving with an identical speed, look like?



3. Using a 2T magnetic field, what electric field strength must be used to get a velocity selector to select only particles which are moving at 100ms^{-1} ?

$$v = \frac{E}{B}$$

$$E = Bv = 2 \times 100 = 200 \text{ Vm}^{-1}$$

4. Some uranium (atomic number 92) ions (charge $+3e$) of various isotopes are put through the velocity selector described in question 3. They then enter an 0.00002T uniform magnetic field. What radius of circular motion would uranium-235 have?

$$\frac{m}{q} = \frac{rBB_{selector}}{E_{selector}}$$

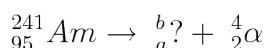
$$\frac{235 \times 1.66 \times 10^{-27}}{3 \times 1.6 \times 10^{-19}} = r \times \frac{2 \times 0.00002}{100} = 0.0000004r$$

$$r = \frac{235 \times 1.66 \times 10^{-27}}{0.0000004 \times 3 \times 1.6 \times 10^{-19}} = 2.03 \text{ m}$$

Radioactive Emissions

You will need a periodic table.

1. Americium-241 is an α emitter. What element, and what isotope, is produced by this decay?

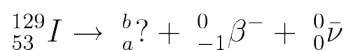


$$b = 241 - 4 = 237$$

$$a = 95 - 2 = 93$$

So, ${}_{93}^{237}\text{Np}$ (Neptunium-237) is produced.

2. Iodine-129 is a β^- emitter. What element, and what isotope, is produced by this decay?



$$b = 129$$

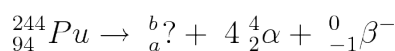
$$a = 53 + 1 = 54$$

So, ${}_{54}^{129}\text{Xe}$ (Xenon-129) is produced.

3. Gamma rays are used to kill microbes in food. Why doesn't the food become radioactive?

Gamma radiation only interacts with the electrons of the atom, not with the nucleus. Radioactivity is due to nuclear properties, not chemical properties.

4. Plutonium-244 decays by emitting an α particle. It does this twice, emits a β^- particle, and then emits a further two α particles. The nucleus becomes a different element each time. What element is produced at the end?

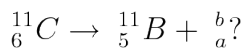


$$b = 244 - 16 = 228$$

$$a = 94 - 8 + 1 = 87$$

So, at the end of this decay chain, ${}_{87}^{228}\text{Fr}$ (Francium-228) is produced.

5. Carbon-11 changes into Boron-11 by a radioactive emission. What was emitted?



$$b = 11 - 11 = 0$$

$$a = 6 - 5 = 1$$

So, a positron (${}_{1}^0\beta^+$) was emitted. However, in order for the lepton numbers to balance, a neutrino (${}_{0}^0\nu$) must also be emitted.

Energy Levels

The following table gives the wavelengths of light given off when electrons change between the energy levels in hydrogen as described in the first row:

Transition of n	3→2	4→2	5→2	6→2	7→2	8→2	9→2	∞→2
Wavelength (nm)	656.3	486.1	434.1	410.2	397.0	388.9	383.5	364.6
Color	Red	Blue-green	Violet	Violet	(Ultraviolet)	(Ultraviolet)	(Ultraviolet)	(Ultraviolet)

1. Calculate the potential energy of an electron at level n=2.

$$c = \lambda f$$

$$3 \times 10^8 = 364.6 \times 10^{-9} \times f$$

$$f = 8.23 \times 10^{14} \text{ Hz}$$

$$\Delta E = -hf = -6.63 \times 10^{-34} \times 8.23 \times 10^{14} = -5.46 \times 10^{-19} \text{ J} = -3.41 \text{ eV}$$

2. Calculate the difference in potential energy between levels n=2 and n=3.

This time, let's derive a general formula:

$$f = \frac{\Delta E}{h}$$

$$c = \frac{\lambda \Delta E}{h}$$

$$\Delta E = \frac{ch}{\lambda} = \frac{3 \times 10^8 \times 6.63 \times 10^{-34}}{656.3 \times 10^{-9}} = 3.03 \times 10^{-19} \text{ J} = 1.89 \text{ eV}$$

3. What is the potential energy of an electron at level n=3?

$$-3.41 - 1.89 = -5.30 \text{ eV}$$

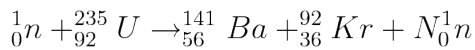
4. If an electron were to jump from n=7 to n=5, what would the wavelength of the photon given off be?

$$\Delta E = \frac{ch}{\lambda_{5,2}} - \frac{ch}{\lambda_{7,2}} = ch \left(\frac{1}{\lambda_{5,2}} - \frac{1}{\lambda_{7,2}} \right) = 3 \times 10^8 \times 6.63 \times 10^{-34} \left(\frac{1}{434.1 \times 10^{-9}} - \frac{1}{397 \times 10^{-9}} \right) = -4.28 \times 10^{-20} \text{ J}$$

$$\lambda = \frac{-ch}{\Delta E} = \frac{3 \times 10^8 \times 6.63 \times 10^{-34}}{4.28 \times 10^{-20}} = 4.65 \text{ } \mu\text{m}$$

Fission

1. A neutron is fired at some Uranium-235. Barium-141 and Krypton-92 are produced:



How many neutrons are produced (ie. what is the value of N)?

Consider the mass numbers:

$$1 + 235 = 141 + 92 + N$$

$$N + 233 = 236$$

$$N = 3$$

2. What proportion of the neutrons produced must be absorbed in order to make the reaction stable?

Two thirds, assuming that Uranium-235 always produces three neutrons when split, which it doesn't.

3. What would happen if too many neutrons were absorbed?

The reaction would stop.

4. Alternatively, Uranium-235 can split into Xenon-140, two neutrons and another element. What is this element? (You will need to use a periodic table.)

Consider the mass numbers:

$$1 + 235 = 140 + N + 2$$

$$N + 142 = 236$$

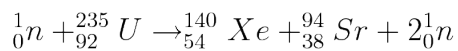
$$N = 94$$

Then consider the atomic numbers:

$$0 + 92 = 54 + n + 2(0)$$

$$n = 38$$

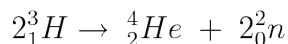
So, the element is strontium-94:



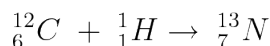
Fusion

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

1. In the Sun, two tritium nuclei (${}^3_1\text{H}$) are fused to produce helium-4 (${}^4_2\text{He}$). What else is produced, apart from energy?



2. In larger stars, carbon-12 (${}^{12}_6\text{C}$) is fused with protium (${}^1_1\text{H}$). What single nucleus does this produce?



3. In this reaction, 1.95 MeV of energy is released. What difference in binding energy, in J, does this correspond to?

$$1.95 \text{ MeV} = 1.95 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 3.12 \text{ MJ}$$

4. If all this energy was emitted as a photon, what would its frequency be?

$$E = hf$$

$$f = \frac{E}{h} = \frac{3.12 \times 10^6}{6.63 \times 10^{-34}} = 4.71 \times 10^{39} \text{ Hz}$$

This is crazily large, so we can conclude that not all the energy is emitted as a photon.

5. In order to contain a fusion reaction, electromagnetism may be used. What other force could be used? Why is this not being used for fusion reactors on Earth?

Gravity, as in stars. However, the amount of mass required to do this is far larger than the Earth, so we cannot do this.

Binding Energy

1. Deuterium (an isotope of Hydrogen with an extra neutron) has a nuclear mass of 2.01355321270 u. What is its binding energy?

$$\text{Total mass of neutron and proton is } 1.00727647 + 1.00866492 = 2.01594139 \text{ u}$$

$$\text{Mass defect} = 2.01594139 - 2.01355321270 = 0.002388178 \text{ u} = 3.965662187 \times 10^{-30} \text{ kg}$$

$$E = \Delta mc^2 = 3.57 \times 10^{-13} \text{ J} = 2.23 \text{ MeV}$$

2. Uranium-235 has a nuclear mass of 235.0439299 u. It contains 92 protons. What is its binding energy?

$$235 - 92 = 143 \text{ neutrons}$$

$$\text{Total mass of baryons is } (92 \times 1.00727647) + (143 \times 1.00866492) = 236.9085188 \text{ u}$$

$$\text{Mass defect} = 236.9085188 - 235.0439299 = 1.8645889 \text{ u} = 3.096222181 \times 10^{-27} \text{ kg}$$

$$E = \Delta mc^2 = 2.79 \times 10^{-10} \text{ J} = 1.74 \text{ GeV}$$

3. How would you expect H-2 and U-235 to be used in nuclear reactors? Why?

H-2 is used in experimental fusion reactors, since it has a smaller nucleus than Iron-56.

U-235 is used in fission reactors, since it has a much larger nucleus than Iron-56.

Risks, Doses and Dose Equivalents

1. A mobile phone emits electromagnetic radiation. 1.2 watts of power are absorbed per. kilogram. Assuming that the radiation is absorbed uniformly across a 5kg head, what dose of radiation would be delivered to the head when making a 10-minute telephone call?

1.2 x 5 = 6 grays are absorbed per. second. 10 minutes = 600 seconds.

$$6 = \frac{\text{dose}}{600}$$

So, the total dose is 3600 Gy.

Note to reader: There is currently something wrong with this question, since 3600 Gy is enough to kill 600 people. Mobile phones don't do that. I'm working on it. --Sjlegg (talk) 14:30, 6 April 2009 (UTC)

2. What dose equivalent does this correspond to?

Quality factor for photons is 1, so dose equivalent is 720 Sv.

3. How many nuclei are there in 1mg of Americium-241?

The mass of 1 Americium-241 nucleus is 241u, which corresponds to $241 \times 1.66 \times 10^{-27} = 400 \times 10^{-27}$ kg.

$$\frac{10^{-3}}{400 \times 10^{-27}} = 2.5 \times 10^{21}$$

4. An ham sandwich becomes contaminated with 1 µg of Americium-241, and is eaten by an 80kg person. The half-life of Americium-241 is 432 years. Given that Americium-241 gives off 5.638 MeV alpha particles, how long would it be before a dose equivalent of 6 Sv is absorbed, making death certain?

First calculate the activity:

$$A = \lambda N$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$$

$$A = \frac{N \ln 2}{t_{\frac{1}{2}}} = \frac{2.5 \times 10^{21} \times \ln 2}{432 \times 365.24 \times 24 \times 60 \times 60} = 1.27 \times 10^{11} \text{ Bq}$$

Then calculate the power (energy per. second):

$$P = 1.27 \times 10^{11} \times 5.638 \times 10^6 \times 1.6 \times 10^{-19} = 0.115 \text{ W}$$

Then calculate the dose per. second:

$$\frac{0.115}{80} = 1.43 \text{ mGys}^{-1}$$

Multiply this by the quality factor for alpha particles (20) to get a dose equivalent per. second of 0.0287 Svs^{-1} .

$$\frac{6}{0.0287} = 209 \text{ s} = 3 \text{ min } 29 \text{ s}$$

5. What assumptions have you made?

We assumed that the Americium was absorbed uniformly throughout the body, and that the activity remained constant. The latter is acceptable since this period of time is relatively short. The former is not acceptable.

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