GCE AS/A level

0984/01

S16-0984-01

MATHEMATICS – S2
Statistics

A.M. WEDNESDAY, 15 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The independent random variables X, Y are such that X has a Poisson distribution with mean 2 and Y has a Poisson distribution with mean 3. Given that $W = XY$, determine

(a) the mean and the variance of W, [6]

(b) $P(W = 4)$. [4]

2. Sue keeps chickens in her garden. She selects, at random, 10 of the eggs produced and weighs them. The results, in grams, are shown below.

62·5 64·2 61·5 65·2 66·2 63·8 60·1 63·2 64·4 66·1

You may assume that this is a random sample from a normal distribution with a standard deviation of 1·9.

(a) Determine a 95% confidence interval for the mean weight of eggs produced by Sue's chickens. [6]

(b) Sue was hoping to obtain a 95% confidence interval of width 1 at most. Calculate the minimum sample size necessary to achieve this. [4]

3. For a certain breed of dog, the weights of the males are normally distributed with mean 40 kg and standard deviation 2·5 kg. The weights of the females are normally distributed with mean 32 kg and standard deviation 1·5 kg.

(a) Calculate the upper quartile of the weights of male dogs of this breed. [2]

(b) A random selection is made of 3 males and 2 females of the breed.
Calculate the probability that

(i) the combined weight of the 5 dogs exceeds 185 kg,

(ii) the combined weight of the 3 males is less than twice the combined weight of the 2 females. [12]

4. The independent random variables X, Y are such that X is N(μ_x, $1{\cdot}5^2$) and Y is N(μ_y, $2{\cdot}5^2$). In order to test the hypotheses

$$H_0 : \mu_x = \mu_y \quad ; \quad H_1 : \mu_x \neq \mu_y$$

a random sample of size 8 is taken from the distribution of X and a random sample of size 12 is taken from the distribution of Y. The means of these two samples are denoted by $\bar{x}$ and $\bar{y}$ respectively. The significance level is to be 10%.

(a) Determine the critical region in the form $\left|\bar{x} - \bar{y}\right| > k$, where the value of k is to be found. [5]

(b) (i) If, in fact, $\mu_x - \mu_y = 0{\cdot}5$, find the probability of incorrectly accepting H_0.

(ii) Comment on your result in (i). [9]

5. A seed manufacturer claims that 70% of seeds of a certain variety will germinate but the manager of a garden centre claims that the germination rate is less than this.

(a) A trial is therefore conducted in which 50 seeds of this variety are planted. It is found that 32 of these seeds germinate.

(i) State suitable hypotheses to test these claims.

(ii) Calculate the *p*-value of this result and state your conclusion in context. [7]

(b) A further trial is conducted in which 500 seeds of this variety are planted and it is found that 329 of these seeds germinate. Carry out a hypothesis test using a normal approximation and state your conclusion in context. [7]

6. A piece of string of length 20 cm is cut at a random point. The length of the longer piece is denoted by X cm and the length of the shorter piece is denoted by Y cm. You may assume that X is uniformly distributed on the interval [10, 20].

(a) Determine $P(Y < 8)$. [2]

(b) (i) Express Y in terms of X.

(ii) Determine $P(XY > 64)$. [6]

(c) Calculate $E(XY)$. [5]

END OF PAPER

