GCE AS/A level

0981/01

MATHEMATICS M2

Mechanics

A.M. TUESDAY, 21 June 2016

1 hour 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

a 12 page answer book;
a Formula Booklet;
a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle of mass $4\,\text{kg}$ moves along the x-axis, starting, when $t = 0$, from the point where $x = 3$

At time $t\,\text{s}$, its velocity $v\,\text{ms}^{-1}$ is given by

$$v = 12t^2 - 7kt + 1$$

where k is constant.

When $t = 2$, the displacement of the particle from the origin is $16\,\text{m}$

(a) Determine the value of k

[5 marks]

(b) Calculate the magnitude of the force acting on the particle when $t = 5$

[4 marks]

2. A particle is projected from horizontal ground with speed $24{\cdot}5\,ms^{-1}$ in a direction inclined at an angle of 30^{o} above the horizontal.

(a) Calculate the horizontal range of the particle.

[6 marks]

(b) Determine the maximum height reached by the particle.

[3 marks]

(c) Write down the speed and the direction of motion of the particle as it hits the ground.

[1 mark]

3. At time $t = 0$ s, the position vector of an object A is $\underline{i}$ m and the position vector of another object B is $3\underline{i}$ m
The constant velocity vector of A is $2\underline{i} + 5\underline{j} - 4\underline{k}$ ms^{-1} and the constant velocity vector of B is $\underline{i} + 3\underline{j} - 5\underline{k}$ ms^{-1}.

Determine the value of t when A and B are closest together and find the least distance between A and B.

[9 marks]

4. By burning a charge, a cannon fires a cannon ball of mass $12\,kg$ horizontally.
As the cannon ball leaves the cannon, its speed is $600\,ms^{-1}$
The recoiling part of the cannon has a mass of $1600\,kg$

(a) Determine the speed of the recoiling part immediately after the cannon ball leaves the cannon.

[3 marks]

(b) Find the energy created by the burning of the charge. State any assumption you have made in your solution.

[4 marks]

(c) Calculate the constant force needed to bring the recoiling part to rest in $1{\cdot}2\,m$

[2 marks]

5. A particle is attached to one end of a light elastic string of natural length l m and modulus of elasticity λ N

The other end of the string is attached to the ceiling.

The particle hangs in equilibrium.

The length of the string is 0·95 m when the weight of the particle is 30 N, and 1·15 m when the weight of the particle is 70 N

Find the value of l and the value of λ

[6 marks]

6. A particle moves on a horizontal plane such that its velocity vector $\underline{v}\ \text{ms}^{-1}$ at time $t\ \text{s}$ is given by

$$\underline{v} = 7\sin 2t\ \underline{i} + 6\cos 3t\ \underline{j}$$

(a) Find the acceleration vector of the particle at time $t\ \text{s}$.

[2 marks]

(b) Given that when $t = 0$, the particle has position vector $(0{\cdot}5\underline{i} + 3\underline{j})\ \text{m}$, find the position vector of the particle when $t = \frac{\pi}{2}$.

[5 marks]

7. The diagram opposite shows two points A and B on a mountain bike track.

The heights of A and B above ground level are $20\,\text{m}$ and $22\,\text{m}$ respectively.

The length of the track between A and B is $16\,\text{m}$

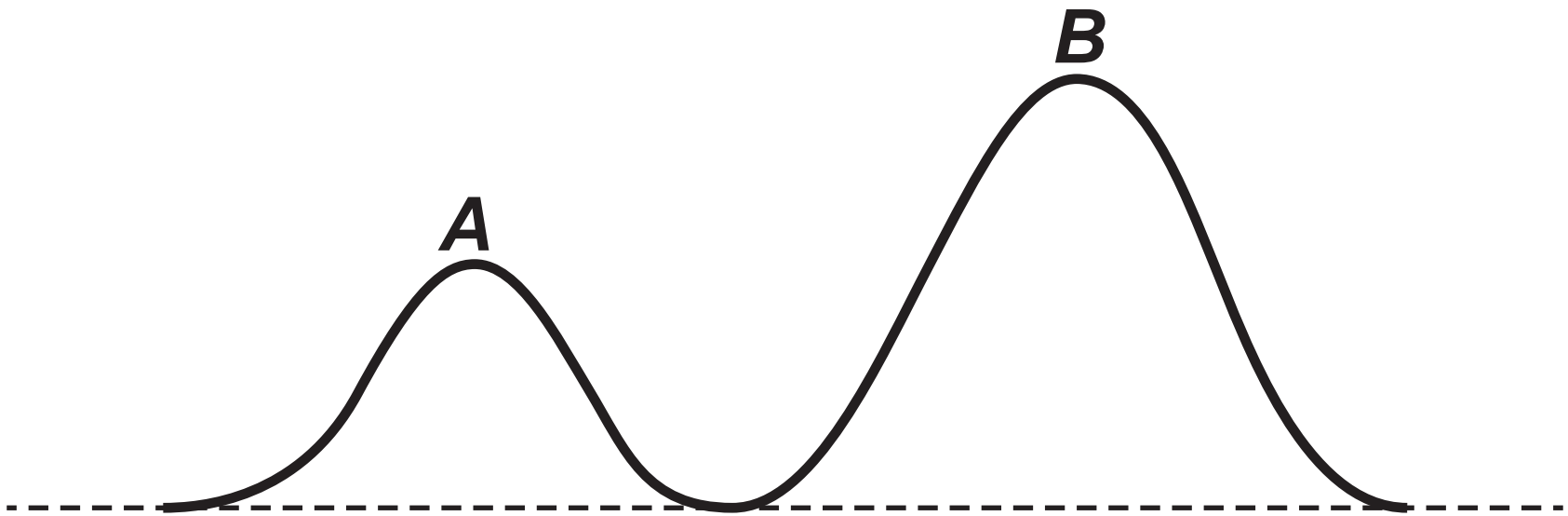
The resistance to motion of a biker on the track may be modelled by a constant force of magnitude $50\,\text{N}$

The total mass of the biker and his bike is $70\,\text{kg}$

The speed of the biker at A is $v\,\text{ms}^{-1}$

Find the minimum value of v if the biker is to reach B without pedalling.

[7 marks]


B
A

8. A rough circular plate rotates horizontally about a smooth fixed vertical axis through its centre O.

A point A on the plate moves with constant speed $v\,\text{ms}^{-1}$, where OA is $1{\cdot}6\,\text{m}$.

A particle of mass $m\,\text{kg}$ lies on the point A on the plate.

The coefficient of friction between the particle and the plate is $0{\cdot}72$.

Given that the particle remains at the point A, find the greatest possible value of v.

Hence write down the greatest possible value of the angular velocity of the particle.

State clearly your units for the angular velocity.

[7 marks]

9. A smooth sphere, with centre O and radius $4\,\text{m}$, is fixed.

A particle P, of mass m, resting on the sphere at its highest point, is given a horizontal speed of magnitude $\sqrt{g}\,\text{ms}^{-1}$

where g is the magnitude of the acceleration due to gravity.

At the instant the line OP makes an angle θ with the upwards vertical, the speed of P is $v\,\text{ms}^{-1}$

(a) Determine an expression for v^2 in terms of g and θ while P remains in contact with the sphere.

[4 marks]

(b) Find, in terms of m, g and θ, the magnitude of the force exerted by the sphere on P. Hence calculate the value of $\cos\theta$ and the value of v^2 when P leaves the surface of the sphere.

[7 marks]

END OF PAPER