GCE AS/A level

0976/01

MATHEMATICS – C4

Pure Mathematics

P.M. FRIDAY, 17 June 2016

1 hour 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen or your usual method.

Answer ALL questions.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{17 + 4x - x^2}{(2x - 1)(x - 3)^2}$$

(a) Express $f(x)$ in terms of partial fractions.

[4 marks]

(b) USE YOUR RESULT TO PART (a) to find an expression for $f'(x)$

[2 marks]

2(a) (i) Expand $\dfrac{1}{\sqrt{1+2x}}$ in ascending powers of x up to and including the term in x^2

(ii) State the range of values of x for which your expansion is valid.

[3 marks]

(b) Use your expansion in part (a) to find an approximate value for one root of the equation

$$\frac{6}{\sqrt{1+2x}} = 4 + 15x - x^2$$

[2 marks]

3. The curve C has equation

$$x^4 + 2x^3y - 3y^4 = 16$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{2x^3 + 3x^2y}{6y^3 - x^3}$

[3 marks]

(b) Show that there are only two points on C where the gradient of the tangent is -2.
Find the coordinates of each of these two points.

[4 marks]

4(a) The angle x is such that $0° \leqslant x \leqslant 180°$, $x \neq 90°$

Given that x satisfies the equation

$$3\tan 2x + 16\cot^2 x = 0$$

(i) show that
$$3\tan^3 x - 8\tan^2 x + 8 = 0$$

(ii) find all possible values of x, giving each answer in degrees, correct to one decimal place.

[8 marks]

(b) Express $24\cos\theta - 7\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$

Hence, find the range of values of k for which the equation

$$24\cos\theta - 7\sin\theta = k$$

has no solutions.

[5 marks]

5. The parametric equations of the curve C are

$$x = \frac{3}{t}, \qquad y = 4t.$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$3y = -4p^2x + 24p.$$

[4 marks]

(b) The tangent to C at the point P passes through the point $(1, 9)$. Show that P can be one of two points. Find the coordinates of each of these two points.

[4 marks]

6(a) Find $\int (2x+1)\mathrm{e}^{-3x}\,\mathrm{d}x$

[4 marks]

(b) Use the substitution $u = 4 + 5\tan x$ to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x}\,\mathrm{d}x.$$

[4 marks]

7. The value, $£V$, of a particular car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^3

(a) Write down a differential equation satisfied by V

[1 mark]

(b) Given that the initial value of the car is $£A$, show that

$$V^2 = \frac{A^2}{bt + 1}$$

where b is a constant.

[4 marks]

(c) When $t = 2$, the value of the car has fallen to a half of its initial value. Find the value of t when the value of the car will have fallen to a quarter of its initial value.

[4 marks]

8. The position vectors of the points A and B are given by

$$\underline{a} = \underline{i} + 3\underline{j} - 3\underline{k}$$
$$\underline{b} = 3\underline{i} + 4\underline{j} - \underline{k}$$

respectively.

(a) (i) Write down the vector $\underline{AB}$

(ii) Find the vector equation of the line AB.

[3 marks]

(b) The vector equation of the line L is given by

$$\underline{r} = -\underline{i} + 8\underline{j} + p\underline{k} + \mu(-2\underline{i} + \underline{j} + 3\underline{k}),$$

where p is a constant.

(i) Given that the lines AB and L intersect, find the value of p.

(ii) Determine whether or not the line L is perpendicular to the vector $6\underline{i} - 4\underline{j} + 5\underline{k}$, giving a reason for your answer.

[7 marks]

9. The region R is bounded by the curve $y = \cos x + \sin x$, the x-axis and the lines $x = \dfrac{\pi}{5}$, $x = \dfrac{2\pi}{5}$. Find the volume of the solid generated when R is rotated through four right angles about the x-axis. Give your answer correct to two decimal places.

[6 marks]

10. Prove by contradiction the following proposition.

When x is real and $x \neq 0$

$$\left| x + \frac{1}{x} \right| \geqslant 2$$

The first two lines of the proof are given below.

[3 marks]

Assume that there is a real value of x such that

$$\left| x + \frac{1}{x} \right| < 2$$

Then squaring both sides, we have:

END OF PAPER