GCE AS/A level

0981/01

MATHEMATICS M2

Mechanics

A.M. FRIDAY, 5 June 2015

1 hour 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

a 12 page answer book;
a Formula Booklet;
a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The vectors $\underline{\mathbf{x}}$ and $\underline{\mathbf{y}}$ are given by

$$\underline{\mathbf{x}} = \sin\theta\,\underline{\mathbf{i}} + 2\cos 2\theta\,\underline{\mathbf{j}}$$

$$\underline{\mathbf{y}} = 2\underline{\mathbf{i}} - \underline{\mathbf{j}}$$

Find the values of θ between 0 and 2π such that $\underline{\mathbf{x}}$ is perpendicular to $\underline{\mathbf{y}}$. [6 marks]

2. An object of mass $50\,\text{kg}$ moves in a straight horizontal line under the action of a constant horizontal force of magnitude $1600\,\text{N}$ acting along the line. The resistance to motion of the object is proportional to time t seconds. At time t seconds, the velocity of the object is $v\,\text{ms}^{-1}$ and at time $t = 2$, it is moving with velocity $41\,\text{ms}^{-1}$ and acceleration $-4\,\text{ms}^{-2}$.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 32 - 18t$$

[4 marks]

(b) Find an expression for v in terms of t and determine the times when the velocity of the object is $28\,\text{ms}^{-1}$. [6 marks]

3. A vehicle of mass $6000\,\text{kg}$ is moving up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{6}{49}$. The vehicle's engine exerts a constant power of $P\,\text{W}$. The constant resistance to motion of the vehicle is $R\,\text{N}$. At the instant the vehicle is moving with velocity $\frac{16}{5}\,\text{ms}^{-1}$, its acceleration is $2\,\text{ms}^{-2}$.

The maximum velocity of the vehicle is $\frac{16}{3}\,\text{ms}^{-1}$.

Determine the value of P and the value of R.

[9 marks]

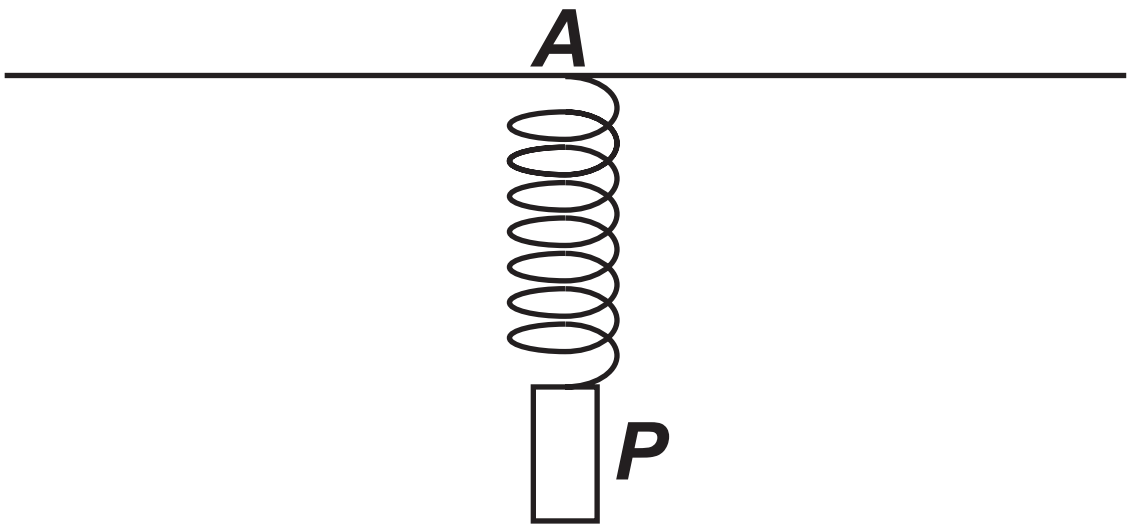
4. A particle of mass $0{\cdot}5\,\text{kg}$ is moving under the action of a single force $\underline{F}\,\text{N}$, where

$$\underline{F} = (4t - 3)\underline{i} + (3t^2 - 5t)\underline{j}.$$

(a) The velocity of the particle at time $t\,\text{s}$ is $\underline{v}\,\text{ms}^{-1}$. When $t = 0$, $\underline{v} = 8\underline{i} - 7\underline{j}$. Find an expression for $\underline{v}$ in terms of t. [5 marks]

(b) When $t = 3$, the particle receives an impulse of $2\underline{i} - 9\underline{j}\,\text{Ns}$. Find the speed of the particle immediately after the impulse. [5 marks]

5. The diagram shows a light spring of natural length $0{\cdot}4\,\text{m}$ and modulus of elasticity $1470\,\text{N}$ with one end A fixed and the other end attached to an object P of mass $15\,\text{kg}$.



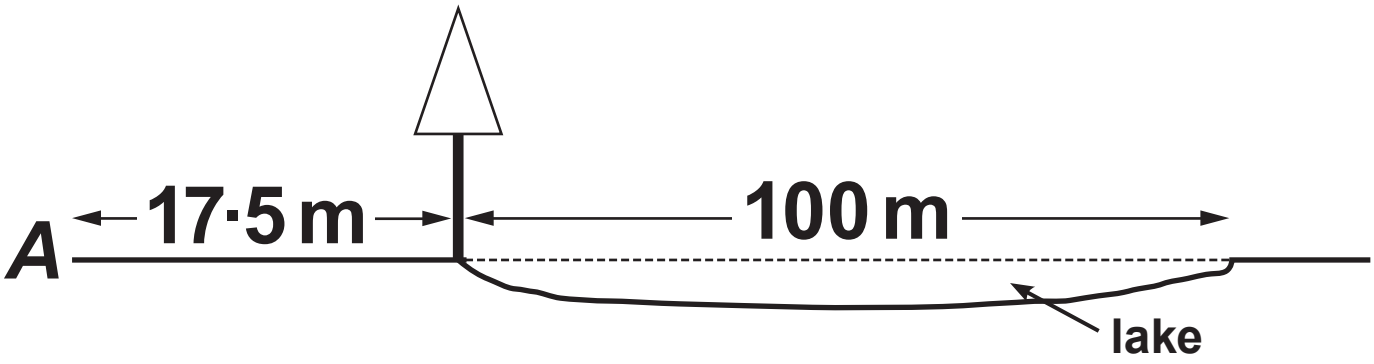
Initially, P hangs in equilibrium with the spring vertical.

(a) Determine the extension of the spring. [3 marks]

The object P is pulled downwards so that the total length of the spring is $0{\cdot}56\,\text{m}$. It is then released.

(b) Calculate the speed of P when it is at a distance $0{\cdot}45\,\text{m}$ from A. [8 marks]

6. A golfer hits a ball from a point A with initial velocity of $35\,\text{ms}^{-1}$ at an angle α above the horizontal where $\sin\alpha = 0{\cdot}8$. The ball passes over a tree which is growing in front of a lake. The lake is $100\,\text{m}$ wide, as shown in the diagram. The tree is at a horizontal distance of $17{\cdot}5\,\text{m}$ from A.



(a) Determine whether or not the golf ball will fall into the lake. [6 marks]

(b) Find the magnitude and direction of the velocity of the ball as it passes over the tree. [7 marks]

7. A car of mass $1200\,\text{kg}$ is moving in a horizontal circle of radius $80\,\text{m}$ on a road banked at an angle of $12°$ to the horizontal. When the car is moving with a constant speed of $v\,\text{ms}^{-1}$, there is no tendency to sideslip. Calculate the normal reaction of the road on the car and find the value of v. [5 marks]

8. One end of a light inextensible string of length $0{\cdot}8\,\text{m}$ is attached to a fixed point. The other end of the string is attached to a particle P of mass $3\,\text{kg}$. Initially P hangs at rest with the string vertical. The particle P is then projected horizontally with speed $5\,\text{ms}^{-1}$, so that it starts to describe a vertical circle. When the string is inclined at an angle θ to the downwards vertical, P has speed $v\,\text{ms}^{-1}$ and the tension in the string is $T\,\text{N}$.

(a) Find, in terms of θ

(i) an expression for v^2,

(ii) an expression for T. [8 marks]

(b) Find the greatest possible value of θ and briefly describe the subsequent motion of P. [3 marks]

END OF PAPER