GCE AS/A level

0978/01

S15-0978-01

MATHEMATICS – FP2
Further Pure Mathematics

P.M. TUESDAY, 16 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Express

$$\frac{5}{(x^2+1)(2-x)}$$

in partial fractions. [4]

(b) Using the substitution $u = \tan x$ and the result in *(a)*, evaluate the integral

$$\int_0^{\frac{\pi}{4}} \frac{5}{2-\tan x}\,\mathrm{d}x.$$

Give your answer correct to three significant figures. [9]

2. The function f is defined by

$$f(x) = ax^3 + bx \qquad \text{for } x \leqslant -1,$$

$$f(x) = x^2 - x + 2 \qquad \text{for } x > -1.$$

(a) Given that f and its derivative are both continuous at $x = -1$, determine the values of the constants a and b. [6]

(b) The equation $f(x) = 0$ has exactly one root. Determine its value. [2]

3. The complex number $z = 2\left(\cos\left(\frac{3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{3\pi}{4}\right)\right)$.

(a) Find the three cube roots of z, giving your answers in the form $x + \mathrm{i}y$, with x and y correct to three decimal places. [6]

(b) Find the smallest positive integer n for which z^n is

(i) real,

(ii) imaginary. [3]

4. Find the general solution to the equation

$$\cos\left(\theta + \frac{\pi}{6}\right) + \cos\left(2\theta + \frac{\pi}{6}\right) + \cos\left(3\theta + \frac{\pi}{6}\right) = 0.$$ [8]

5. Differentiate the following integrals with respect to x.

(a) $\displaystyle\int_0^x \mathrm{e}^{\sqrt{u}}\,\mathrm{d}u$ [1]

(b) $\displaystyle\int_0^{x^2} \mathrm{e}^{\sqrt{u}}\,\mathrm{d}u$ [3]

(c) $\displaystyle\int_x^{x^2} \mathrm{e}^{\sqrt{u}}\,\mathrm{d}u$ [2]

6. The point $P(x, y)$ moves in such a way that its distance from the point $(0, 3)$ is equal to its distance from the line $y + 3 = 0$.

(a) Show that the locus of P is the curve C with equation $x^2 = 12y$. [2]

(b) (i) Show that the point $(6t, 3t^2)$ lies on C for all values of t.

(ii) Show that the equation of the tangent to C at the point $(6t, 3t^2)$ is

$$y = tx - 3t^2.$$

(iii) Find the values of t for which the tangent passes through the point $(0, -12)$.

(iv) Hence find the angle between the two tangents to C from the point $(0, -12)$. [9]

7. The function f is defined by

$$f(x) = \frac{1}{x-1} - \frac{4}{x-2}.$$

(a) Write down the equations of the vertical asymptotes on the graph of f. [1]

(b) Find the points of intersection of the graph of f with the coordinate axes. [3]

(c) Find the coordinates of the stationary points on the graph of f and classify each point as a maximum or a minimum. [8]

(d) Sketch the graph of f. [2]

(e) The set $S = [-1, 0]$. Determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

END OF PAPER