GCE AS/A level

0976/01

MATHEMATICS – C4

Pure Mathematics

A.M. FRIDAY, 12 June 2015

1 hour 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

a 12 page answer book;
a Formula Booklet;
a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$f(x) = \frac{2x^2 + 5x + 25}{(x+3)^2(x-1)}$$

(a) express $f(x)$ in terms of partial fractions,

[4 marks]

(b) evaluate

$$\int_3^{10} f(x)\,\mathrm{d}x$$

giving your answer correct to two decimal places.

[3 marks]

2. The curve C has equation

$$x^4 + 3x^2y - 2y^2 = 34$$

(a) Show that

$$\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$$

[3 marks]

(b) Find the coordinates of each of the points on C where the tangent is parallel to the y-axis.

[4 marks]

3(a) Find all values of x in the range

$0° \leqslant x \leqslant 180°$ satisfying

$$\tan(x + 45°) = 8\tan x$$

[5 marks]

(b) (i) Express $\sqrt{13}\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Find all values of θ in the range

$0° \leqslant \theta \leqslant 360°$ satisfying

$$\sqrt{13}\sin\theta - 6\cos\theta = -4$$

[6 marks]

4. The line L has equation $y = mx$, where $m > 0$. The region R is bounded by L, the x-axis and the line $x = a$, where $a > 0$.

(a) Using integration, find an expression, in terms of a and m, for the volume V generated when R is rotated through four right angles about the x-axis. **[3 marks]**

(b) The point with coordinates (a, b) lies on L.

(i) Rewrite the expression for the volume V found in part (a) in terms of a and b.

(ii) Give a geometrical interpretation of your answer. **[3 marks]**

5. Expand $\left(1 + \frac{x}{8}\right)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2.

State the range of values of x for which your expansion is valid.

Hence, by writing $x = 1$ in your expansion, find an approximate value for $\sqrt{2}$ in the form $\frac{a}{b}$,

where a and b are integers whose values are to be found. **[5 marks]**

6. The parametric equations of the curve C are

$$x = at^2,\ y = 2at,$$

where a is a positive constant. The points P and Q lie on C and have parameters p and q respectively.

(a) Simplifying your answer in each case, find

(i) the gradient of the tangent to C at the point P

(ii) the equation of the tangent to C at the point P [4 marks]

(b) (i) Find an expression, in its simplest form, for the gradient of the line PQ

(ii) Explain how you could use the answer of (b)(i) to derive the gradient of the tangent to C at the point P [4 marks]

7(a) Use the substitution $u = 12 - x^3$ to evaluate

$$\int_0^2 \frac{x^2}{\left(12 - x^3\right)^2}\,\mathrm{d}x$$

[4 marks]

(b) (i) Find $\displaystyle\int x\cos 2x\,\mathrm{d}x$

(ii) Use the result of (b)(i) to find

$$\int x\sin^2 x\,\mathrm{d}x$$

[7 marks]

8. The position vectors of the points A and B are given by

$$\underline{a} = 5\underline{i} - \underline{j} - \underline{k}$$

$$\underline{b} = 4\underline{i} - 3\underline{j} + 6\underline{k}$$

respectively.

(a) (i) Write down the vector $\underline{AB}$

(ii) Find the vector equation of the line AB

[3 marks]

(b) The vector equation of the line L is given by

$$\underline{r} = 2\underline{i} - 3\underline{j} - 4\underline{k} + \mu(\underline{i} + \underline{j} - \underline{k})$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6 marks]

9. A bookseller values a rare book at £A on August 1st 2010. The value, £P, of the book t years after this date may be modelled as a continuous variable. The rate of increase of P may be assumed to be directly proportional to P^2.

(a) Write down a differential equation satisfied by P [1 mark]

(b) Show that

$$\frac{1}{k}\left(\frac{P-A}{PA}\right)=t$$

where k is a constant. [4 marks]

(c) The value of the book is £800 on August 1st 2013 and £900 on August 1st 2014.
Find the value of A. [3 marks]

10. Prove by contradiction the following proposition.

If a and b are odd integers such that 4 is a factor of $a - b$, then 4 is NOT a factor of $a + b$.

The first lines of the proof are given below.

Assume that 4 is a factor of $a + b$.

Then there exists an integer c such that $a + b = 4c$. [3 marks]

END OF PAPER