GCE AS/A level

0975/01

MATHEMATICS – C3

Pure Mathematics

A.M. WEDNESDAY, 3 June 2015

1 hour 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

a 12 page answer book;
a Formula Booklet;
a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1(a) **Use Simpson's Rule with five ordinates to find an approximate value for the integral**

$$\int_0^{\frac{4\pi}{9}} \ln(\cos x)\, dx$$

Show your working and give your answer correct to four decimal places. **[4 marks]**

(b) **USE YOUR ANSWER TO PART (a) to deduce an approximate value for the integral**

$$\int_0^{\frac{4\pi}{9}} \ln(\sec x)\, dx$$

[1 mark]

2(a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$7 \operatorname{cosec}^2 \theta - 4 \cot^2 \theta = 16 + 5 \operatorname{cosec} \theta$$

[6 marks]

(b) Without carrying out any calculations, explain why there are no values of ϕ in the range $0° \leqslant \phi \leqslant 90°$ which satisfy the equation

$$4 \sec \phi + 3 \operatorname{cosec} \phi = 6$$

[1 mark]

3(a) The curve C_1 is defined by

$$x^3 + 2x\cos y + y^2 = 1 + \frac{\pi^2}{4}$$

Find the value of $\frac{dy}{dx}$ at the point $(1, \frac{\pi}{2})$.

[4 marks]

(b) The curve C_2 is such that

$$\frac{dy}{dx} = x^2 y$$

Find an expression for $\frac{d^2y}{dx^2}$ in terms of x and y. Simplify your answer. [3 marks]

4. Given that $x = \tan^{-1} t$, $y = \ln t$, where $t > 0$,

(a) find an expression for $\dfrac{dy}{dx}$ in terms of t,

[4 marks]

(b) find the value of x for which $\dfrac{d^2y}{dx^2} = 0$

[5 marks]

5(a) On the same diagram, sketch the graphs of

$y = \cos^{-1} x$ and $y = 5x - 1$

[2 marks]

(b) YOU MAY ASSUME that the equation

$$\cos^{-1} x - 5x + 1 = 0$$

has a root α between $0{\cdot}4$ and $0{\cdot}5$
The recurrence relation

$$x_{n+1} = \frac{1}{5}(1 + \cos^{-1} x_n)$$

with $x_0 = 0{\cdot}4$ can be used to find α.
Find and record the values of x_1, x_2, x_3, x_4.
Write down the value of x_4 correct to four decimal places and prove that this is the value of α correct to four decimal places. [5 marks]

6(a) Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\ln(4x^2 - 3x - 5)$

(ii) $e^{\sqrt{x}}$

(iii) $\dfrac{a + b\sin x}{a - b\sin x}$ where a, b are constants.

[7 marks]

(b) By first writing $\cot x = (\tan x)^{-1}$ and assuming the derivative of $\tan x$, find an expression for $\dfrac{d}{dx}(\cot x)$.

Simplify your answer.

[3 marks]

7(a) **Find each of the following integrals, simplifying your answer wherever possible.**

(i)
$$\int \frac{(7x^2 - 2)}{x}\, dx$$

(ii)
$$\int \sin\left(\frac{2x}{3} - \pi\right) dx$$

[5 marks]

(b) **Evaluate**
$$\int_3^6 \frac{1}{\sqrt[4]{(5x - 14)}}\, dx$$

[4 marks]

8(a) Find all values of x satisfying the inequality

$$|3x - 5| \leqslant 1$$

[3 marks]

(b) USE YOUR ANSWER TO PART (a) to find all values of y satisfying the inequality

$$\left|\frac{3}{y} - 5\right| \leqslant 1$$

[2 marks]

9. Given that $f(x) = \ln x$, sketch, on the same diagram, the graphs of $y = f(x)$ and

$$y = \frac{2}{3} f(x + 4)$$

Label the coordinates of the point of intersection of EACH of the graphs with the x-axis.
Indicate the behaviour of EACH of the graphs for large positive and negative values of y.

[5 marks]

10(a) Show, by counter-example, that the following statement is false.

'If two functions h and k are such that their derivatives h' and k' are equal, then the functions h and k must themselves be equal.'

[2 marks]

10(b) The functions f and g have domains $[7, 60]$ and $[9, \infty)$ respectively and are defined by

$$f(x) = 2\ln(4x + 5) + 3$$
$$g(x) = e^x$$

(i) Find an expression for $f^{-1}(x)$.

(ii) Write down the domain of f^{-1}, giving the end-points of your domain correct to the nearest integer.

(iii) Write down an expression for $gf(x)$ and simplify your answer.

[9 marks]

END OF PAPER