GCE MARKING SCHEME

MATHEMATICS
AS/Advanced

JANUARY 2014

INTRODUCTION

The marking schemes which follow were those used by WJEC for the January 2014 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics C1 January 2014

Solutions and Mark Scheme

Final Version

1. (*a*) (i) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^3/_2$ (or equivalent) A1

(ii) Use of gradient $L_1 \times$ gradient $AB = -1$ M1

A correct method for finding the equation of L_1 using candidate's gradient for L_1 M1

Equation of L_1: $y - 1 = {}^2/_3\,(x - 4)$ (or equivalent)

(f.t. candidate's gradient for AB) A1

(*b*) (i) An attempt to solve equations of L_1 and L_2 simultaneously M1

$x = -2, y = -3$ (convincing) A1

(ii) A correct method for finding the coordinates of the mid-point of AC M1

Mid-point of AC has coordinates $(2, -2{\cdot}5)$ (c.a.o.) A1

(iii) A correct method for finding the length of $AB(BC)$ M1

$AB = \sqrt{13}$ A1

$BC = \sqrt{52}$ (or equivalent) A1

A correct method for finding the area of triangle ABC m1

Area of triangle $ABC = 13$ (c.a.o.) A1

2. $\dfrac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} = \dfrac{(3\sqrt{3} - 2\sqrt{5})(2\sqrt{3} - \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5})}$ M1

Numerator: $6 \times 3 - 3 \times \sqrt{3} \times \sqrt{5} - 4 \times \sqrt{5} \times \sqrt{3} + 10$ A1

Denominator: $12 - 5$ A1

$\dfrac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} = 4 - \sqrt{15}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $2\sqrt{3} + \sqrt{5}$

3. An attempt to differentiate, at least one non-zero term correct M1

$\dfrac{dy}{dx} = 20 \times -1 \times x^{-2} + 4x$ A1

An attempt to substitute $x = 2$ in candidate's derived expression for $\dfrac{dy}{dx}$ m1

Value of $\dfrac{dy}{dx}$ at $P = 3$ (c.a.o.) A1

Gradient of normal = $\dfrac{-1}{\text{candidate's derived value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - 7 = -\dfrac{1}{3}(x - 2)$ (or equivalent)

(f.t. candidate's value for $\dfrac{dy}{dx}$ provided all three method marks are awarded) A1

4. Either $p = 0{\cdot}8$ or a sight of $(x + 0{\cdot}8)^2$ B1

A convincing argument to show that the value 25 is correct B1

$x^2 + 1{\cdot}6x - 24{\cdot}36 = 0 \Rightarrow (x + 0{\cdot}8)^2 = 25$ (f.t. candidate's value for p) M1

$x = 4{\cdot}2$ (f.t. candidate's value for p) A1

$x = -5{\cdot}8$ (f.t. candidate's value for p) A1

5. (*a*) $(1 + \sqrt{6})^5 = (1)^5 + 5(1)^4(\sqrt{6}) + 10(1)^3(\sqrt{6})^2 + 10(1)^2(\sqrt{6})^3$
$+ 5(1)(\sqrt{6})^4 + (\sqrt{6})^5$ (five or six terms correct) B2

(**If B2 not awarded**, award B1 for four correct terms)

$(1 + \sqrt{6})^5 = 1 + 5\sqrt{6} + 60 + 60\sqrt{6} + 180 + 36\sqrt{6}$
(six terms correct) B2

(**If B2 not awarded**, award B1 for four or five correct terms)

$(1 + \sqrt{6})^5 = 241 + 101\sqrt{6}$ (f.t. one error) B1

(*b*) ${}^{n}C_2 \times 3^k = 495$ ($k = 1$, **2**) M1

Either $9n^2 - 9n - 990 = 0$ or $n^2 - n - 110 = 0$ or $n(n - 1) = 110$ A1

$n = 11$ (c.a.o.) A1

6. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = 8^2 - 4 \times (2k - 3) \times (2k + 3)$ A1

Putting $b^2 - 4ac <(\leq) 0$ m1

$100 - 16k^2 < 0$ (o.e.) (c.a.o.) A1

Finding critical values $k = -5/2$, $k = 5/2$
(o.e.) (f.t. candidate's values for m, n) B1

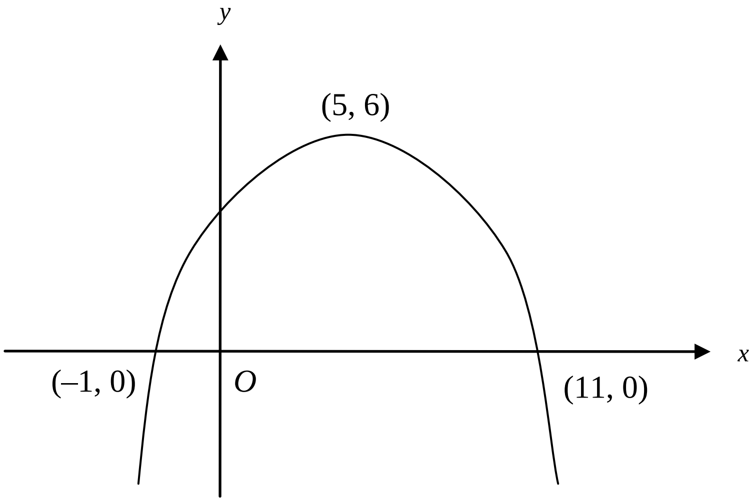
$k < -5/2$ or $5/2 < k$ (o.e.) (f.t. only critical values of $-a$ and a) B1

Each of the following errors earns a final B0

the use of non-strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*)



Concave down curve and y-coordinate of maximum = 6	B1
x-coordinate of maximum = 5	B1
Both points of intersection with x-axis	B1

(*b*) $y = f(-2x)$ B2

(**If B2 not awarded**, award B1 for either $y = f(-{}^{1}/_{2}\,x)$ or $y = f(2x)$)

8. (*a*)

$y + \delta y = 7(x + \delta x)^2 - 6(x + \delta x) - 3$	B1
Subtracting y from above to find δy	M1
$\delta y = 14x\delta x + 7(\delta x)^2 - 6\delta x$	A1
Dividing by δx and letting $\delta x \to 0$	M1
$\dfrac{dy}{dx} = \lim_{\delta x \to 0} \dfrac{\delta y}{\delta x} = 14x - 6$	(c.a.o.) A1

(*b*)

$\dfrac{dy}{dx} = a \times \dfrac{4}{3} \times x^{1/3} + 24 \times \dfrac{1}{2} \times x^{-1/2}$ B1, B1

Attempting to substitute $x = 64$ in candidate's expression for $\dfrac{dy}{dx}$ and putting expression equal to $\dfrac{11}{2}$ M1

(The M1 is only awarded if at least one B1 has been awarded)

$a = \dfrac{3}{4}$ (c.a.o.) A1

9. (*a*) Use of $f(-3) = -39$ — M1

$-27a + 117 + 30 - 24 = -39 \Rightarrow a = 6$ (convincing) — A1

(*b*) Attempting to find $f(r) = 0$ for some value of r — M1

$f(-2) = 0 \Rightarrow x + 2$ is a factor — A1

$f(x) = (x + 2)(6x^2 + ax + b)$ with one of a, b correct — M1

$f(x) = (x + 2)(6x^2 + x - 12)$ — A1

$f(x) = (x + 2)(2x + 3)(3x - 4)$ (f.t. only $6x^2 - x - 12$ in above line) — A1

$x = -2, -{}^3/_2, {}^4/_3$ (f.t. for factors $2x \pm 3$, $3x \pm 4$) — A1

Special case

Candidates who, after having found $x + 2$ as one factor, then find just one of the remaining factors by using e.g. the factor theorem, are awarded B1 for the final 4 marks

10. (*a*) $\frac{dy}{dx} = -6x^2 + 24x - 18$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = 1, 3$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(1, -3)$ and $(3, 5)$ (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

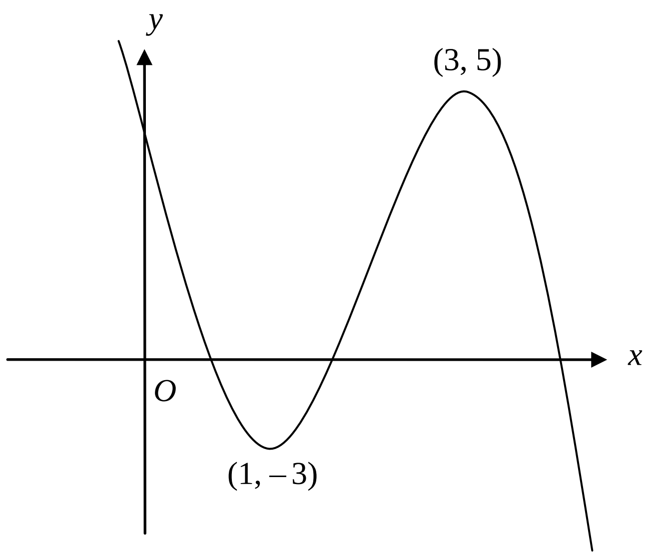
either $(1, -3)$ is a minimum point

or $(3, 5)$ is a maximum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a negative cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) Use of both $k = -3$, $k = 5$ to find the range of values for k

(f.t. candidate's y-values at stationary points) M1

$-3 < k < 5$ (f.t. candidate's y-values at stationary points) A1

Mathematics C2 January 2014

Solutions and Mark Scheme

Final Version

1.

2	2		
2·5	1·843908891		
3	1·732050808		
3·5	1·647508942		
4	1·58113883	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}5$ M1

$I \approx \frac{0{\cdot}5}{2} \times \{2 + 1{\cdot}58113883 + 2(1{\cdot}843908891 + 1{\cdot}732050808 + 1{\cdot}647508942)\}$

$I \approx 14{\cdot}02807611 \times 0{\cdot}5 \div 2$

$I \approx 3{\cdot}507019028$

$I \approx 3{\cdot}507$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}4$

2	2		
2·4	1·870828693		
2·8	1·772810521		
3·2	1·695582496		
3·6	1·632993162		
4	1·58113883	(all values correct)	B1

Correct formula with $h = 0{\cdot}4$ M1

$I \approx \frac{0{\cdot}4}{2} \times \{2 + 1{\cdot}58113883 + 2(1{\cdot}870828693 + 1{\cdot}772810521 + 1{\cdot}695582496 + 1{\cdot}632993162)\}$

$I \approx 17{\cdot}52556857 \times 0{\cdot}4 \div 2$

$I \approx 3{\cdot}505113715$

$I \approx 3{\cdot}505$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) $8\cos^2\theta - 7(1 - \cos^2\theta) = 4\cos\theta - 3$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - 4\cos\theta - 4 = 0 \Rightarrow (5\cos\theta + 2)(3\cos\theta - 2) = 0$

$\Rightarrow \cos\theta = \frac{2}{3}$, $\cos\theta = -\frac{2}{5}$ (c.a.o.) A1

$\theta = 48{\cdot}19°, 311{\cdot}81°$ B1

$\theta = 113{\cdot}58°, 246{\cdot}42°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) $X = 114°$ B1

$Y - Z = 20°$ B1

$114° + Y + Z = 180°$ (f.t. only for an obtuse value for X) M1

$Y = 43°, Z = 23°$ (f.t. one error) A1

3. (*a*) $a + 2d + a + 7d = 0$ B1

$a + 4d + a + 6d + a + 9d = 22$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = -18, d = 4$ (both values) (c.a.o.) A1

(*b*) $S_n = \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$ B1

$S_{2n} = \frac{2n}{2}[2 \times 9 + (2n - 1) \times 2]$ B1

$\frac{2n}{2}[2 \times 9 + (2n - 1) \times 2] = k \times \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$ (k = **3**, $^1/_3$)

(f.t. candidate's quadratic expressions for S_{2n}, S_n provided at least one of the first two B marks is awarded) M1

An attempt to solve this equation including dividing both sides by n to reach a linear equation in n. m1

$n = 8$ (c.a.o.) A1

4. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) (i) $ar^3 = -108$ and $ar^6 = 4$ B1

$r^3 = \dfrac{4}{-108}$ (o.e.) M1

$r = -{}^1/_3$ (c.a.o.) A1

(ii) $a \times (-{}^1/_3)^3 = -108 \Rightarrow a = 2916$

(f.t. candidate's derived value for r) B1

$S_\infty = \dfrac{2916}{1 - (-{}^1/_3)}$ (use of formula for sum to infinity)

(f.t. candidate's derived values for r and a) M1

$S_\infty = 2187$ (c.a.o.) A1

5. (*a*) (i) **Either**: $5^2 = 3^2 + x^2 - 2 \times 3 \times x \times \cos ADB$ (o.e.)

Or: $6^2 = 1^2 + x^2 - 2 \times 1 \times x \times \cos ADC$ (o.e.)

(at least one correct use of cos rule) M1

$\cos ADB = \dfrac{x^2 - 16}{6x}$ (convincing) A1

$\cos ADC = \dfrac{x^2 - 35}{2x}$ A1

(ii) $\dfrac{x^2 - 16}{6x} + \dfrac{x^2 - 35}{2x} = 0$ (o.e.)

(f.t. candidate's derived expression for $\cos ADC$) M1

$4x^2 = 121$ (f.t. candidate's derived expression for $\cos ADC$ providing it is of similar form) A1

$x = 5{\cdot}5$ (convincing) (c.a.o.) A1

(*b*) $ADB = 64{\cdot}42°$ B1

Area of triangle $ADB = \dfrac{5{\cdot}5 \times 3 \times \sin 64{\cdot}42°}{2}$

(f.t. candidate's derived value for angle ADB) M1

Area of triangle $ADB = 7{\cdot}44 \text{ cm}^2$ (c.a.o.) A1

6. (*a*) $5 \times \dfrac{x^{-2}}{-2} - 2 \times \dfrac{x^{4/3}}{4/3} - 4x + c$ B1, B1, B1

(–1 if no constant term present)

(*b*) Area $= \displaystyle\int_2^6 \left(3x^2 - \frac{1}{4}x^3\right) \mathrm{d}x$ (use of integration) M1

$\dfrac{3x^3}{3} - \dfrac{1}{4 \times 4}x^4$ (correct integration) B1

Area = (216 – 81) – (8 – 1)

(correct method for substituting limits) m1

Area = 128 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indices) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) **Either:**

$(5 - 4x) \log_{10} 7 = \log_{10} 11$

(taking logs on both sides and using the power law) M1

$x = \dfrac{5 \log_{10} 7 - \log_{10} 11}{4 \log_{10} 7}$ A1

$x = 0{\cdot}942$ (f.t. one slip, see below) A1

Or:

$5 - 4x = \log_7 11$ (rewriting as a log equation) M1

$x = \dfrac{5 - \log_7 11}{4}$ A1

$x = 0{\cdot}942$ (f.t. one slip, see below) A1

Note: an answer of $x = -0{\cdot}942$ from $x = \dfrac{\log_{10} 11 - 5 \log_{10} 7}{4 \log_{10} 7}$

earns M1 A0 A1

an answer of $x = 1{\cdot}558$ from $x = \dfrac{\log_{10} 11 + 5 \log_{10} 7}{4 \log_{10} 7}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

(*c*) $\log_8 x = -\dfrac{1}{3} \Rightarrow x = 8^{-1/3}$

(rewriting log equation as power equation) M1

$x = 8^{-1/3} \Rightarrow x = \dfrac{1}{2}$ A1

8. (*a*) (i) $A(2, -4)$ B1

(ii) Gradient $AP = \dfrac{\text{inc in } y}{\text{inc in } x}$ M1

Gradient $AP = \dfrac{(-7) - (-4)}{6 - 2} = -\dfrac{3}{4}$

(f.t. candidate's coordinates for A) A1

Use of $m_{\tan} \times m_{\text{rad}} = -1$ M1

Equation of tangent is:

$y - (-7) = \dfrac{4}{3}(x - 6)$ (f.t. candidate's gradient for AP) A1

(*b*) An attempt to substitute $(x + 3)$ for y in the equation of the circle and form quadratic in x M1

$x^2 + (x + 3)^2 - 4x + 8(x + 3) - 5 = 0 \Rightarrow 2x^2 + 10x + 28 = 0$ A1

An attempt to calculate value of discriminant m1

Discriminant $= 100 - 224 < 0 \Rightarrow$ no points of intersection

(f.t. one slip) A1

9. Denoting $A\hat{O}B$ by θ,

Area of sector $AOB = \dfrac{1}{2} \times 7^2 \times \theta$

Area of sector $COD = \dfrac{1}{2} \times 4^2 \times \theta$ (at least one correct) M1

$\dfrac{1}{2} \times 7^2 \times \theta - \dfrac{1}{2} \times 4^2 \times \theta = 23{\cdot}1$ (f.t candidate's expressions for the areas of the sectors) m1

$\theta = 1{\cdot}4$ (c.a.o.)

A1

$CD = 5{\cdot}6$ cm, $AB = 9{\cdot}8$ cm (both values, f.t candidate's value for θ) B1

Use of perimeter of $ACDB = AC + CD + DB + BA$ M1

Perimeter of $ACDB = 21{\cdot}4$ cm (c.a.o.) A1

10. (*a*) $t_2 = {}^3/_4$ B1

$t_3 = -{}^1/_3, \; t_4 = 4$ B1

(*b*) The sequence repeats itself every third term B1

$t_{50} = {}^3/_4$ B1

Mathematics C3 January 2014

Solutions and Mark Scheme

Final Version

1. (*a*)

0	0		
$\pi/12$	0·071796769		
$\pi/6$	0·333333333		
$\pi/4$	1		
$\pi/3$	3	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = \pi/12$ — M1

$I \approx \frac{\pi/12}{3} \times \{0 + 3 + 4(0{\cdot}071796769 + 1) + 2(0{\cdot}333333333)\}$

$I \approx 7{\cdot}953853742 \times (\pi/12) \div 3$

$I \approx 0{\cdot}69410468$

$I \approx 0{\cdot}6941$ (f.t. one slip) — A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^{\pi/3} \sec^2 x \, dx = \int_0^{\pi/3} 1 \, dx + \int_0^{\pi/3} \tan^2 x \, dx$ — M1

$\int_0^{\pi/3} \sec^2 x \, dx = 1{\cdot}7413$ (f.t. candidate's answer to (*a*)) — A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) Choice of x satisfying $75° \leq x < 90°$ and one correct evaluation B1
Both evaluations correct B1

(*b*) $15(1 + \cot^2\theta) + 2\cot\theta = 23$
(correct use of $\text{cosec}^2\theta = 1 + \cot^2\theta$) M1
An attempt to collect terms, form and solve quadratic equation in $\cot\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cot\theta + b)(c\cot\theta + d)$, with $a \times c$ = candidate's coefficient of $\cot^2\theta$ and $b \times d$ = candidate's constant m1

$15\cot^2\theta + 2\cot\theta - 8 = 0 \Rightarrow (5\cot\theta + 4)(3\cot\theta - 2) = 0$

$\Rightarrow \cot\theta = \frac{2}{3}, \cot\theta = -\frac{4}{5}$

$\Rightarrow \tan\theta = \frac{3}{2}, \tan\theta = -\frac{5}{4}$ (c.a.o.) A1

$\theta = 56{\cdot}31°, 236{\cdot}31°$ B1

$\theta = 128{\cdot}66°, 308{\cdot}66°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\tan\theta = +, -$, f.t. for 3 marks, $\tan\theta = -, -$, f.t. for 2 marks
$\tan\theta = +, +$, f.t. for 1 mark

3. $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(3) = 0$ B1

$\frac{d}{dx}(-2x^2y) = -2x^2\frac{dy}{dx} - 4xy$ B1

$\frac{d}{dx}(3y^2) = 6y\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{-4}{-14} = \frac{2}{7}$ (c.a.o.) B1

4. (*a*) $\dfrac{dx}{dt} = 6t^2$ B1

(*b*) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = 2 + 12t^2$ B1

Use of $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \div \dfrac{dx}{dt}$ M1

$\dfrac{d^2y}{dx^2} = \dfrac{2 + 12t^2}{6t^2}$ (c.a.o.) A1

$\dfrac{d^2y}{dx^2} = 2 \Rightarrow 2 + 12t^2 = 12t^2$ ($\Rightarrow 2 = 0$) $\Rightarrow$ no such t exists E1

(*c*) Use of $\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$ M1

$\dfrac{dy}{dt} = 12t^3 + 24t^5$ (f.t. candidate's expression for $\dfrac{dx}{dt}$) A1

Use of a valid method of integration to find y m1

$y = 3t^4 + 4t^6$ (+ c) (f.t. one error in candidate's $\dfrac{dy}{dt}$) A1

$y = 3t^4 + 4t^6 + 3$ (c.a.o.) A1

5. $x_0 = 1$

$x_1 = 0{\cdot}612372435$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 0{\cdot}62777008$

$x_3 = 0{\cdot}627136142$

$x_4 = 0{\cdot}627162204 = 0{\cdot}62716$ (x_4 correct to 5 decimal places) B1

Let $h(x) = x^3 + 7x^2 - 3$

An attempt to check values or signs of $h(x)$ at $x = 0{\cdot}627155$,

$x = 0{\cdot}627165$ M1

$h(0{\cdot}627155) = -6{\cdot}15 \times 10^{-5} < 0$, $h(0{\cdot}627165) = 3{\cdot}81 \times 10^{-5} > 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}62716$ correct to five decimal places A1

6. (*a*) $\frac{dy}{dx} = 10 \times (5x^3 - x)^9 \times f(x)$ $\quad (f(x) \neq 1)$ M1

$\frac{dy}{dx} = 10(5x^3 - x)^9(15x^2 - 1)$ A1

(*b*) **Either** $\frac{dy}{dx} = \frac{f(x)}{\sqrt{1-(x^3)^2}}$ (including $f(x) = 1$) **or** $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1-x^5}}$ M1

$\frac{dy}{dx} = \frac{3x^2}{\sqrt{1-x^6}}$ A1

(*c*) $\frac{dy}{dx} = x^4 \times f(x) + \ln(2x) \times g(x)$ M1

$\frac{dy}{dx} = x^4 \times f(x) + \ln(2x) \times g(x)$ $\quad$ (either $f(x) = 2 \times \frac{1}{2x}$ or $g(x) = 4x^3$) A1

$\frac{dy}{dx} = x^3 + 4x^3 \ln(2x)$ $\quad$ (all correct) A1

(*d*) $\frac{dy}{dx} = \frac{(2x+3)^6 \times k \times e^{4x} - e^{4x} \times 6 \times (2x+3)^5 \times m}{[(2x+3)^6]^2}$

with either $k =$ **4**, $m =$ **2** or $k = 4$, $m = 1$ or $k = 1$, $m = 2$ M1

$\frac{dy}{dx} = \frac{(2x+3)^6 \times 4 \times e^{4x} - e^{4x} \times 6 \times (2x+3)^5 \times 2}{[(2x+3)^6]^2}$ A1

$\frac{dy}{dx} = \frac{8xe^{4x}}{(2x+3)^7}$ $\quad$ (correct numerator) A1

(correct denominator) A1

7. (*a*) (i) $\int e^{5x/6}\,dx = k \times e^{5x/6} + c$ $(k = 1, {}^5/_6, \mathbf{{}^6/_5})$ M1

$\int e^{5x/6}\,dx = {}^6/_5 \times e^{5x/6} + c$ A1

(ii) $\int (8x + 1)^{1/3}\,dx = \dfrac{k \times (8x + 1)^{4/3}}{4/3} + c$ $(k = 1, 8, \mathbf{{}^1/_8})$ M1

$\int (8x + 1)^{1/3}\,dx = {}^3/_{32} \times (8x + 1)^{4/3} + c$ A1

(iii) $\int \sin(1 - x/3)\,dx = k \times \cos(1 - x/3) + c$
$(k = -1, \mathbf{3}, -3, {}^1/_3)$ M1

$\int \sin(1 - x/3)\,dx = 3 \times \cos(1 - x/3) + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \dfrac{1}{4x-1}\,dx = k \times \ln(4x - 1)$ $(k = 1, 4, \mathbf{{}^1/_4})$ M1

$\int \dfrac{1}{4x-1}\,dx = 1/4 \times \ln(4x - 1)$ A1

$k \times [\ln(4a - 1) - \ln 7] = 0{\cdot}284$ $(k = 1, 4, \mathbf{{}^1/_4})$ m1

$\dfrac{4a - 1}{7} = e^{1{\cdot}136}$ (o.e.) (c.a.o.) A1

$a = 5{\cdot}7$ (f.t. $a = 2{\cdot}6$ for $k = 1$ and $a = 2{\cdot}1$ for $k = 4$) A1

8. Trying to solve $3x + 4 = 2(x - 3)$ M1
Trying to solve $3x + 4 = -2(x - 3)$ M1
$x = -10$, $x = 0{\cdot}4$ (c.a.o.) A1

Alternative mark scheme
$(3x + 4)^2 = [2(x - 3)]^2$ (squaring both sides) M1
$5x^2 + 48x - 20 = 0$ (at least two coefficients correct) A1
$x = -10$, $x = 0{\cdot}4$ (c.a.o.) A1

9. (*a*) $y - 1 = \dfrac{2}{\sqrt{3x - 5}}$ B1

An attempt to isolate $3x - 5$ by crossmultiplying and squaring M1

$x = \dfrac{1}{3}\left(5 + \dfrac{4}{(y-1)^2}\right)$ (c.a.o.) A1

$f^{-1}(x) = \dfrac{1}{3}\left(5 + \dfrac{4}{(x-1)^2}\right)$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = (1, 1{\cdot}5]$ B1 B1

10. (*a*) $g'(x) = \dfrac{4}{(x+1)^2}$ B1

$g'(x) > 0 \Rightarrow g$ is an increasing function B1

(*b*) $R(g) = (0, 4)$ B1 B1

(*c*) $D(fg) = (-\infty, -2)$ B1

$R(fg) = (\sqrt{5}, \sqrt{21})$ (f.t. candidate's $R(g)$) B1

(*d*) (i) $fg(x) = \left(\left(\dfrac{-4}{x+1}\right)^2 + 5\right)^{1/2}$ B1

(ii) Putting expression for $fg(x)$ equal to 3 and squaring both sides M1

$\left(\dfrac{-4}{x+1}\right)^2 = 4$ (o.e.) (c.a.o.) A1

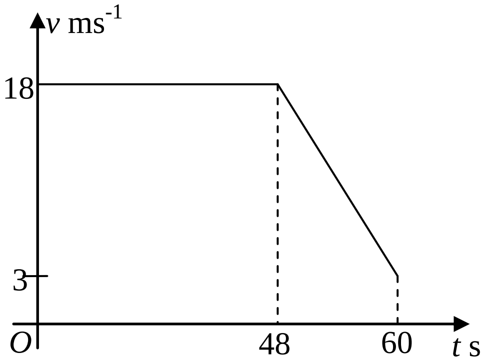
$x = -3, 1$ (two values, f.t. one slip) A1

Rejecting $x = 1$ and thus $x = -3$ (c.a.o.) A1

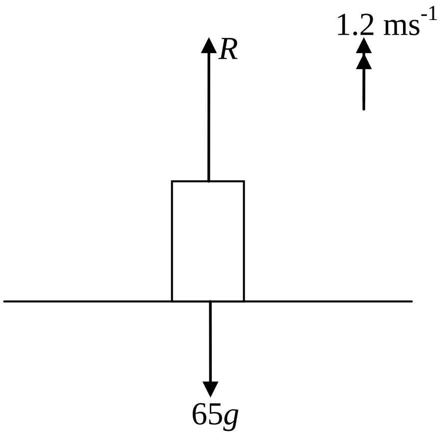
Mathematics M1 January 2014

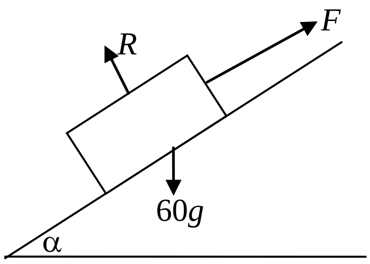
Solutions and Mark Scheme

Final Version

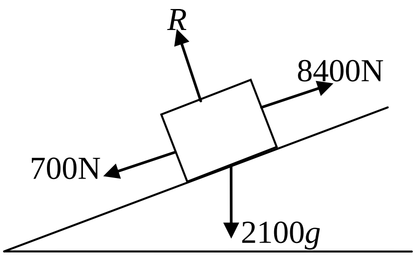
Q	Solution	Mark	Notes
1(a)		B1 B1	(0, 18) to (48, 18) Or (48, 18) to (60, 3) graph all correct, with units, labels.
1(b)	magnitude of deceleration = $\frac{18-3}{12}$ = $\underline{1.25\ (\text{ms}^{-2})}$	M1 A1	 A0 if negative
1(c)	Distance = area under graph Distance = $48\times18 + 0.5(18 + 3)\times12$ Distance = $\underline{990\ (\text{m})}$	M1 B1 A1	attempt at total area. one correct area seen cao

Q	Solution	Mark	Notes
2(a)	Use of $v = u+at$, $v=0$, $u=(\pm)7$, $a=(\pm)9.8$ $0 = 7 - 9.8t$ $t = \frac{7}{9\cdot 8} = \frac{5}{7}$ (s)	M1 A1	oe correct equ solvable for t A1
2(b)	Use of $s= ut+0.5at^2$, $u=(\pm)7$, $a=(\pm)9.8$, $t=4$ $s = 7\times4 + 0.5(-9.8)\times4^2$ $s = 28 - 4.9\times16$ $s = -50.4$ Height of cliff is <u>50.4 (m)</u>	M1 A1 A1	 if staged method, one correct distance cao, allow –ve

Q	Solution	Mark	Notes
3			
	N2L applied to man	M1	dim correct and R and $65g$ opposing.
	$R - 65g = 65a$	A1	
	$R = 65\times1.2 + 65\times9.8$		
	$R = \underline{715\ (\text{N})}$	A1	cao

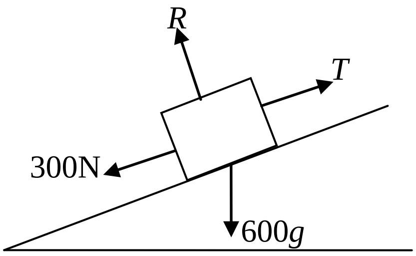
Q	Solution	Mark	Notes
4(a)(i)			
	$R = 60g\cos\alpha$ $F = \mu R$ $F = 60\times9.8\cos\alpha \times 0.3$ $F = \underline{159.87\ (\text{N})}$	B1 B1	
4(a)(ii)	N2L applied to object $60g\sin\alpha - F = 60a$ $60a = 60\times9.8\sin25^\circ - 159.87$ $a = \underline{1.48\ (\text{ms}^{-2})}$	M1 A1 A1	all forces, dim correct. ft F
4(b)	If object remains stationary, component Of weight down slope $\leq$ Friction $60g\sin\alpha \leq \mu\times60g\cos\alpha$ $\therefore$ least $\mu = \tan 25^\circ$ $= 0.4663$ $= \underline{0.47\ (\text{to 2 d.p.})}$	 M1 A1 A1	 si

Q	Solution	Mark	Notes
5	Resolve in Q direction $Q = 9\sin60^{\circ}$ $= 9 \times \frac{\sqrt{3}}{2} = \underline{7.794}$	M1 A1	equation required cao
	Resolve in P direction $P + 9\cos60^{\circ} = 6$ $P = 6 - 9 \times 0.5$ $P = \underline{1.5}$	M1 A1 A1	equation required, all forces correct equation cao

<table>
<tr><th>Q</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>6(a)</td><td>
</td><td></td><td></td></tr>
<tr><td></td><td>N2L on whole system</td><td>M2

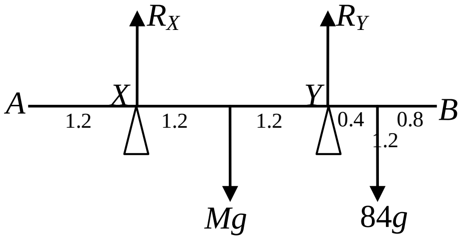
(M1</td><td>all forces in same dir, dim correct. 8400N and resistance opposing.
one force missing but must have comp of wt. and resistance.)</td></tr>
<tr><td></td><td>$8400 - 700 - 2100g\sin\alpha = 2100a$
$8400 - 700 - 5762.4 = 2100a$
$a = \underline{0.923\ (\text{ms}^{-2})}$</td><td>A2

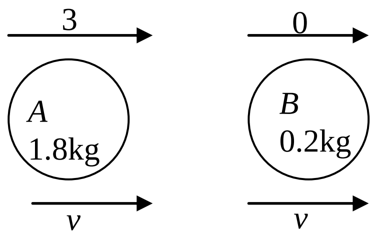
A1</td><td>-1 each error

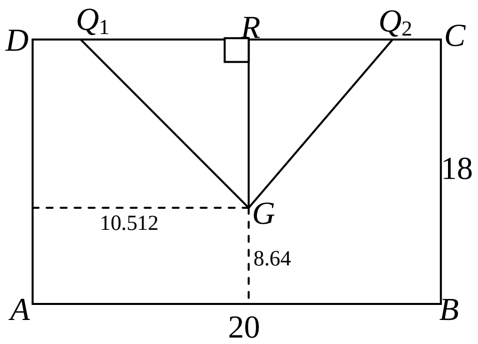
cao 3 dp required.</td></tr>
<tr><td>6(b)</td><td>
</td><td></td><td></td></tr>
<tr><td></td><td>N2L applied to trailer</td><td>M1</td><td>all forces, no extra.
Dim correct. Either resist. or comp wt opposing</td></tr>
<tr><td></td><td>$T - 300 - 600g\sin\alpha = 600a$
$T - 300 - 600\times9.8\times\frac{7}{25} = 600\times\frac{346}{375}$
$T = \underline{2500\ (\text{N})}$</td><td>A2

A1</td><td>-1 each error

ft a.
answers rounding to 2500</td></tr>
</table>

Q	Solution	Mark	Notes
7(a)			
7(a)(i)	Moments about Y	M1	dim. Correct, all forces, equation, oe
		B1	any correct moment.
	$Mg\times1.2 = R_X\times2.4 + 84g\times0.4$		
	$(9.8\times1.2)M = 2.4\times156.8 + 84\times9.8\times0.4$		
	$M = \underline{60}$	A1	
7(a)(ii)	Resolve vertically	M1	all forces
	$R_X + R_Y = Mg + 84g$	A1	
	$R_Y = 144\times9.8 - 156.8$		
	$R_Y = \underline{1254.4\ (\text{N})}$	A1	ft M
7(b)(i)	When plank about to tilt about Y		
	$R_Y = 0$	M1	si
	Resolve vertically	M1	all forces
	$R_X = 60g + 84g$		
	$R_X = \underline{1411.2\ (\text{N})}$	A1	ft M
7(b)(ii)	Moments about X	M1	dim correct
	$84g \times x = 60g \times 1.2$		
	$x = \frac{6}{7} = \underline{0.86}$	A1	ft M
	Distance of the person from X = 0.86 (m)		

Q	Solution	Mark	Notes
8(a)(i)			
	Conservation of momentum	M1	allow different v's
	$1.8\times3 + 0.2\times0 = 1.8v + 0.2v$	A1	
	$2v = 5.4$		
	$v = \underline{2.7\ (\text{ms}^{-1})}$	A1	convincing
8(a)(ii)	$e = \underline{0}$	B1	
8(b)(i)	N2L applied to combined object	M1	dim correct
	$-8 = 2a$		
	$a = -4\ \text{ms}^{-2}$		
	$\lvert a\rvert = \underline{4\ (\text{ms}^{-2})}$	A1	
8(b)(ii)	Use of $v = u+at$, u=2.7, a=(±)4, t=0.5	M1	oe
	$v = 2.7 - 4\times0.5$	A1	ft a if <0.
	$v = \underline{0.7\ (\text{ms}^{-1})}$	A1	ft a if <0.
8(b)(iii)	Use of $v^2=u^2+2as$, u=2.7, v=2, a=(±)4	M1	oe
	$2^2 = 2.7^2 - 2\times4s$	A1	ft a if <0.
	$s = \underline{0.41(125\ \text{m})}$	A1	ft a if <0.

Q	Solution	Mark	Notes
9(a)	Area, from AD, from AB		
	$ABCD$: 360, 10, 9	B1	
	Circle: 21, 6, 12	B1	
	XYZ: 36, 13, 7	B1	
	Lamina: 375, x, y	B1	all 4 correct areas
9(a)(i)	Moments about AD	M1	consistent use of signs for areas and moments.
	$360\times10 + 36\times13 = 375x + 21\times6$	A1	ft table if +XYZ and -circ
	x = <u>10.5(12 cm)</u>	A1	cao
9(a)(ii)	Moments about AB	M1	consistent use of signs for areas and moments.
	$360\times9 + 36\times7 = 375y + 21\times12$	A1	ft table if +XYZ and -circ
	y = <u>8.6(4 cm)</u>	A1	cao
9(b)			
	Consider triangle $RQ_{1/2}G$ Angle RGQ = angle RQG = 45° $\therefore$ $RQ = RG$		
	Let $DQ_1 = x$		
	$10.512 - x = 18 - 8.64$	M1	
	$x = 10.512 - 9.36$		
	DQ_1 = <u>1.1(52 cm)</u>	A1	ft x, y
	$DQ_2 = 10.512 + (18 - 8.64)$	M1	
	DQ_2 = <u>19.8(72 cm)</u>	A1	ft x, y

Mathematics S1 January 2014

Solutions and Mark Scheme

Final Version

Ques	Solution	Mark	Notes
1(a)(i)	$P(A \cap B) = P(B)P(A \mid B)$	**M1**	Award M1 for using formula
	$= 0.08$	**A1**	
(ii)	$P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$	**M1**	Award M1 for using formula
	$= 0.16$	**A1**	FT their $P(A \cap B)$ unless independence assumed
(b)	Considering any valid expression, eg $P(A \cap B) > 0$, $P(A \mid B) > 0$, $P(B \mid A) > 0$, $P(A \cup B) < P(A) + P(B)$, the events are not mutually exclusive	**B1**	FT previous work Conclusion must be justified
2(a)	P(1 of each) =		
	$\dfrac{6}{12} \times \dfrac{4}{11} \times \dfrac{2}{10} \times 6$ or $\binom{6}{1} \times \binom{4}{1} \times \binom{2}{1} \div \binom{12}{3}$	**M1A1**	M1A0 if 6 omitted or incorrect factor used
	$= \dfrac{12}{55}$ (0.218)	**A1**	
(b)	P(3 Els) = $\dfrac{6}{12} \times \dfrac{5}{11} \times \dfrac{4}{10}$ or $\binom{6}{3} \div \binom{12}{3}$	**M1**	
	$= \dfrac{1}{11}$ (0.091)	**A1**	
(c)	P(3 Gala) = $\dfrac{4}{12} \times \dfrac{3}{11} \times \dfrac{2}{10}$ or $\binom{4}{3} \div \binom{12}{3}$		
	$= \dfrac{1}{55}$ (0.018) si	**B1**	
	P(3 the same) = $\dfrac{1}{11} + \dfrac{1}{55} = \dfrac{6}{55}$ (0.109)	**M1A1**	FT previous values
3(a)	P(C wins 1st shot) = P(R misses)P(C hits)	**M1**	
	$= 0.7 \times 0.4$	**A1**	
	$= 0.28$		
(b)	P(C wins 2nd shot) = $0.7 \times 0.6 \times 0.7 \times 0.4$	**M1**	
	$= 0.42 \times 0.28$ ($k = 0.42$)	**A1**	
(c)	P(C wins) = $0.28 + 0.42 \times 0.28 + \ldots$	**M1**	FT their value of k if between 0 and 1
	$= \dfrac{0.28}{1 - 0.42}$	**A1**	
	$= 0.483$ (14/29)	**A1**	

Ques	Solution	Mark	Notes
4(a)(i)	$P(X=6) = \binom{20}{6} \times 0.2^6 \times 0.8^{14} = 0.109$	M1A1	M0 if no working shown
(ii)	Prob = 0.9900 – 0.0692 or 0.9308 – 0.0100 = 0.921 cao	B1B1 B1	B0B0B0 if no working shown
(b)	B(200,0.0123) is approx Po(2.46)	B1	
	$P(Y=3) = \dfrac{e^{-2.46} \times 2.46^3}{3!} = 0.212$	M1A1	M0 if no working shown Do not accept use of tables
5(a)	$P(2G) = \frac{1}{3}\times 1 + \frac{1}{3}\times\frac{3}{4}\times\frac{2}{3} + \frac{1}{3}\times\frac{2}{4}\times\frac{1}{3}$	M1A3	M1 Use of Law of Total Prob (Accept tree diagram)
	$= \frac{5}{9}$ cao	A1	
(b)	$P(A\|2G) = \dfrac{1/3}{5/9}$	B1B1	FT denominator from (a) B1 num, B1 denom
	$= \frac{3}{5}$ cao	B1	
6(a)(i)	X is B(10,0.75) si $E(X) = 7.5$, $\text{Var}(X) = 1.875$	B1 B1 B1	
(ii)	Attempt to evaluate either P(X = 7) or P(X = 8) P(X = 7) = 0.250 ; P(X = 8) = 0.282 So try P(X = 9) = 0.188 Most likely value = 8	M1 A1 A1 A1	Award the final A1 only if the previous A1 was awarded
(b)(i)	$W = 10X - 2(10 - X) = 12X - 20$	B1	
(ii)	E(W) = 12 × 7.5 – 20 = 70 Var(W) = 12^2 × Var(X) = 270	B1 M1A1	FT their mean and variance from (a) and FT their derived values of a and b provided that $a \neq 1$ and $b \neq 0$
7(a)	$E(X) = 0.1\times1 + 0.2\times2 + 0.3\times3 + 0.1\times4 + 0.3\times5$ $= 3.3$	M1 A1	
	$E(X^2) = 0.1\times1 + 0.2\times4 + 0.3\times9 + 0.1\times16$ $+ 0.3\times25$ (12.7)	B1	.
	$\text{Var}(X) = 12.7 - 3.3^2 = 1.81$	M1A1	FT their $E(X^2)$
(b)(i)	The possibilities are (1,1,2); (1,2,1); (2,1,1) $P(S=4) = 0.1^2 \times 0.2 \times 3 = 0.006$	B1 M1A1	Award M1 if only one correct possibility given
(ii)	The only extra possibility is (1,1,1) so $P(S=3) = 0.1^3$ (0.001) Therefore $P(S \le 4) = 0.007$	B1 B1 B1	FT from (b)(i) if M1 awarded

Ques	Solution	Mark	Notes
8(a)(i)	Prob = $\dfrac{e^{-15}\times 15^{12}}{12!}$ or 0.2676 – 0.1848 = 0.083 or 0.8152 – 0.7324	**M1** **A1**	M0 if no working shown
(ii)	We require $P(X \geq 20)$ = 1 – 0.8752 = 0.1248	**M1** **A1**	Award M1A0 for use of adjacent row or column
(b)	(Using tables, the number required is) 25	**M1A1**	Award M1A0 for 24 or 26
9(a)(i)	Using F(2) = 1 $1 = k(8-2)$ k =1/6 (convincing)	**M1** **A1**	
(ii)	$P(1.25 \leq X \leq 1.75) = F(1.75) - \mathrm{F}(1.25)$ = 0.6015… – 0.1171.. si = 0.484 (31/64)	**M1** **A1** **A1**	
(b)(i)	$f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x^3 - x}{6}\right)$ $= \dfrac{3x^2 - 1}{6}$	**M1** **A1**	
(ii)	$\mathrm{E}(X) = \displaystyle\int_1^2 x\left(\dfrac{3x^2-1}{6}\right)\mathrm{d}x$ $= \left[\dfrac{x^4}{8} - \dfrac{x^2}{12}\right]_1^2$ = 1.625 cao	**M1A1** **A1** **A1**	M1 for the integral of $xf(x)$, A1 for completely correct with or without limits FT on their f if previous M1 awarded Limits must appear here if not before M0 if no working shown

Mathematics FP1 January 2014

Solutions and Mark Scheme

Final Version

Ques	Solution	Mark	Notes
1	$f(x+h)-f(x)=\dfrac{x+h}{1+x+h}-\dfrac{x}{1+x}$	M1A1	
	$=\dfrac{(x+h)(1+x)-x(1+x+h)}{(1+x+h)(1+x)}$	A1	
	$=\dfrac{h}{(1+x+h)(1+x)}$	A1	
	$f'(x)=\lim_{h\to 0}\dfrac{h}{h(1+x+h)(1+x)}$	M1	
	$=\dfrac{1}{(1+x)^2}$ cso	A1	
2	$S_n=\sum_{r=1}^{n}r(r+1)^2=\sum_{r=1}^{n}r^3+2\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r$	M1A1	
	$=\dfrac{n^2(n+1)^2}{4}+\dfrac{2n(n+1)(2n+1)}{6}+\dfrac{n(n+1)}{2}$	A1A1	Award A1 for 2 correct
	$=\dfrac{n(n+1)}{12}\left(3n^2+3n+8n+4+6\right)$	A1	
	$=\dfrac{n(n+1)(n+2)(3n+5)}{12}$	A1	
3(a)	$(1+2\mathrm{i})^4=1+4.2\mathrm{i}+6(2\mathrm{i})^2+4(2\mathrm{i})^3+(2\mathrm{i})^4$	M1	Award M1 for use of binomial theorem (oe)
	$= 1 + 8\mathrm{i} - 24 - 32\mathrm{i} + 16 = -7 - 24\mathrm{i}$	A1	
(b)(i)	Let $f(x) = x^4+12x-5$ $f(1 + 2\mathrm{i}) = -7 - 24\mathrm{i} + 12 + 24\mathrm{i} - 5 = 0$ (showing that 1 + 2i is a root)	M1A1	
(ii)	Another root is $1 - 2\mathrm{i}$	B1	
	EITHER It follows that x^2-2x+5 is a factor of $f(x)$	B1	
	$x^4+12x-5 = (x^2-2x+5)(x^2+2x-1)$	M1A1	
	The other two roots are $-1\pm\sqrt{2}$	M1A1	
	OR $(1 + 2\mathrm{i})(1 - 2\mathrm{i}) = 5$ $(1 + 2\mathrm{i}) + (1 - 2\mathrm{i}) = 2$	B1	
	Therefore if α, β denote the other two roots $\alpha + \beta = -2$ and $\alpha\beta = -1$	B1	
	So α, β are the roots of the equation $x^2+2x-1=0$	B1	
	The other two roots are $-1\pm\sqrt{2}$	M1A1	

Ques	Solution	Mark	Notes
4	$\alpha+\beta=\frac{3}{2}, \alpha\beta=2$	B1	
	$\alpha^2\beta+\alpha\beta^2+\alpha\beta=\alpha\beta(\alpha+\beta+1)=5$	M1A1	FT one slip in line above in sign or in their two values.
	$\alpha^3\beta^3+\alpha^2\beta^3+\alpha^3\beta^2=\alpha^2\beta^2(\alpha\beta+\alpha+\beta)=14$	M1A1	
	$\alpha\beta^2\times\alpha^2\beta\times\alpha\beta=\alpha^4\beta^4=16$	M1A1	
	The required equation is $x^3-5x^2+14x-16=0$ cao	B1	FT their three values
5(a)	Ref matrix $=\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$	B1	
	Translation matrix $=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$	B1	
	Rotation matrix $=\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$	B1	
	$\mathbf{T}=\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}=$	M1	
	$\begin{bmatrix}0 & 1 & 2\\ -1 & 0 & -1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$ or $\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & -1 & 1\\ -1 & 0 & 2\\ 0 & 0 & 1\end{bmatrix}$	A1	
	$=\begin{bmatrix}-1 & 0 & 2\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}$		
(b)	The general point on the line is $(\alpha, 2\alpha-1)$.	M1	
	Consider $\begin{bmatrix}-1 & 0 & 2\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ 2\alpha-1\\ 1\end{bmatrix}=\begin{bmatrix}-\alpha+2\\ 2\alpha-2\\ 1\end{bmatrix}$	m1	
	$x=-\alpha+2, y=2\alpha-2$	A1	
	Eliminating α, the equation of the image is $y=2-2x$	M1A1	

Ques	Solution	Mark	Notes
6(a)	Putting n = 1, the formula gives $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ which is correct so the result is true for n = 1	**B1**	
	Assume formula is true for $n = k$, ie $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}^k = \begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$	**M1**	
	Consider, for $n = k + 1$, $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}^{k+1} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}^k\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ or $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}^k$	**M1**	
	$=\begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ or $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$	**A1**	
	$=\begin{bmatrix}1 & 2+3(3^k - 1)\\0 & 3^{k+1}\end{bmatrix}$ or $\begin{bmatrix}1 & 3^k - 1 + 2.3^k\\0 & 3^{k+1}\end{bmatrix}$	**A1**	This line must be seen
	$=\begin{bmatrix}1 & 3^{k+1} - 1\\0 & 3^{k+1}\end{bmatrix}$	**A1**	Award this A1 only if previous A1 awarded
	Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	**A1**	Award final A1 only if all six previous marks have been awarded
(b)	The formula gives $\mathbf{A}^{-1} = \begin{bmatrix}1 & -2/3\\0 & 1/3\end{bmatrix}$	**B1**	
	EITHER Consider $\begin{bmatrix}1 & -2/3\\0 & 1/3\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ OR $\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix}3 & -2\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & -2/3\\0 & 1/3\end{bmatrix}$	**B1**	
	The formula is therefore correct for n = –1	**B1**	

Ques	Solution	Mark	Notes
7(a)(i)	Cofactor matrix = $\begin{bmatrix}-1 & 2 & -1\\ -9 & 6 & 0\\ 7 & -5 & 1\end{bmatrix}$ si	M1 A1	Award the M1 if at least 5 of the elements are correct
	Adjugate matrix = $\begin{bmatrix}-1 & -9 & 7\\ 2 & 6 & -5\\ -1 & 0 & 1\end{bmatrix}$	A1	
(ii)	Determinant = 3	B1	
	Inverse matrix = $\frac{1}{3}\begin{bmatrix}-1 & -9 & 7\\ 2 & 6 & -5\\ -1 & 0 & 1\end{bmatrix}$	B1	FT their adjugate matrix
(b)	$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{3}\begin{bmatrix}-1 & -9 & 7\\ 2 & 6 & -5\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}13\\ 13\\ 19\end{bmatrix}$	M1	FT their inverse matrix
	$= \begin{bmatrix}1\\ 3\\ 2\end{bmatrix}$	A1	
8(a)	Taking logs, $\ln f(x) = \sqrt{x}\ln\left(\frac{1}{x}\right)$	B1	
	Differentiating, $\frac{f'(x)}{f(x)} = \frac{1}{2\sqrt{x}}\ln\left(\frac{1}{x}\right) + \sqrt{x}.-\frac{1}{x}$	B1B1	B1 for each side
	$f'(x) = f(x)\left(\frac{-2-\ln x}{2\sqrt{x}}\right)$	B1	Award this B1 only if ln(1/x) has been simplified to – ln$x$ and the two terms are over a common denom.
(b)	Putting $f'(x) = 0$, ln(x) = – 2 so $x = \mathrm{e}^{-2}$ = 0.135 $y = \mathrm{e}^{2/\mathrm{e}}$ =2.09	M1 A1 A1	
(c)	$f'(x) > 0$ for $0 < x < \mathrm{e}^{-2}$; $f'(x) < 0$ for $x > \mathrm{e}^{-2}$ cao	B1	Accept $x < \mathrm{e}^{-2}$
	It is a maximum	B1	Award this B1 if the answer is consistent with a previous line containing two sets of values of x even if incorrect.

Ques	Solution	Mark	Notes
9(a)	Putting $z = 0$, we see that LHS =RHS = 2 hence locus passes through (0,0)	**M1** **A1**	Accept alternative arguments that do not depend upon the result obtained in (b)
(b)	Putting $z = x + iy$, $\lvert x - 2 + iy\rvert = 2\lvert x + i(y + 1)\rvert$ $(x-2)^2 + y^2 = 4\left(x^2 + (y+1)^2\right)$ $x^2 - 4x + 4 + y^2 = 4x^2 + 4y^2 + 8y + 4$ $3x^2 + 3y^2 + 4x + 8y = 0$ (This shows that the locus of P is a circle.) Consider the equation in the form $x^2 + y^2 + \frac{4}{3}x + \frac{8}{3}y = 0$ The centre is $\left(-\frac{2}{3}, -\frac{4}{3}\right)$ cao The radius is $\frac{2\sqrt{5}}{3}$ (1.49) cao	**M1** **A1** **A1** **A1** **B1** **B1** **B1**	

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