GCE AS/A level

0983/01

MATHEMATICS – S1
Statistics

A.M. MONDAY, 27 January 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The events A and B are such that

$$P(A) = 0{\cdot}5,\ P(B) = 0{\cdot}2,\ P(A|B) = 0{\cdot}4.$$

(a) Calculate

(i) $P(A \cap B)$,

(ii) $P(B|A)$. [4]

(b) Giving a reason, state whether or not A and B are mutually exclusive. [1]

2. Gwyn has three varieties of apples in his fruit bowl, 6 Elstar, 4 Gala and 2 Regent. He decides to select 3 of these 12 apples at random to take to work.
Calculate the probability that he selects

(a) 1 apple of each variety, [3]

(b) 3 Elstar apples, [2]

(c) 3 apples all of the same variety. [3]

3. When Catrin shoots an arrow at a target, she hits it with probability 0·4. When Rhiannon shoots an arrow at the target, she hits it with probability 0·3. Successive shots are independent. One morning, they decide to shoot arrows alternately at the target, starting with Rhiannon. The winner is the first to hit the target.

(a) Show that the probability that Catrin wins with her first shot is 0·28. [2]

(b) Show that the probability that Catrin wins with her second shot can be written in the form $k \times 0{\cdot}28$, and state the value of k. [2]

(c) Hence, by summing an infinite geometric series, find the probability that Catrin wins. [3]

4. *(a)* The random variable X has the binomial distribution B(20, 0·2).

(i) Without the use of tables, calculate $P(X = 6)$,

(ii) Determine $P(2 \leqslant X \leqslant 8)$. [5]

(b) The random variable Y has the binomial distribution B(200, 0·0123).
Use the Poisson distribution to determine the approximate value of $P(Y = 3)$. [3]

5. Three drawers each contain 4 coins. Drawer A contains 4 gold coins. Drawer B contains 3 gold coins and 1 silver coin. Drawer C contains 2 gold coins and 2 silver coins. David selects one of these drawers at random and then selects 2 coins at random from that drawer without replacement.

(a) Determine the probability that he selects 2 gold coins. [5]

(b) Given that he selects 2 gold coins, determine the probability that Drawer A was selected. [3]

6. Jim takes part in a quiz in which he has to answer 10 questions on his chosen topic. You may assume that he answers each question correctly with probability 0·75 and that answers to successive questions are independent.
Let X denote the number of questions that he answers correctly.

(a) (i) Find the mean and the variance of X.

(ii) Find the most likely value of X. [7]

(b) Jim wins £10 for each question answered correctly but loses £2 for each question not answered correctly. Let £W denote the total amount that Jim wins.

(i) Show that $W = aX - b$, where a, b are positive integers whose values are to be found.

(ii) Find the mean and the variance of W. [4]

7. The discrete random variable X has the following probability distribution.

x	1	2	3	4	5
$P(X = x)$	0·1	0·2	0·3	0·1	0·3

(a) Determine the mean and the variance of X. [5]

(b) Three independent observations X_1, X_2, X_3 are taken from the distribution of X and $S = X_1 + X_2 + X_3$. Calculate

(i) $P(S = 4)$,

(ii) $P(S \leqslant 4)$. [6]

TURN OVER

8. A newsagent sells the Daily Bugle newspaper. You may assume that the daily demand for this newspaper has a Poisson distribution with mean 15. The newsagent begins each day with 20 copies of the newspaper.

(a) Calculate the probability that, on a randomly chosen day, the newsagent sells

(i) 12 copies of the newspaper,

(ii) all 20 copies of the newspaper. [4]

(b) Determine the minimum number of copies of the Daily Bugle that the newsagent should buy each day in order to satisfy the demand with a probability of at least 0·99. [2]

9. The continuous random variable X has cumulative distribution function F given by

$$
\begin{aligned}
F(x) &= 0 && \text{for } x < 1,\\
F(x) &= k(x^3 - x) && \text{for } 1 \leqslant x \leqslant 2,\\
F(x) &= 1 && \text{for } x > 2,
\end{aligned}
$$

where k is a constant.

(a) (i) Show that $k = \frac{1}{6}$.

(ii) Evaluate $P(1{\cdot}25 \leqslant X \leqslant 1{\cdot}75)$. [5]

(b) (i) Find an expression for $f(x)$, valid for $1 \leqslant x \leqslant 2$, where f denotes the probability density function of X.

(ii) Hence determine $E(X)$. [6]

END OF PAPER