GCE AS/A level

0974/01

MATHEMATICS – C2
Pure Mathematics

P.M. FRIDAY, 17 January 2014

1 hour 30 minutes

Suitable for Modified Language Candidates

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_2^4 \sqrt{1+\frac{6}{x}}\,\mathrm{d}x.$$

Show your working. Give your answer correct to three decimal places. [4]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$8\cos^2\theta - 7\sin^2\theta = 4\cos\theta - 3.$$ [6]

(b) The angles X, Y and Z are the three angles of a triangle. Given that $\tan X = -2{\cdot}246$ and that $\tan(Y - Z) = 0{\cdot}364$, find the values of X, Y and Z. Give each angle correct to the nearest degree. [4]

3. *(a)* The sum of the third and eighth terms of an arithmetic series is zero. The sum of the fifth, seventh and tenth terms of the series is 22. Find the first term and the common difference of the series. [4]

(b) The first term of another arithmetic series is 9 and the common difference is 2. The sum of the first $2n$ terms of this arithmetic series is 3 times the sum of the first n terms of the series. Find the value of n. [5]

4. *(a)* A geometric series has first term a and common ratio r. Prove that the sum of the first n terms is given by

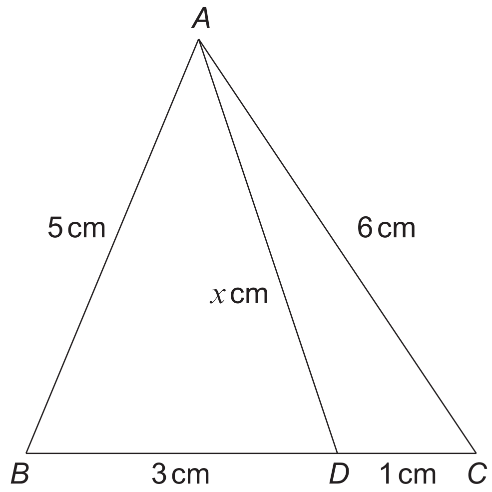
$$S_n = \frac{a\left(1-r^n\right)}{1-r}.$$ [3]

(b) The fourth term of a geometric series is -108 and the seventh term is 4.

(i) Find the common ratio of the series.

(ii) Find the sum to infinity of the series. [6]

5. The diagram below shows a sketch of the triangle *ABC* with *AB* = 5 cm and *AC* = 6 cm. The point *D* is on *BC* such that *BD* = 3 cm, *DC* = 1 cm and *AD* = x cm.



(a) (i) By applying the cosine rule in each of the triangles *ADB* and *ADC*, show that

$\cos A\hat{D}B = \dfrac{x^2 - 16}{6x}$ and find a similar expression for $\cos A\hat{D}C$.

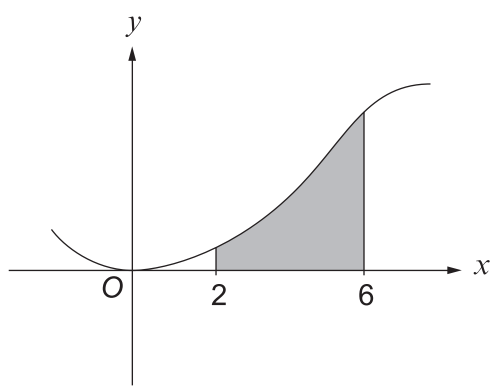
(ii) Noting that $A\hat{D}B$ and $A\hat{D}C$ are angles on a straight line, use the expressions derived in part (i) to write down an equation satisfied by x. Hence show that $x = 5{\cdot}5$. [6]

(b) Find the area of triangle *ADB*. Give your answer correct to two decimal places. [3]

6. *(a)* Find $\displaystyle\int\left(\frac{5}{x^3} - 2x^{\frac{1}{3}} - 4\right)\mathrm{d}x.$ [3]

(b) The diagram below shows a sketch of the curve with equation $y = 3x^2 - \dfrac{1}{4}x^3$.

The shaded region is bounded by the curve, the x-axis and the lines $x = 2$, $x = 6$.
Find the area of this shaded region. [4]



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7. *(a)* Given that $x > 0$, show that

$$\log_a x^n = n \log_a x.$$ [3]

(b) Solve the equation

$$7^{5-4x} = 11.$$

Show your working. Give your answer correct to three decimal places. [3]

(c) Solve the equation

$$\log_8 x = -\frac{1}{3}.$$ [2]

8. The circle *C* has centre *A* and equation

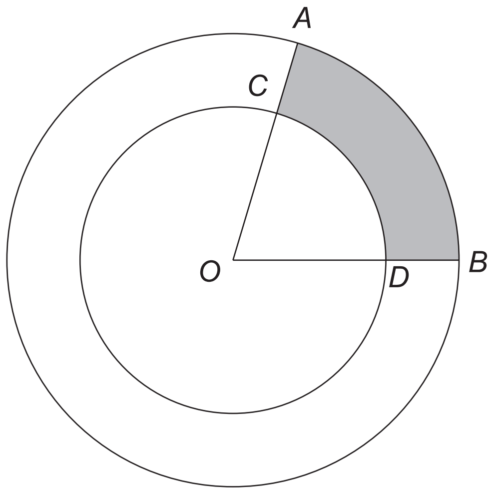
$$x^2 + y^2 - 4x + 8y - 5 = 0.$$

(a) (i) Write down the coordinates of *A*.

(ii) The point *P* has coordinates (6, –7) and lies on *C*. Find the equation of the tangent to *C* at *P*. [5]

(b) The line *L* has equation $y = x + 3$. Show that *L* and *C* do not intersect. [4]

9.



The diagram shows two concentric circles with a common centre *O*. The radius of the larger circle is 7 cm. The radius of the smaller circle is 4 cm. The points *A* and *B* lie on the larger circle. *OA* and *OB* cut the smaller circle at the points *C* and *D* respectively. The area of the shaded region *ACDB* is 23·1 cm^2. Find the perimeter of *ACDB*. [6]

10. The nth term of a number sequence is denoted by t_n. The $(n + 1)$th term of the sequence satisfies

$$t_{n+1} = 1 - \frac{1}{t_n},$$

for all positive integers n. Given that $t_1 = 4$,

(a) evaluate t_2, t_3, and t_4, [2]

(b) describe the behaviour of the sequence and hence, without carrying out any further calculation, write down the value of t_{50}. [2]