GCE AS/A level

0981/01

MATHEMATICS M2
Mechanics

A.M. TUESDAY, 10 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

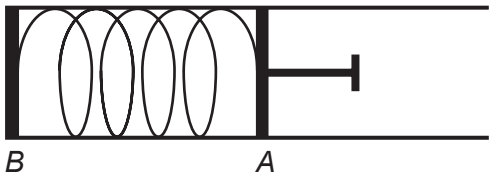
Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The diagram shows a piston, of mass 0·8 kg, enclosed in a horizontal tube and attached to a light spring of natural length 0·2 m and modulus of elasticity 625 N. The other end of the spring is fixed to the end of the tube at point *B*.



Initially, the piston is held at rest at a point *A* with the spring compressed a distance of 0·1 m, so that *AB* is the compressed length of the spring.

(a) Calculate the elastic energy stored in the spring. [2]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude 46 N.

(b) Determine the velocity of the piston when the spring reaches its natural length. [5]

2. A particle of mass 5 kg moves under the action of a horizontal force given by $F = 30t^{-2} - 30$ N at time t s, where $t > 0$. It also experiences a constant resistance to motion of magnitude 120 N.

(a) Show that the motion of the particle satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t^{-2} - 30,$$

where v ms^{-1} is the velocity of the particle at time t s. [2]

(b) Calculate the value of t when the acceleration of the particle is 24 ms^{-2}. [2]

(c) Given that the velocity of the particle is 18 ms^{-1} when $t = \frac{1}{3}$, find an expression for v in terms of t. Hence find the values of t when v = 10. [6]

3. A vehicle of mass 4000 kg is travelling up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{2}{49}$. The engine of the vehicle is working at a constant rate of 90 kW.

(a) Calculate the resistance to the motion of the vehicle at the instant when its speed is 4·8 ms^{-1} and its acceleration is 1·2 ms^{-2}. [6]

(b) Determine the maximum velocity of the vehicle when the resistance to motion has magnitude 12 800 N. [4]

4. At time t = 0, an aeroplane A has position vector $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})$ m and is flying with constant velocity $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ ms^{-1}.
At time t = 0, another aeroplane B has position vector $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})$ m, and is flying with constant velocity $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ ms^{-1}.

(a) Find expressions for the position vector of A and the position vector of B at time t s. [3]

(b) Determine an expression for AB^2, where AB is the distance between A and B at time t s. [4]

(c) Given that the shortest distance between A and B occurs at t = 5, calculate the value of x. [3]

5. A player kicks a ball from a point A on horizontal ground so that 2·5 seconds later the ball just clears a bar at a point B. The point B is 3 m above the ground. The horizontal distance of B from A is 42 m.

(a) Calculate the horizontal and vertical components of the initial velocity of the ball. [4]

(b) Find the magnitude of the velocity of the ball and the angle that the direction of the velocity makes with the horizontal as it passes the point B. [6]

(c) Determine the horizontal distance from B to the point where the ball first hits the ground again. [3]

6. A particle of mass 3 kg moves on a horizontal plane. At time t = 0, the particle has position vector $-2\mathbf{i} + 3\mathbf{j}$ m, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the x-axis and y-axis respectively. At time t s, the particle moves with velocity $\mathbf{v}$ ms^{-1} given by

$$\mathbf{v} = 4\sin 2t\,\mathbf{i} + 15\cos 5t\,\mathbf{j}.$$

(a) Find the magnitude of the force acting on the particle at time $t = \frac{3\pi}{2}$ s. [5]

(b) Determine the position vector of the particle at time t s. [4]

(c) Calculate the time and the distance of the particle from the origin when it crosses the y-axis for the first time. [4]

7. One end of a light rod of length l metres is freely jointed to a fixed point O and the other end is attached to a particle of mass m kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, u ms^{-1}, is a quarter of its speed at the lowest point of the circle.

(a) Show that $u^2 = \frac{4}{15}gl$. [3]

(b) When the rod is inclined at an angle θ to the **downward** vertical,
(i) find an expression for the tension in the rod in terms of m, g and θ.
(ii) determine the value of θ when the tension in the rod becomes zero. [9]

END OF PAPER

