GCE AS/A level

# 0981/01

# MATHEMATICS – M2

## Mechanics

## A.M. TUESDAY, 10 June 2014

## 1 hour 30 minutes plus your additional time allowance

## ADDITIONAL MATERIALS

**In addition to this examination paper, you will need:**

**a 12 page answer book;**
**a Formula Booklet;**
**a calculator.**

## INSTRUCTIONS TO CANDIDATES

**Use black ink, black ball-point pen or your usual method.**

**Answer ALL questions.**

**Take $g$ as $9{\cdot}8\,\text{ms}^{-2}$**

**Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.**

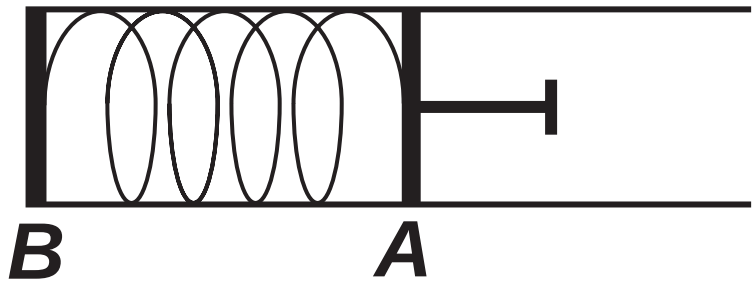
## INFORMATION FOR CANDIDATES

**The number of marks is given in brackets at the end of each question or part-question.**

**You are reminded of the necessity for good English and orderly presentation in your answers.**

1. The diagram shows a piston, of mass $0{\cdot}8\,\text{kg}$, enclosed in a horizontal tube and attached to a light spring of natural length $0{\cdot}2\,\text{m}$ and modulus of elasticity $625\,\text{N}$.

The other end of the spring is fixed to the end of the tube at point $B$




Initially, the piston is held at rest at a point $A$ with the spring compressed a distance of $0{\cdot}1\,\text{m}$, so that $AB$ is the compressed length of the spring.

(a) Calculate the elastic energy stored in the spring. [2 marks]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude $46\,\text{N}$

(b) Determine the velocity of the piston when the spring reaches its natural length. [5 marks]

2. A particle of mass $5\,\text{kg}$ moves under the action of a horizontal force given by

$F = 30t^{-2} - 30\,\text{N}$ at time $t\,\text{s}$, where $t > 0$. It also experiences a constant resistance to motion of magnitude $120\,\text{N}$

(a) Show that the motion of the particle satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t^{-2} - 30$$

where $v\,\text{ms}^{-1}$ is the velocity of the particle at time $t\,\text{s}$ [2 marks]

(b) Calculate the value of $t$ when the acceleration of the particle is $24\,\text{ms}^{-2}$ [2 marks]

2(c) Given that the velocity of the particle is $18\,\text{ms}^{-1}$ when $t = \frac{1}{3}$, find an expression for $v$ in terms of $t$. Hence find the values of $t$ when $v = 10$ [6 marks]

3. A vehicle of mass $4000\,kg$ is travelling up a slope inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha = \frac{2}{49}$. The engine of the vehicle is working at a constant rate of $90\,kW$

(a) Calculate the resistance to the motion of the vehicle at the instant when its speed is $4{\cdot}8\,ms^{-1}$ and its acceleration is $1{\cdot}2\,ms^{-2}$ [6 marks]

(b) Determine the maximum velocity of the vehicle when the resistance to motion has magnitude $12\,800\,N$ [4 marks]

4. At time $t = 0$, an aeroplane $A$ has position vector $(3\underline{i} + 5\underline{j} + 20\underline{k})$ m and is flying with constant velocity $(-\underline{i} + 2\underline{j} + \underline{k})\,\text{ms}^{-1}$

At time $t = 0$, another aeroplane $B$ has position vector $(-2\underline{i} + x\underline{j} + 15\underline{k})$ m, and is flying with constant velocity

$(3\underline{i} - 4\underline{j} + 2\underline{k})\,\text{ms}^{-1}$

(a) Find expressions for the position vector of $A$ and the position vector of $B$ at time $t$ s [3 marks]

(b) Determine an expression for $AB^2$, where $AB$ is the distance between $A$ and $B$ at time $t$ s [4 marks]

(c) Given that the shortest distance between $A$ and $B$ occurs at $t = 5$, calculate the value of $x$ [3 marks]

**5.** A player kicks a ball from a point $A$ on horizontal ground so that $2{\cdot}5$ seconds later the ball just clears a bar at a point $B$. The point $B$ is $3\,\text{m}$ above the ground. The horizontal distance of $B$ from $A$ is $42\,\text{m}$

**(a)** Calculate the horizontal and vertical components of the initial velocity of the ball. **[4 marks]**

**(b)** Find the magnitude of the velocity of the ball and the angle that the direction of the velocity makes with the horizontal as it passes the point $B$ **[6 marks]**

**(c)** Determine the horizontal distance from $B$ to the point where the ball first hits the ground again. **[3 marks]**

6. A particle of mass $3\,\text{kg}$ moves on a horizontal plane. At time $t = 0$, the particle has position vector $-2\underline{i} + 3\underline{j}\,\text{m}$, where $\underline{i}$ and $\underline{j}$ are unit vectors along the $x$-axis and $y$-axis respectively. At time $t\,\text{s}$, the particle moves with velocity $\underline{v}\,\text{ms}^{-1}$ given by

$$\underline{v} = 4\sin 2t\,\underline{i} + 15\cos 5t\,\underline{j}$$

(a) Find the magnitude of the force acting on the particle at time $t = \frac{3\pi}{2}\,\text{s}$ [5 marks]

(b) Determine the position vector of the particle at time $t\,\text{s}$ [4 marks]

(c) Calculate the time and the distance of the particle from the origin when it crosses the $y$-axis for the first time. [4 marks]

7. One end of a light rod of length $l$ metres is freely jointed to a fixed point $O$ and the other end is attached to a particle of mass $m$ kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, $u$ ms$^{-1}$, is a quarter of its speed at the lowest point of the circle.

(a) Show that $u^2 = \frac{4}{15} gl$ [3 marks]

(b) When the rod is inclined at an angle $\theta$ to the DOWNWARD vertical,

(i) find an expression for the tension in the rod in terms of $m$, $g$ and $\theta$

(ii) determine the value of $\theta$ when the tension in the rod becomes zero. [9 marks]

END OF PAPER