



**GCE AS/A level**

**0975/01**

**MATHEMATICS – C3**

**Pure Mathematics**

**P.M. FRIDAY, 6 June 2014**

**1 hour 30 minutes plus your additional time allowance**

## **ADDITIONAL MATERIALS**

**In addition to this examination paper, you will need:**

**a 12 page answer book;  
a Formula Booklet;  
a calculator.**

## **INSTRUCTIONS TO CANDIDATES**

**Use black ink, black ball-point pen or your usual method.**

**Answer ALL questions.**

**Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.**

## **INFORMATION FOR CANDIDATES**

**The number of marks is given in brackets at the end of each question or part-question.**

**You are reminded of the necessity for good English and orderly presentation in your answers.**

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^3 \ln(8 + e^x) dx$$

Show your working and give your answer correct to two decimal places. [4 marks]

- (b) USE YOUR ANSWER TO PART (a) to deduce an approximate value for the integral

$$\int_0^3 \ln(16 + 2e^x) dx$$

[2 marks]

2. Find all values of  $\theta$  in the range

$$0^\circ \leq \theta \leq 360^\circ \text{ satisfying}$$

$$8\tan^2\theta - 5\sec^2\theta = 7 + 4\sec\theta$$

[6 marks]

3. The curve  $C$  is defined by

$$y^4 - 2x^2 + 8xy^2 + 9 = 0$$

- (a) Show that  $\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$  [4 marks]

- (b) Show that there is no point on  $C$  at which  $\frac{dy}{dx} = 0$  [4 marks]

4. Given that

$$x = 2e^t - 5, \quad y = 8e^{-t} + 3e^t - 4,$$

find the value of  $t$  when  $\frac{dy}{dx} = -1$

Give your answer correct to three decimal places.

[7 marks]

5. (a) Show that

$$f(x) = \ln(3x^2 - 2x - 1) - 4x^2$$

has a stationary value when  $X$  satisfies

$$12x^3 - 8x^2 - 7x + 1 = 0$$

[4 marks]

- (b) YOU MAY ASSUME that the equation  
 $12x^3 - 8x^2 - 7x + 1 = 0$   
 has a root  $\alpha$  between  $-1$  and  $0$

The recurrence relation

$$x_{n+1} = \left( \frac{8x_n^2 + 7x_n - 1}{12} \right)^{\frac{1}{3}}$$

with  $x_0 = -0.6$  can be used to find  $\alpha$

Find and record the values of

$x_1, x_2, x_3, x_4$ . Write down the value of  $x_4$  correct to four decimal places and

show this is the value of  $\alpha$  correct to four decimal places.

[5 marks]

6. (a) Differentiate each of the following with respect to  $X$ , simplifying your answer wherever possible.

(i) 
$$\frac{1}{\sqrt[4]{9 - 4x^5}}$$

(ii) 
$$\frac{3 + 2x^3}{7 - x^3}$$

[5 marks]

- (b) (i) Sketch the graph of  $y = \sin^{-1}x$  for values of  $X$  satisfying  $-1 \leq x \leq 1$

- (ii) By first rewriting

$y = \sin^{-1}x$  as  $x = \sin y$ , find an expression for  $\frac{dy}{dx}$  in terms of  $X$ . You

should justify any choice of sign that you

make.

[6 marks]

7. (a) Find each of the following, simplifying your answer wherever possible.

(i)  $\int \cos(2 - 5x) \, dx$

(ii)  $\int \frac{4}{e^{3x-2}} \, dx$

(iii)  $\int \frac{5}{\frac{1}{6}x - 3} \, dx$

[6 marks]

(b) Evaluate  $\int_2^6 \sqrt{4x + 1} \, dx$

[4 marks]

8. (a) Show, by counter-example, that the statement

$$|2a + 3b| \equiv 2|a| + 3|b|$$

is false.

[2 marks]

- (b) Solve the equation

$$|3x - 2| = 7|x|$$

[3 marks]

9. The function  $f$  has domain  $(-\infty, 4)$  and is defined by

$$f(x) = x^2 - 8x + 7$$

- (a) Express  $f(x)$  in the form

$$f(x) = (x + a)^2 + b$$

where  $a, b$  are constants whose values are to be found.

[1 mark]

- (b) Hence or otherwise, find an expression for

$$f^{-1}(x)$$

[4 marks]



10. The functions  $f$  and  $g$  have domains  $[-2, \infty)$  and  $[2, \infty)$  respectively and are defined by

$$f(x) = x^2 + kx - 8$$
$$g(x) = kx - 4$$

where  $k$  is a positive constant.

- (a) Write down, in terms of  $k$ , the range of  $g$  [1 mark]
- (b) (i) Find the least value of  $k$  so that the function  $fg$  can be formed.
- (ii) Write down an expression in terms of  $k$  for  $fg(x)$
- (iii) Given that  $fg(3) = 0$ , find the value of  $k$  [7 marks]

END OF PAPER