GCE AS/A level

0982/01

MATHEMATICS – M3
Mechanics

A.M. FRIDAY, 21 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. At time $t = 0$, a particle of mass 6 kg is projected vertically upwards from a point A with a speed of $24{\cdot}5\,\text{ms}^{-1}$. The resistance acting on the particle has magnitude $3v$ N, where $v\,\text{ms}^{-1}$ is the speed of the particle at time t s.

(a) (i) Show that v satisfies the equation

$$2\frac{\mathrm{d}v}{\mathrm{d}t} = -19{\cdot}6 - v.$$

(ii) Find an expression for v in terms of t. [8]

(b) Determine the time when the particle reaches its maximum height. [2]

(c) Find an expression for x in terms of t, where x m is the distance of the particle from A at time t s. [4]

2. A particle P moves in a straight line with Simple Harmonic Motion about a fixed centre O with period 2 s. At time $t = 0$ s, P is at a point A where $OA = 0{\cdot}5$ m and its velocity is zero.

(a) Write down the amplitude of the motion. [1]

(b) Find the maximum magnitude of the acceleration of P and state the positions of P when this occurs. [4]

(c) Find the smallest positive value of time t for which AP is $0{\cdot}75$ m. [3]

(d) Determine the speed of P when it is $0{\cdot}3$ m from O. [3]

3. *(a)* A particle P, of mass 2 kg, moves along the horizontal x-axis under the action of a force directed towards the origin O. The magnitude of the force is equal to $8x$ N, where x m is the displacement of P from O. The particle is also subjected to a resistive force which is equal to $10v$ N, where $v\,\text{ms}^{-1}$ is the speed of P at time t s. When $t = 0$ s, the particle P is at $x = 2$ m and it is moving away from O with speed $3\,\text{ms}^{-1}$.

(i) Show that the equation of motion of the particle is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4x - 5\frac{\mathrm{d}x}{\mathrm{d}t}.$$

(ii) Find an expression for x in terms of t. [10]

(b) Find the general solution of the second order differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 4x = 12t - 3.$$ [4]

4. Two particles A and B, of masses 5 kg and 3 kg respectively, rest on a smooth horizontal surface. Particle A lies at the edge of the surface and particle B lies a distance of 0·2 m from the edge such that the line AB is perpendicular to the edge of the surface. The two particles are connected by a light inextensible string of length 1·8 m. Particle A is then allowed to drop from rest from the edge of the surface. Calculate the speed of B immediately after the string becomes taut and find the impulsive tension in the string. [9]

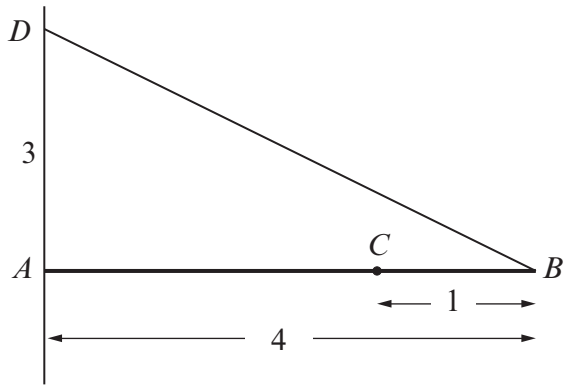
5. A particle P, of mass 0·25 kg, moves away from the origin along the positive x-axis under the action of a force of magnitude $\frac{5}{2x+1}$ N directed away from the origin O. When P is at the origin, its speed is 4 ms^{-1}.

(a) Find an expression for x in terms of the speed v ms^{-1}. [8]

(b) Find the value of x when the speed of P is 6 ms^{-1}. [2]

(c) Find the speed of P when the acceleration of P is 5 ms^{-2}. [4]

6. The diagram shows a uniform rod AB, of mass 6 kg and length 4 m, held in a horizontal position by means of a light inextensible string BD, where D is a point 3 m vertically above A. The end A of the rod rests against a rough vertical wall. A particle of mass 3 kg is attached to the rod at C, where BC = 1 m. The rod is in limiting equilibrium in a vertical plane perpendicular to the wall.



(a) Calculate the tension in the string. [4]

(b) Find the vertical component and the horizontal component of the force exerted by the wall on the rod.
Hence find

(i) the magnitude of the resultant force exerted by the wall on the rod,

(ii) the coefficient of friction between the rod and the wall. [9]

